Schedulability analysis of mixed-criticality systems with multiple frequency specifications

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ABSTRACT

In mixed-criticality systems functionalities of different *criticalities*, that need to have their correctness validated to different levels of assurance, co-exist upon a shared platform. Multiple specifications at differing levels of assurance may be provided for such systems; the specifications that are trusted at very high levels of assurance tend to be more conservative than those at lower levels of assurance. Prior research on the scheduling of such mixed-criticality systems has primarily focused upon the case where multiple estimates of the worst-case execution time (WCET) of pieces of code are provided; in this paper, a model is considered in which multiple estimates are instead provided for the rate at which event-triggered processes are executed. An algorithm is derived for scheduling such systems upon a preemptive uniprocessor; the effectiveness of this algorithm is demonstrated quantitatively via the speedup factor metric.

Keywords

HI and LO criticality; uniprocessor scheduling; Liu & Layland tasks; multiple period estimations; speedup bound

1. INTRODUCTION

Recurrent task models are widely used to model event-triggered phenomena that occur repeatedly: each event gives rise to a *job* that needs to be executed. If these triggering events occur arbitrarily frequently then it is not possible to offer deterministic guarantees as to when each job will receive execution. Often, however, lower bounds can be estimated beforehand on the minimum amount of time that must elapse between successive occurrences of these events, and performance guarantees made under the assumption that these bounds are correct. Some commonly-used recurrent task models include the *Liu & Layland (LL)* model [10] and the *3-parameter sporadic tasks* model [11]. In this paper, we will primarily focus upon the Liu & Layland (henceforth called the LL) model. Each task τ_i in the LL model

is characterized by two parameters: a *worst-case execution time* (WCET) C_i , and a *minimum inter-arrival separation* T_i (often simply called the *period* of the task). Such a task generates a potentially infinite sequence of jobs at run-time, with successive job-arrivals assumed to be separated by at least T_i time units and each job needing to execute for at most C_i time units by a deadline that occurs T_i time units after its arrival time.

Mixed-criticality scheduling theory arose out of a recognition of the reality that for many systems the true values of the parameters characterizing a task (*i*) are very difficult to determine precisely, and (*ii*) may be extremely conservative in the sense that they are far more pessimistic than the typical or average value. (For instance, advanced architectural features found in modern processors, such as memory hierarchies, deep pipelining, speculative out-of-order execution, etc., often result in the worst-case execution time of a task upon such a processor being multiple orders of magnitude greater than the execution time experienced by the overwhelmingly vast majority of jobs of the task.) Under mixed-criticality scheduling, one may choose to specify multiple *estimates* of a single parameter of a task, the different estimates being made at different levels of assurance. Each task is assigned a *criticality*, and the performance guarantee is that all the tasks will execute correctly if the least conservative parameter estimates are correct, while less critical tasks are not required to execute correctly if the less conservative parameter estimates turn out to be incorrect — see Example 1 for an illustration.

Much prior research on mixed-criticality scheduling (see, e.g., the survey [8] by Burns & Davis) has focused upon models in which each task is characterized by multiple WCET parameters, the different parameters representing estimates, at different levels of assurance, of the actual WCET of the task. It has also been observed [4, 8] that it sometimes makes sense to specify multiple period parameters for tasks that are modeling event-triggered phenomena, since the precise pattern of triggering events is typically unknown beforehand and the period parameter denotes an *estimate* of the minimum duration between successive triggering events. A few papers [1, 7] have indeed considered this possibility; in this research we provide a comprehensive analysis of this aspect of mixed criticalities. That is, we primarily focus our attention on just the period parameter, and analyze situations in which *multiple estimates are provided for the period parameter of each task, the different estimates being made at different levels of assurance*. We illustrate via an example:

EXAMPLE 1. Consider a system in which two indepen-

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EMSOFT'16, October 01-07, 2016, Pittsburgh, PA, USA

© 2016 ACM. ISBN 978-1-4503-4485-2/16/10...\$15.00

DOI: <http://dx.doi.org/10.1145/2968478.2968488>

dent pieces of event-triggered code are to be implemented upon a shared preemptive processor. The system developer models these pieces of code as LL tasks τ_1 and τ_2 , and determines that these tasks have WCET parameters of two and five respectively (i.e., $C_1 = 2$ and $C_2 = 5$). The system developer further estimates that successive triggerings of τ_1 will be at least twenty time units apart, and successive triggerings of τ_2 will be at least six time units apart. From the system developer’s perspective, therefore, $T_1 = 20$ and $T_2 = 6$ and this system thus has a *utilization* [10] of $\frac{2}{20} + \frac{5}{6} \approx 0.93$. The system developer consequently concludes, by the well-known utilization-based Earliest Deadline First (EDF) schedulability test [10] that the system would meet all deadlines if scheduled using EDF.

Now, let us suppose that task τ_1 is responsible for some highly safety-critical function, and is therefore subject to certification by a Certification Authority (CA) — τ_2 is not subject to such certification. But the CA makes more conservative assumptions than the system developer does and assumes that successive events triggering τ_1 may occur as frequently as ten time units apart.¹ The CA hence requires that the safety-critical task τ_1 be modeled as having a period of 10, rather than 20. The utilization of the system, from the CA’s perspective, is $\frac{2}{10} + \frac{5}{6} \approx 1.03$, and EDF cannot guarantee to meet all deadlines (indeed, since the utilization is > 1 no algorithm can guarantee to do so). An EDF-scheduled implementation will therefore not pass certification by the CA.

An alternative implementation would assign higher scheduling priority to τ_1 — this would pass certification since each job of τ_1 is now guaranteed execution immediately upon arrival. But consider a situation in which one job of each task arrives simultaneously: τ_1 ’s job would execute first for two time units, thereby denying τ_2 ’s job an opportunity to execute for five time units by its deadline regardless of when the next job of τ_1 may arrive. Thus, although this system design is accepted by the CA, deadlines are missed even when the system behaves as expected by the system developer (i.e., as a system with effective utilization ≈ 0.93). A preferred scheduling policy would be one that has all the jobs meet their deadlines if the system developer’s assumptions are correct, and all jobs of τ_1 meet their deadlines (while those of τ_2 need not) if the system developer’s assumptions are incorrect but the CA’s assumptions are correct. We will derive such a scheduling policy in this paper.

(We point out here that in order to pass certification, the system developer must ensure that each of τ_1 ’s jobs completes within a deadline that is 10 time units after its arrival (since the jobs are assumed to have deadlines after a duration of one period after their arrival time), even if the jobs do arrive 20 time units apart. Hence from the system developer’s perspective the task is better modeled as a 3-parameter sporadic task [11] — this task model is described in Section 2.1 — with WCET 2, relative deadline 10, and period 20.) $\square$

¹The CA may also make a more conservative estimate of the WCET C_1 than the system developer does; we will assume in this paper that both the system developer and the CA make the same estimate of all WCETs, thus allowing us to focus on variations in the period estimates. Sec. 6 discusses extensions that enable the concurrent consideration of uncertainties along both the WCET and period dimensions.

This research. Most prior research (see [8] for a survey) on mixed-criticality scheduling has looked at systems specified with multiple WCET parameters. The research described in this document is, by contrast, focused on dealing with pessimism in the period perimeter; this work thus complements the large body of prior work dealing with pessimism along the WCET dimension. For MC systems in which certification pessimism is expressed along the period dimension we present, prove the correctness of, and provide a quantitative metric of efficacy for, an algorithm for scheduling such systems upon a preemptive uniprocessor platform. As is fairly routine in the mixed-criticality scheduling theory literature, we primarily restrict our attention here to systems in which two criticality levels, designated LO and HI, are defined; we briefly address, in Section 6, possible extension to more levels.

Related work. We will not review here the vast body of research that exists concerning the mixed-criticality scheduling of task systems with multiple WCET specifications — see [8] for a comprehensive survey. Preemptive uniprocessor EDF-based scheduling of task systems specified with pessimism expressed along the frequency dimension — the subject of this paper — was first studied in [1], and subsequently extended to fixed-priority scheduling in [5]. Burns and Davis [7] considered the scheduling of streams of periodic real-time traffic on controller-area networks (CANs), with the traffic streams being specified by multiple period parameters at different levels of pessimism.

The research described in this manuscript may be considered to be an extension (and improvement) of the results reported in [1]. The scheduling algorithm that was presented in [1] was proved correct there, but no quantitative metrics of its efficacy were derived. In this paper, we derive and prove correct a scheduling algorithm that strictly dominates the one in [1] — this is formally shown in Section 5.2. We have additionally derived, in Sections 5.3–5.4, a quantitative metric of its effectiveness by showing that this algorithm has a speedup bound of $(1 + \sqrt{3}/2)$ or ≈ 1.866 .

Organization. The mixed-criticality model studied in this paper is formally defined in Section 2; some additional definitions that will prove useful later in this paper are also presented here. The main technical contribution in this paper — an algorithm for scheduling mixed-criticality task systems upon preemptive uniprocessor platforms — is presented in Section 3; a proof of its correctness is derived in Section 4. Section 5 characterizes the effectiveness of this algorithm by both showing that it strictly dominates the prior state of the art, and by deriving a speedup bound for it.

2. MODEL & DEFINITIONS

We start out formally defining the task model that we will be using. We characterize a *mixed-criticality Liu & Layland (MC LL) task* by a 4-tuple of parameters:

$$\tau_i = (\chi_i, C_i, T_i(\text{HI}), T_i(\text{LO})),$$

where

- $\chi_i \in \{\text{LO}, \text{HI}\}$ denotes the criticality of the task (with HI being “more” critical than LO);
- C_i denotes the WCET of the task;

- $T_i(\text{HI})$ denotes a conservative estimate of the minimum amount of time that will elapse between the arrival times of successive jobs of τ_i ; and
- $T_i(\text{LO})$ denotes a less conservative (equivalently, more optimistic) estimate of the minimum amount of time that will elapse between the arrival times of successive jobs of τ_i .

We make the assumption that $T_i(\text{HI}) \leq T_i(\text{LO})$ for all tasks τ_i . We also assume that for each LO-criticality task τ_i , $T_i(\text{HI}) = T_i(\text{LO})$; i.e., we will not admit jobs of such tasks more frequently than in accordance with their LO-criticality periods.

A MC LL task system τ is a finite collection of independent MC tasks: $\tau = \{\tau_1, \tau_2, \dots, \tau_n\}$. We study the problem of scheduling such MC LL task systems on a fully preemptive uniprocessor.

In this notation, the task system of Example 1 is specified as follows:

$$\tau = \{\tau_1 = (\text{HI}, 2, 10, 20), \tau_2 = (\text{LO}, 5, 6, 6)\} \quad (1)$$

Behaviors. Any execution of the system satisfying the property that for each task τ_i , successive jobs of τ_i arrive at least $T_i(\text{LO})$ time units apart, is said to exhibit a *LO-criticality behavior*. Any execution of the system that does not exhibit LO-criticality behavior but which satisfies the property that for each task τ_i , successive jobs of τ_i arrive at least $T_i(\text{HI})$ time units apart is said to exhibit a *HI-criticality behavior*. Any execution that does not exhibit HI-criticality or LO-criticality behavior is said to be *erroneous*.

Correctness. An algorithm for scheduling MC LL task systems is said to be *correct* if and only if it satisfies the property that

- Each job of each task τ_i completes execution within $T_i(\text{HI})$ units of arrival, in all LO-criticality behaviors of the system²; and
- Each job of each HI-criticality task τ_i (i.e., all τ_i with $\chi_i = \text{HI}$) completes execution within $T_i(\text{HI})$ units of arrival, in all HI-criticality behaviors of the system.

2.1 The 3-parameter sporadic task model

The correctness requirement of MC LL tasks in LO-criticality behaviors is conveniently expressed using the 3-parameter sporadic task model [11], which generalizes the Liu & Layland (LL) model. A 3-parameter sporadic task $\tau_i = (C_i, D_i, T_i)$ is characterized by three parameters: a WCET C_i and a period T_i as in the LL model, plus an additional *relative deadline* parameter D_i . Such a task generates an unbounded sequence of jobs, with each job having an execution requirement $\leq C_i$ and successive arrivals at least T_i time units apart. However, each job is required to complete by a deadline that is D_i (rather than T_i , as in the LL model) time units after its arrival time.

The EDF-schedulability-analysis of systems of 3-parameter sporadic tasks on preemptive uniprocessors has been extensively studied (see, e.g. [10, 6]); such study has yielded exact

²Note that the deadlines for the jobs are $T_i(\text{HI})$ time-units after arrival and not $T_i(\text{LO})$; this is a consequence of the fact that jobs of LL tasks are expected to complete execution within one period of their arrival and the period, conservatively estimated, is $T_i(\text{HI})$. (For LO-criticality tasks τ_i we have $T_i(\text{LO}) = T_i(\text{HI})$.)

preemptive uniprocessor EDF-schedulability analysis algorithms for 3-parameter sporadic task systems. These algorithms make use of the concept of the *demand bound function* [6] of 3-parameter sporadic tasks; the demand bound function $\text{DBF}(\tau_i, t)$ of 3-parameter sporadic task τ_i for an interval-length t is defined as follows:

$$\text{DBF}(\tau_i, t) \stackrel{\text{def}}{=} \max \left(0, \left(\left\lfloor \frac{t - D_i}{T_i} \right\rfloor + 1 \right) \times C_i \right) \quad (2)$$

The *load* of a 3-parameter sporadic task system τ is defined as follows:

$$\text{LOAD}(\tau) \stackrel{\text{def}}{=} \max_{t \geq 0} \frac{\sum_{\tau_i \in \tau} \text{DBF}(\tau_i, t)}{t} \quad (3)$$

It has been shown [6] that a necessary and sufficient condition for 3-parameter sporadic task system τ to be EDF-schedulable on a unit-speed preemptive uniprocessor is that

$$\text{LOAD}(\tau) \leq 1 \quad (4)$$

Pseudo-polynomial algorithms are known [6, 13, 14] for computing $\text{LOAD}(\tau)$, for task systems τ possessing the additional property that the quantity $(\sum_{\tau_i \in \tau} C_i / T_i)$ is bounded from above by a constant < 1 . Polynomial-time approximation schemes (PTAS's) have also been derived that are able to compute an approximation to $\text{LOAD}(\tau)$ in polynomial time, to any desired degree of accuracy [9].

The following definition specifies a transformation from a mixed-criticality LL task system to a 3-parameter sporadic task system; we will be using this transformation in later sections of this paper.

DEFINITION 1. Let x denote a positive real number, $0 < x \leq 1$. For any mixed-criticality LL task τ_i , we can define a corresponding 3-parameter sporadic task $\hat{\tau}_i(x)$ as follows :

$$\hat{\tau}_i(x) = (\hat{C}_i, \hat{D}_i, \hat{T}_i) \stackrel{\text{def}}{=} \begin{cases} (C_i, T_i(\text{HI}), T_i(\text{LO})), & \text{if } \chi_i = \text{LO} \\ (C_i, xT_i(\text{HI}), T_i(\text{LO})), & \text{if } \chi_i = \text{HI} \end{cases} \quad (5)$$

For MC LL task system τ , let $\hat{\tau}(x)$ denote the 3-parameter sporadic task system obtained from τ by applying the $\hat{\tau}_i(x)$ transformation to each task in τ :

$$\hat{\tau}(x) \stackrel{\text{def}}{=} \{\hat{\tau}_1(x), \hat{\tau}_2(x), \dots, \hat{\tau}_n(x)\} \quad (6)$$

□

In words, $\hat{\tau}_i(x) = (\hat{C}_i, \hat{D}_i, \hat{T}_i)$ is a 3-parameter sporadic task that has the same WCET as τ_i ($\hat{C}_i = C_i$), and the same period as the LO-criticality period of τ_i ($\hat{T}_i = T_i(\text{LO})$). If τ_i is of LO criticality, then $\hat{\tau}_i$'s relative deadline is equal to its period ($\hat{D}_i = T_i(\text{LO})$, which equals $T_i(\text{LO})$). However, if τ_i is a HI-criticality task, then $\hat{\tau}_i$ is assigned a relative deadline parameter that is the HI-criticality period parameter of τ_i scaled by the factor x (i.e., $\hat{D}_i = xT_i(\text{HI}) \leq T_i(\text{HI})$). In this notation, letting τ denote the task system in Expression (1), we would have

$$\begin{aligned} \hat{\tau}(1) &= \{\hat{\tau}_1(1) = (\text{HI}, 2, 10, 20), \hat{\tau}_2(1) = (\text{LO}, 5, 6, 6)\} \\ \hat{\tau}(0.5) &= \{\hat{\tau}_1(0.5) = (\text{HI}, 2, 5, 20), \hat{\tau}_2(0.5) = (\text{LO}, 5, 6, 6)\} \end{aligned}$$

Using this notation, the correctness requirement for any mixed-criticality LL task τ_i in a LO-criticality behavior may be modeled by the 3-parameter sporadic task $\hat{\tau}_i(1)$.

3. THE ALGORITHM

Let $\tau = \{\tau_1, \tau_2, \dots, \tau_n\}$ denote the MC LL task system that is to be scheduled on a unit-speed preemptive processor. Suppose that at some time-instant t^* a job of some task τ_j arrives less than $T_j(\text{LO})$ time units after the arrival of the previous job of τ_j . The behavior is no longer compliant with LO-criticality specifications, and we assert that *a criticality level change has occurred at time-instant t^** . Henceforth we are not required to demonstrate correct execution of LO-criticality tasks and can discard all LO-criticality jobs. Informally, we would like to ensure for each HI-criticality task τ_i that is active – i.e., has arrived but has not completed execution – at time t^* that there is enough capacity available on the processor to ensure that τ_i 's currently-active job completes execution by its deadline, provided only HI-criticality jobs execute henceforth. To achieve this, we will ensure that as long as the system is behaving according to its LO-criticality specifications, τ_i 's job would have completed well before its actual deadline. We will thus ensure that if it is active at time-instant t^* there is sufficient computing capacity freed up by the discarded LO-criticality tasks to schedule τ_i 's job to completion by its actual deadline.

Our scheduling algorithm for achieving this can be thought of operating in three phases. The **first** phase is occurs prior to run-time: a schedulability test is applied to the system and if successful, a *scaling parameter* x is computed. This readies the system for the **second**, run-time phase of the algorithm. During this second phase, jobs are initially dispatched under the assumption that the system will behave according to the LO-criticality specifications. If this is violated, a criticality level change occurs and the algorithm enters a **third** phase.

3.1 The pre-runtime phase

We seek to determine, prior to run-time, whether a given mixed-criticality LL task system τ can be successfully scheduled by our algorithm; if so, we compute a scaling parameter x by which we will initially shrink the deadlines of all HI-criticality jobs in order to ensure that they all complete well before their actual deadlines.

We first introduce some notation:

- The term $U_{\text{LO}}^{\text{LO}}$ ($U_{\text{HI}}^{\text{HI}}$, respectively) denotes the cumulative utilization of the LO-criticality (HI-criticality, resp.) tasks assuming the LO-criticality (HI-criticality, resp.) period estimates:

$$U_{\text{LO}}^{\text{LO}} \stackrel{\text{def}}{=} \sum_{\tau_i \in \tau \wedge \chi_i = \text{LO}} \frac{C_i}{T_i(\text{LO})}$$

$$U_{\text{HI}}^{\text{HI}} \stackrel{\text{def}}{=} \sum_{\tau_i \in \tau \wedge \chi_i = \text{HI}} \frac{C_i}{T_i(\text{HI})}$$

- The term ℓ_{LO} denotes the *load* of the system (see Expression (3)) assuming that the LO-criticality period estimates are correct, and each HI-criticality job must complete within an interval of duration equal to the HI-criticality period estimate of the task that generated it:

$$\ell_{\text{LO}} \stackrel{\text{def}}{=} \text{LOAD}(\hat{\tau}(1))$$

(Recall that, as stated in both Footnote 2 and after Definition 1, the correctness requirement for any

mixed-criticality LL task τ_i in a LO-criticality behavior is modeled as the 3-parameter sporadic task $\hat{\tau}_i(1)$. Hence ℓ_{LO} represents the maximum load that may be generated by τ in any LO-criticality behavior.)

The pre-processing algorithm is depicted in pseudo-code form in Algorithm 1; in Section 4 we will show that if Algorithm 1 returns success for a given task system τ , then τ is scheduled correctly by the run-time dispatcher described in Section 3.2 below. As can be seen from the pseudo-code in Algorithm 1, $U_{\text{LO}}^{\text{LO}}$ and $U_{\text{HI}}^{\text{HI}}$ are first computed; if they sum to at most one (this is tested in Line C1), then x is assigned the value one. Else, ℓ_{LO} is computed and an additional condition checked (Line C2); if this condition evaluates to true, then x is assigned a value specified in Line L2. If the condition in Line C2 evaluates to false, a form of search is conducted for a suitable value of x — the test of Line C3 determines whether this search has yielded success (line L4) or failure (Line L5). This pre-processing phase is deemed successful if and only if the pre-processing algorithm returns success, in which case the computed value of x is used as the scaling factor for HI-criticality deadlines in the subsequent, run-time phase.

3.2 Run-time dispatching

Dispatching of jobs during run-time occurs according to the following rules:

1. There is a *criticality level indicator* Γ , initialized to LO.
2. While ($\Gamma \equiv \text{LO}$), (this corresponds to the **second** phase of our algorithm as described in the overview.)
 - (a) If a job of task $\tau_i \in \tau$ arrives at time t , it is assigned a scheduling deadline as follows

$$\begin{cases} t + T_i(\text{LO}) & \text{if } \chi_i = \text{LO} \\ t + x \times T_i(\text{HI}) & \text{if } \chi_i = \text{HI}. \end{cases}$$

That is, each LO-criticality job is assigned a deadline equal to its arrival time plus its period, while each HI-criticality job is assigned a deadline equal to its arrival time plus its HI-criticality period scaled by the scaling factor x .

- (b) At each instant the earliest-deadline waiting job is selected for execution (ties broken arbitrarily).
 - (c) If a job of some task τ_i arrives earlier than $T_i(\text{LO})$ time units after the arrival of the previous job of the same task, then $\Gamma \leftarrow \text{HI}$.
3. Once ($\Gamma \equiv \text{HI}$) — this corresponds to the **third** phase of our algorithm, which may or may not execute during any given run of the system
 - (a) All currently-active LO-criticality jobs are immediately discarded. The deadline of each currently active HI-criticality job is changed to become equal to its arrival time plus the HI-criticality period (the $T_i(\text{HI})$) of the task that generated it). That is, if a job of τ_i that was released at some time t is active, its deadline, for scheduling purposes, is henceforth $t + T_i(\text{HI})$.
 - (b) If a job of τ_i arrives at some time t , it is assigned a deadline equal to $t + T_i(\text{HI})$ if $\chi_i = \text{HI}$, and discarded if $\chi_i = \text{LO}$.

Algorithm 1: The preprocessing algorithm

Input: Mixed-criticality Liu & Layland task system τ **Output:** A scaling factor x ; *success* or *failure***begin**

```
C1  compute  $U_{LO}^{LO}$  and  $U_{HI}^{HI}$ 
L1  if  $(U_{LO}^{LO} + U_{HI}^{HI} \leq 1)$  then
    |  $x \leftarrow 1$ ; return success                                /* See Section 4.1 */
    end
    else
        compute  $\ell_{LO}$                                           /* Must be  $\leq 1$ ; else trivially infeasible */
C2  if  $U_{HI}^{HI} \leq 4(1 - \ell_{LO})/(4 - \ell_{LO})$  then
L2  |  $x \leftarrow 1 - U_{HI}^{HI}/2$ ; return success                    /* See Section 4.2 */
    end
    else
L3  |  $x \leftarrow$  the smallest value of  $v$  for which the 3-parameter sporadic task system  $\hat{\tau}(v)$  is EDF-schedulable
C3  | if (the 3-parameter sporadic task system  $\bigcup_{\chi_i=HI} \{\hat{\tau}_i(1-x)\}$  is EDF-schedulable) then
L4  | | return success                                          /* See Section 4.3 */
    | end
    | else
L5  | | return failure
    | end
    end
end
end
end
```

(c) At each instant the earliest-deadline waiting job is selected for execution (ties broken arbitrarily).

4. An additional rule could specify the circumstances when Γ gets reset to LO. This could happen, for instance, if no HI-criticality jobs are active at some instant in time. (We will not discuss the process of resetting $\Gamma \leftarrow LO$ any further in this document.)

4. PROOF OF CORRECTNESS

We now show that if Algorithm 1 declares *success* for a given task system τ , then τ is scheduled correctly by the run-time dispatcher of Section 3.2. Observe that Algorithm 1 may declare *success* at three places: Lines L1, L2, and L4; we consider each case separately, in Sections 4.1–4.3.

4.1 Returns success at Line L1

This is the simplest case; informally, it corresponds to the situation where the task system is successfully scheduled using an isolation-based approach. (That is, the regular (i.e., non-MC) LL task system obtained when each task is modeled as a regular LL task with period equal to its period estimate at its own criticality level is determined to be schedulable using EDF.)

LEMMA 1. *If the algorithm returns success at Line L1 for a given task system τ , then τ is scheduled correctly by the run-time dispatcher described in Section 3.2.*

Proof: By setting the scaling factor x equal to one, we are in effect *over-approximating* the computational demand of each HI-criticality task $\tau_i = (HI, C_i, T_i(HI), T_i(LO))$ as a regular LL task with WCET C_i and period $T_i(HI)$, and representing each LO-criticality task $\tau_i = (HI, C_i, T_i(HI), T_i(LO))$ as a regular LL task with WCET C_i and period $T_i(LO)$. The

utilization of the resulting regular LL task system is

$$\begin{aligned} \sum_{i:\chi_i=LO} \frac{C_i}{T_i(LO)} + \sum_{i:\chi_i=HI} \frac{C_i}{T_i(HI)} \\ = (U_{LO}^{LO} + U_{HI}^{HI}) \leq 1 \end{aligned}$$

This task system is therefore schedulable by EDF to always meet all deadlines. It follows that all deadlines of all tasks are met under both LO-criticality and HI-criticality behaviors when scheduled using EDF. $\square$

4.2 Returns success at Line L2

This case turns out to be the most challenging to analyze. We proceed in two steps. The first step – proving that no deadlines are missed in any LO-criticality behavior – is fairly straightforward; this is shown in Lemma 2. We then adapt and apply some recently-introduced techniques and ideas from [3] to prove, in Lemma 3, that no deadlines are missed in any HI-criticality behavior as well.

FACT 1. *If the conditional expression of Line C2 evaluates to true, then*

$$U_{HI}^{HI} \leq 2(1 - \ell_{LO}) \tag{7}$$

Proof: Suppose that the conditional expression of Line C2 evaluates to true; i.e., we have

$$U_{HI}^{HI} \leq \frac{4(1 - \ell_{LO})}{4 - \ell_{LO}}$$

We will show that Inequality 7 must also hold by showing below that

$$\frac{4(1 - \ell_{LO})}{4 - \ell_{LO}} \leq 2(1 - \ell_{LO}).$$

Observe that

$$\begin{aligned} & 2(1 - \ell_{LO}) - \frac{4(1 - \ell_{LO})}{4 - \ell_{LO}} \\ &= 2(1 - \ell_{LO}) \left(1 - \frac{2}{4 - \ell_{LO}}\right) \\ &= 2(1 - \ell_{LO}) \left(\frac{2 - \ell_{LO}}{4 - \ell_{LO}}\right) \end{aligned}$$

which is ≥ 0 for all $\ell_{LO} \in [0, 1]$. We have thus shown that

$$2(1 - \ell_{LO}) - \frac{4(1 - \ell_{LO})}{4 - \ell_{LO}} \geq 0,$$

from which it follows that

$$2(1 - \ell_{LO}) \geq \frac{4(1 - \ell_{LO})}{4 - \ell_{LO}}$$

□

LEMMA 2. *If the algorithm returns success at Line L2, then no deadline is missed by the run-time dispatcher described in Section 3.2 upon any LO-criticality behavior.*

Proof: We saw in Fact 1 above that $U_{HI}^{HI} \leq 2(1 - \ell_{LO})$ if Line L2 is executed; since x gets assigned the value $\left(1 - (U_{HI}^{HI}/2)\right)$ in Line L2, this value of x is guaranteed to be $\geq \ell_{LO}$.

Consider now any LO-criticality behavior of τ . For any time interval $[t_1, t_2]$, let $\Delta(t_1, t_2)$ ($\Delta'(t_1, t_2)$, respectively) denote the sum of the WCETs of jobs that have both arrival times and deadlines (*scaled* deadlines, respectively)³ within this interval in this behavior. Consider a particular time-interval $[t_o, t_o + t]$. Let J_k denote any job that contributes to $\Delta'(t_o, t_o + t)$, and let a_k, d_k denote the arrival time and relative deadline (prior to scaling) of J_k . We have

$$\begin{aligned} a_k + xd_k &\leq t_o + t \\ \Leftrightarrow xd_k &\leq t - (a_k - t_o) \\ \Leftrightarrow d_k &\leq \frac{t}{x} - \frac{(a_k - t_o)}{x} \\ \Rightarrow d_k &\leq \frac{t}{x} - (a_k - t_o) \quad (\text{Since } x \leq 1) \\ \Rightarrow a_k + d_k &\leq \frac{t}{x} + t_o \end{aligned}$$

That is, any job contributing to $\Delta'(t_o, t_o + t)$ also contributes to $\Delta(t_o, t_o + \frac{t}{x})$; we therefore have

$$\begin{aligned} \Delta'(t_o, t_o + t) &= \Delta(t_o, t_o + \frac{t}{x}) \\ &\leq \frac{t}{x} \times \ell_{LO} \\ &\leq t \quad (\text{Since } x \geq \ell_{LO}) \end{aligned}$$

We have thus shown that for any time interval $[t_o, t_o + t]$, the demand $\Delta'(t_o, t_o + t)$ is no larger than the duration of the interval. Since the run-time dispatcher described in Section 3.2 effectively schedules any LO-criticality behavior using EDF with the scaled deadlines, it follows from the optimality of EDF that no deadlines will be missed in any LO-criticality behavior. □

By Lemma 2 above, no deadlines are missed in any LO-criticality behavior. Let us now derive conditions that must

³Recall that a job of HI-criticality task τ_i arriving at time-instant t_o has its deadline at $(t_o + T_i)$ and scaled deadline at $(t_o + x T_i)$.

necessarily be satisfied by the task system τ in order for a deadline miss to occur in some HI-criticality behavior.

Let I denote a minimal⁴ collection of jobs generated by τ for which a deadline miss occurs, and let t_f denote the earliest instant at which this happens — see Figure 1. It follows from Lemma 2 that the algorithm has set the criticality level indicator Γ to HI prior to time-instant t_f , and that the job to miss its deadline at t_f is a HI-criticality job. Let us denote the time-instant at which Γ was first assigned the value HI as t^* .

FACT 2. *All jobs receiving any execution in the interval $[t^*, t_f]$ must have (absolute) deadlines no larger than t_f .*

Proof: Suppose there is a job that has deadline larger than t_f and receives some execution between t^* and t_f , say in the interval $[t_1, t_2]$. This means that during $[t_1, t_2]$ there are no active jobs awaiting execution that have deadline at most t_f . Hence the collection of jobs obtained by considering only jobs with arrival time at least t_2 will also miss a deadline at t_f , which contradicts the assumed minimality of I . □

Let S^* denote the collection of jobs in I that receive execution after t^* . Let J_0 denote the job that has the earliest arrival time in S^* . Let a_0 and d_0 denote its arrival time and absolute deadline, respectively; let $\tilde{d}_0 \stackrel{\text{def}}{=} a_0 + x(d_0 - a_0)$ denote its scaled deadline.

FACT 3. *Any LO-criticality job in I must have its absolute deadline $\leq a_0 + x(t_f - a_0)$.*

Proof: Note that no LO-criticality job with absolute deadline $> \tilde{d}_0$ will execute at or after a_0 — since such a job will only receive execution time before t^* , this would imply that J_0 has completed execution by time-instant t^* , hereby contradicting the definition of J_0 .

Suppose there is a LO-criticality job with absolute deadline $> \tilde{d}_0$ that executes prior to time-instant a_0 ; consider the latest time-instant during which it executes. It must be the case that there are no active jobs with deadline at most $\tilde{d}_0$ awaiting execution at this time-instant, and the job sequence obtained by only considering jobs released after this time-instant also misses a deadline. That contradicts the assumed minimality of I . Therefore any LO-criticality job in I must have an absolute deadline $\leq d_0 = a_0 + x(d_0 - a_0) \leq a_0 + x(t_f - a_0)$, applying the result of Fact 2 to conclude that $d_0 \leq t_f$. □

FACT 4. *Any HI-criticality job in I that only receives execution prior to time-instant t^* must have its absolute scaled deadline $\leq a_0 + x(t_f - a_0)$.*

Proof: If a HI-criticality job $J_k \in I$ with arrival time a_k , absolute deadline d_k , and scaled absolute deadline $\tilde{d}_k$ does not receive execution after t^* , there are two possibilities:

1. $a_k \geq a_0$. In this case $\tilde{d}_k \leq \tilde{d}_0$, since otherwise J_k would not receive any execution after a_0 , implying that it does not receive any execution before t^* , which

⁴By *minimal*, we mean that some deadline is missed when I is scheduled, but no deadline miss would occur if any proper subset of I were being scheduled.

S^* : jobs executing over $[t^*, t_f]$.
 S_1 : LO-criticality jobs executing prior to t^* .
 S_2 : HI-criticality jobs executing prior to t^* .
 C^*, C_1, C_2 : cumulative execution requirements of jobs in S^*, S_1, S_2 .

Figure 1: Notation. S^* : jobs executing over $[t^*, t_f]$. S_1 : LO-criticality jobs executing prior to t^* . S_2 : HI-criticality jobs executing prior to t^* .

combined with J_k not receiving any execution after t^* , would mean that the instance I without J_k would also see a deadline miss of J at t_f (thus contradicting the assumed minimality of I).

2. $a_k < a_0$. If $d_k \leq t_f$, then $\tilde{d}_k = a_k + x(d_k - a_k) \leq a_0 + x(t_f - a_0)$. Otherwise, if $d_k > t_f$, then again $\tilde{d}_k \leq \tilde{d}_0$ must hold, since otherwise we can remove J_k from I without affecting the deadline miss of J , thus contradicting the minimality of I .

The proof is completed by noting that $\tilde{d}_k \leq \tilde{d}_0 = a_0 + x(d_0 - a_0) \leq a_0 + x(t_f - a_0)$.

Observe that $a_k + xd_k = \tilde{d}_k \leq a_0 + x(t_f - a_0)$ implies that $d_k \leq a_0/x + (t_f - a_0)$ — this may be formally shown in a manner analogous to the derivation in the proof of Lemma 2. $\square$

Let us define S_1 to be the collection of LO-criticality jobs of I , and S_2 to be the collection of HI-criticality jobs of I that only receive execution before t^* . Therefore, the collection of jobs I is the disjoint union of S_1, S_2 and S^* .

Let C_1, C_2 , and C^* denote respectively the sum of execution requirements of all the jobs in S_1, S_2 , and S^* . Let $\delta \stackrel{\text{def}}{=} t_f - a_0$, the length of the time interval in which there are pending jobs from S^* .

Facts 3 and 4 yield the following two inequalities:

$$\begin{cases} C_1 & \leq \ell_{\text{LO}} \times (a_0 + x\delta) \\ C_1 + C_2 & \leq \ell_{\text{LO}} \times (a_0/x + \delta). \end{cases} \quad (8)$$

By definition of $U_{\text{HI}}^{\text{HI}}$, we also obtain another set of inequalities

$$\begin{cases} C^* & \leq U_{\text{HI}}^{\text{HI}} \delta \\ C^* + C_2 & \leq U_{\text{HI}}^{\text{HI}} \times (a_0/x + \delta). \end{cases} \quad (9)$$

Since we have assumed that J misses its deadline $t_f = a_0 + \delta$, it must be the case that $C_1 + C_2 + C^* > t_f$, which is equivalent to $C_1 + C_2 + C^* - a_0 > \delta$. Hence the maximum value of $C_1 + C_2 + C^* - a_0$ over all possible a_0 must be greater than δ . We will now attempt to derive conditions necessary for this to be the case. For notational convenience, let us henceforth refer to a_0 as α , and study the function

$$o(\alpha) \stackrel{\text{def}}{=} C_1 + C_2 + C^* - \alpha$$

From inequalities (8) and (9), we get the following two upper

bounds, $f(\alpha)$ and $g(\alpha)$, on the function $o(\alpha)$:

$$f(\alpha) \stackrel{\text{def}}{=} \ell_{\text{LO}} \times (\alpha + x\delta) + U_{\text{HI}}^{\text{HI}} \times (\alpha/x + \delta) - \alpha \quad (10)$$

$$g(\alpha) \stackrel{\text{def}}{=} \ell_{\text{LO}} \times \left(\frac{\alpha}{x} + \delta\right) + U_{\text{HI}}^{\text{HI}} \times \delta - \alpha. \quad (11)$$

As observed at the beginning of the proof of Lemma 2, the value assigned to x at Line L2 of our algorithm is $\geq \ell_{\text{LO}}$, which in turn implies that $\ell_{\text{LO}}/x \leq 1$. Using this fact in conjunction with the observation that

$$(\ell_{\text{LO}} + U_{\text{HI}}^{\text{HI}}) \geq (U_{\text{LO}}^{\text{LO}} + U_{\text{HI}}^{\text{HI}}) > 1$$

(since the condition of Line C1 had evaluated to false), we obtain the following bounds on the derivatives of f and g

$$\begin{aligned} \frac{\partial f(\alpha)}{\partial \alpha} &= \ell_{\text{LO}} + U_{\text{HI}}^{\text{HI}}/x - 1 \geq \ell_{\text{LO}} + U_{\text{HI}}^{\text{HI}} - 1 > 0 \\ \frac{\partial g(\alpha)}{\partial \alpha} &= \ell_{\text{LO}}/x - 1 \leq 1 - 1 = 0, \end{aligned}$$

Hence $f(\alpha)$ increases and $g(\alpha)$ does not increase, with increasing α for $\alpha \in (0, +\infty)$. Thus $\max_{\alpha} o(\alpha)$ is always bounded from above by the values of f and g at the point where the lines $f(\alpha)$ and $g(\alpha)$ intersect. Let γ denote the value of α for which this happens: $f(\gamma) = g(\gamma)$. Straightforward algebraic simplification yields

$$\gamma = \frac{\ell_{\text{LO}} \times (1 - x)\delta}{U_{\text{HI}}^{\text{HI}} - \ell_{\text{LO}} \times (1 - x)}, \quad (12)$$

and therefore

$$\begin{aligned} \max_{\alpha} o(\alpha) &\leq \\ &\left(\frac{U_{\text{HI}}^{\text{HI}^2} + \ell_{\text{LO}} U_{\text{HI}}^{\text{HI}} \times x - \ell_{\text{LO}} \times x + \ell_{\text{LO}} \times x^2}{U_{\text{HI}}^{\text{HI}} - \ell_{\text{LO}} + \ell_{\text{LO}} \times x} \delta \right) \end{aligned} \quad (13)$$

Hence, if J misses its deadline as we assumed, we must have $o(\gamma) > \delta$; that is,

$$\ell_{\text{LO}} \times x^2 - (2 - U_{\text{HI}}^{\text{HI}}) \ell_{\text{LO}} \times x + \ell_{\text{LO}} - U_{\text{HI}}^{\text{HI}} + U_{\text{HI}}^{\text{HI}^2} > 0. \quad (14)$$

We have just shown that Inequality 14 represents a *necessary* condition for a deadline miss. Taking its negation, we conclude

FACT 5. *If the following condition is satisfied*

$$\ell_{\text{LO}} \times x^2 - (2 - U_{\text{HI}}^{\text{HI}}) \ell_{\text{LO}} \times x + \ell_{\text{LO}} - U_{\text{HI}}^{\text{HI}} + U_{\text{HI}}^{\text{HI}^2} \leq 0 \quad (15)$$

then no deadline is missed in any HI-criticality behavior. $\square$

We now have the pieces in place to prove that our algorithm schedules every HI-criticality behavior correctly, if it declares success at Line L2.

Figure 2: Deadlines of HI-criticality task τ_i after a criticality level change occurs at t^* . The first deadline is at least $(1-x)T_i(\text{HI})$ time away; successive deadlines are at least $T_i(\text{HI})$ time apart.

LEMMA 3. *If the algorithm returns success at Line L2, then no deadline is missed by the run-time dispatcher described in Section 3.2 upon any HI-criticality behavior.*

Proof: For Line L2 of Algorithm 1 to have executed, it must be the case that the condition C2 evaluates to true; i.e., that

$$\begin{aligned}
 U_{\text{HI}}^{\text{HI}} &\leq \frac{4(1 - \ell_{\text{LO}})}{4 - \ell_{\text{LO}}} \\
 \Leftrightarrow 4U_{\text{HI}}^{\text{HI}} - U_{\text{HI}}^{\text{HI}}\ell_{\text{LO}} &\leq 4 - 4\ell_{\text{LO}} \\
 \Leftrightarrow 4U_{\text{HI}}^{\text{HI}} - U_{\text{HI}}^{\text{HI}}\ell_{\text{LO}} - 4 + 4\ell_{\text{LO}} &\leq 0 \\
 \Leftrightarrow 4\left(U_{\text{HI}}^{\text{HI}} - \frac{U_{\text{HI}}^{\text{HI}}\ell_{\text{LO}}}{4} - 1 + \ell_{\text{LO}}\right) &\leq 0 \\
 \Leftrightarrow U_{\text{HI}}^{\text{HI}} - \frac{U_{\text{HI}}^{\text{HI}}\ell_{\text{LO}}}{4} - 1 + \ell_{\text{LO}} &\leq 0 \quad (16)
 \end{aligned}$$

Let us now simplify the LHS of Inequality 15 when x is assigned the value $\left(\frac{2 - U_{\text{HI}}^{\text{HI}}}{2}\right)$:

$$\begin{aligned}
 &\ell_{\text{LO}} \times x^2 - (2 - U_{\text{HI}}^{\text{HI}})\ell_{\text{LO}} \times x + \ell_{\text{LO}} - U_{\text{HI}}^{\text{HI}} + U_{\text{HI}}^{\text{HI}2} \\
 &= \ell_{\text{LO}} \times \left(\frac{2 - U_{\text{HI}}^{\text{HI}}}{2}\right)^2 - (2 - U_{\text{HI}}^{\text{HI}})\ell_{\text{LO}} \times \left(\frac{2 - U_{\text{HI}}^{\text{HI}}}{2}\right) \\
 &\quad + \ell_{\text{LO}} - U_{\text{HI}}^{\text{HI}} + U_{\text{HI}}^{\text{HI}2} \\
 &= \frac{\ell_{\text{LO}}}{4} (2 - U_{\text{HI}}^{\text{HI}})^2 - \frac{\ell_{\text{LO}}}{2} (2 - U_{\text{HI}}^{\text{HI}}) + \ell_{\text{LO}} - U_{\text{HI}}^{\text{HI}} + U_{\text{HI}}^{\text{HI}2} \\
 &= -\frac{\ell_{\text{LO}}}{4} (2 - U_{\text{HI}}^{\text{HI}})^2 + \ell_{\text{LO}} - U_{\text{HI}}^{\text{HI}} + U_{\text{HI}}^{\text{HI}2} \\
 &= -\frac{\ell_{\text{LO}}}{4} (4 - 4U_{\text{HI}}^{\text{HI}} + U_{\text{HI}}^{\text{HI}2}) + \ell_{\text{LO}} - U_{\text{HI}}^{\text{HI}} + U_{\text{HI}}^{\text{HI}2} \\
 &= -\ell_{\text{LO}} + \ell_{\text{LO}}U_{\text{HI}}^{\text{HI}} - \frac{\ell_{\text{LO}}U_{\text{HI}}^{\text{HI}2}}{4} + \ell_{\text{LO}} - U_{\text{HI}}^{\text{HI}} + U_{\text{HI}}^{\text{HI}2} \\
 &= U_{\text{HI}}^{\text{HI}}(\ell_{\text{LO}} - \frac{\ell_{\text{LO}}U_{\text{HI}}^{\text{HI}}}{4} - 1 + U_{\text{HI}}^{\text{HI}}) \\
 &\leq 0 \quad (\text{By Inequality 16 above})
 \end{aligned}$$

and hence by Fact 5, no HI-criticality deadlines are missed. $\square$

4.3 Returns success at Line L4

We now turn our attention to the third and last point at which Algorithm 1 may declare success. This case – the pseudo-code between the lines labeled L3 and L5 in Algorithm 1 – essentially emulates the algorithm that was derived and proved correct in [1]; its correctness therefore follows from the proofs in [1]. We outline the essence of those proofs in the proof of Lemma 4 below.

LEMMA 4. *If the algorithm returns success at Line L4 for a given task system τ , then τ is scheduled correctly by the run-time dispatcher described in Section 3.2.*

Proof: In Line L3, we seek the *smallest* value of x such that the (regular) sporadic task system $\hat{\tau}(x)$ is EDF-schedulable. It is straightforward to show, using techniques analogous to those used in the proof of Lemma 2, that $\hat{\tau}(\ell_{\text{LO}})$ is indeed EDF-schedulable; hence the value of x we seek lies in the range $[0, \ell_{\text{LO}}]$. It can be obtained to any desired degree of accuracy by bisection search (“binary search”) for the smallest value of x such that task system $\hat{\tau}(x)$ is deemed EDF-schedulable by one of the schedulability tests described in [6, 13, 14].

It is evident that for this value of x , no deadline is missed by the run-time dispatcher described in Section 3.2 upon any LO-criticality behavior. It remains to determine whether any deadlines may be missed in any HI-criticality behavior. To check this, we test (line C3) whether the regular 3-parameter sporadic task system,

$$\bigcup_{\chi_i = \text{HI}} \{\hat{\tau}_i(1-x)\} \quad (17)$$

is EDF-schedulable or not. If so, τ passes our schedulability test; otherwise, we declare failure and return. Figure 2 briefly illustrates why testing the EDF-schedulability of the 3-parameter sporadic task system depicted in Equation 17 above is sufficient for ensuring that no deadline is missed by the run-time dispatcher described in Section 3.2 upon any HI-criticality behavior. It is evident from inspection of this figure that the demand bound function for a HI-criticality task τ_i after the instant t^* at which the criticality level change occurs is exactly equal to the demand bound function of a 3-parameter sporadic task with WCET C_i , relative deadline $(1-x)T_i(\text{HI})$, and period T_i . Hence, validating that the 3-parameter sporadic task system depicted in Equation 17 above is EDF-schedulable is sufficient for ensuring that no deadlines are missed by our EDF-based run-time dispatcher in any HI-criticality behavior. $\square$

5. PROPERTIES

In Section 4 above, we proved that Algorithm 1 of Section 3.1, when used in conjunction with the run-time dispatcher of Section 3.2, represents a correct scheduling strategy for MC LL task systems in which multiple period parameters are specified for each task. In this section, we provide an evaluation of our scheduling algorithm.

5.1 Run-time complexity

The preprocessing step (Algorithm 1) starts out requiring that $U_{\text{LO}}^{\text{LO}}$ and $U_{\text{HI}}^{\text{HI}}$ be computed — this can be done in time linear in the number of tasks in the system. The quantity ℓ_{LO} may additionally need to be computed; for task systems for which LO-criticality utilization is bounded from above by a constant strictly less than one, this can be done in time pseudo-polynomial in the representation of the task system [6, 13, 14], or approximately to any desired degree of accuracy in polynomial time [9] — see the discussion in Section 2.1. If Algorithm 1 fails to return at Lines L1 or L2, then an iterative process of computing the scaling factor x , and of validating this x , would require multiple calls to a pseudo-polynomial time exact or polynomial-time approximate algorithm, the number of calls being determined by the desired degree of accuracy.

In Sections 5.3 and 5.4 below, we will provide quantitative characterizations of the efficacy of this algorithm. These characterizations continue to hold even if the preprocessing

Figure 3: Feasibility region for Algorithm 1. The x axis denotes the LO-criticality load ℓ_{LO} ; the y axis, the HI-criticality utilization U_{HI}^{HI} . The curved line satisfies $U_{HI}^{HI} = 4(1 - \ell_{LO})/(4 - \ell_{LO})$. Any instance with ℓ_{LO} and U_{HI}^{HI} bounded between this line and the axes is guaranteed schedulable by Algorithm 1.

algorithm (Algorithm 1) simply declares failure upon failing to return at Lines L1 or L2; hence, it is safe to declare that the algorithm being evaluated quantitatively in Sections 5.3-5.4 has pseudo-polynomial run-time complexity.

The run-time dispatcher of Section 3.2 is essentially implementing preemptive uniprocessor EDF, for which very efficient implementations are known that have logarithmic run-time complexity per scheduling decision (see, e.g., [12]). A naive implementation of the run-time dispatcher could result in linear time-complexity if the criticality level-change occurs; however, efficient implementations of this operation are known as well [2, 3] that implement such a change in logarithmic time.

5.2 Dominance over the algorithm in [1]

Prior to the work reported in this document, the state of the art for scheduling MC LL task systems with multiple periods specified for the tasks was the algorithm in [1]. Algorithm 1 strictly generalizes this algorithm by including it as a “catch-all” in lines L2-L5; hence Algorithm 1 will trivially successfully schedule any instance that was successfully scheduled by the algorithm of [1].

5.3 Feasibility region

It is evident that any instance τ with $\ell_{LO} > 1$ or $U_{HI}^{HI} > 1$ cannot possibly be scheduled on a unit-speed processor to always meet all deadlines; hence,

$$(\ell_{LO} \leq 1 \wedge U_{HI}^{HI} \leq 1)$$

is a necessary condition for schedulability. If

$$U_{HI}^{HI} \leq \frac{4(1 - \ell_{LO})}{4 - \ell_{LO}},$$

then Algorithm 1 will deem the instance schedulable (and return success in Line L2). The curve $U_{HI}^{HI} = 4(1 - \ell_{LO})/(4 - \ell_{LO})$ is depicted in Figure 3; any instance τ for which ℓ_{LO} and U_{HI}^{HI} falls below this curve is successfully scheduled by

Algorithm 1 in conjunction with the run-time dispatcher of Section 3.2.

5.4 Speedup

A scheduling algorithm A is said to have a speedup bound b , where b is a positive real number ≥ 1 , if any task system schedulable by an optimal clairvoyant scheduler upon a unit-speed processor is also schedulable by algorithm A upon a speed- b processor. Speedup factor is thus one metric of effectiveness for scheduling algorithms – the smaller the speedup factor, the better the algorithm (with optimal algorithms having speedup factor 1).

As stated in Section 5.3 above, any instance with $U_{HI}^{HI} \leq 4(1 - \ell_{LO})/(4 - \ell_{LO})$ is guaranteed schedulable by Algorithm 1. If we choose $\ell_{LO} \leftarrow (4 - 2\sqrt{3})$, we have that the corresponding upper bound in U_{HI}^{HI} is

$$\begin{aligned} & \frac{4(1 - (4 - 2\sqrt{3}))}{4 - (4 - 2\sqrt{3})} \\ &= \frac{4(-3 + 2\sqrt{3})}{2\sqrt{3}} \\ &= \frac{4\sqrt{3}(2 - \sqrt{3})}{2\sqrt{3}} \\ &= 2(2 - \sqrt{3}) \\ &= (4 - 2\sqrt{3}) \end{aligned}$$

Observing that $\frac{1}{(4 - 2\sqrt{3})} = 1 + \frac{\sqrt{3}}{2}$,

we conclude that

LEMMA 5. *Any instance that is schedulable correctly upon a particular preemptive uniprocessor platform by an optimal clairvoyant scheduler is scheduled correctly by Algorithm 1 upon a preemptive uniprocessor platform that is $(1 + \frac{\sqrt{3}}{2})$ times as fast.* $\square$

That is, Algorithm 1 has a speedup bound $(1 + \frac{\sqrt{3}}{2})$.

6. CONCLUSIONS

Mixed-criticality scheduling has recently emerged as an exciting and important area of research in the real-time systems community. Most of the prior research on mixed-criticality scheduling has focused upon dealing with uncertainty in WCET estimates. However, epistemic uncertainty regarding external events that trigger a system response is an additional important form of uncertainty in many safety-critical systems; in this work we have attempted to understand how to deal with such uncertainty when it is characterized by multiple estimates, at differing levels of assurance, of the minimum inter-arrival separation between successive triggering events. We have derived, and proved the correctness of, an algorithm for scheduling collections of such tasks upon preemptive uniprocessor platforms. We have also provided quantitative characterizations of the efficacy of our algorithm; to our knowledge, this is the first quantitatively characterized algorithm for scheduling mixed-criticality systems with multiple frequency specifications.

In this paper, we have focused solely upon multiple frequency estimations, completely decoupling this from the issue of multiple WCET estimations. In fact our results generalize to concurrent consideration of both sources of uncertainty — i.e., a model in which multiple estimations of both

the frequency and the WCET are provided; however, the presentation of this generalized algorithm (and particularly its proof) is considerably more messy than the presentation here, and so for pedantic reasons we have chosen to only discuss the frequency dimension.

It would be interesting and important to extend our techniques and results to systems with more than two criticality levels specified. Although we know how to generalize the algorithm to this manner, we have not been able to obtain closed-form speedup bounds for the generalized algorithm — indeed, obtaining such closed-form speedup bounds remains open even for the simpler, and far more widely studied, problem in which multiple specifications are provided only along the WCET problem. We leave this as future work.

Acknowledgements

This research has been supported in part by NSF grants CNS 1115284, CNS 1218693, CNS 1409175, and CPS 1446631, AFOSR grant FA9550-14-1-0161, ARO grant W911NF-14-1-0499, and a grant from General Motors Corp.

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