An Adaptive Control Law for Controlled Lagrangian Particle Tracking

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ABSTRACT

Controlled Lagrangian particle tracking (CLPT) is a method that evaluates the accuracy of ocean models employed for the navigation of autonomous underwater vehicles (AUVs). The accuracy of ocean models can be represented by the discrepancy between the predicted and true trajectories of AUVs, called controlled Lagrangian prediction error (CLPE). To reduce CLPE, we develop an adaptive control law that enables AUVs to follow the predicted trajectory in the true flow field. Because CLPE is exponentially increasing and navigation performance is significantly degraded when previous controllers are used, we propose the adaptive control law that makes CLPE converges to zero. Although true flows are unknown, the proposed control law identifies the true flow field so that AUVs follows the predicted trajectory. We prove that CLPE is ultimately bounded under bounded disturbances. The proposed control law is verified by simulation results.

CCS Concepts

•Computer systems organization → Robotic control; Robotic autonomy;

Keywords

Controlled Lagrangian Particle Tracking; Autonomous Underwater Vehicle; Navigation; Flow Model; Adaptive Control

1. INTRODUCTION

Autonomous underwater vehicles (AUVs) serve as mobile sensors for sensing and mapping environmental fields in the ocean. [19, 11, 18]. The success of such missions requires both reliable sensors and control of AUVs that are capable of persistently collecting information-rich data in the ocean. Control of AUVs is an area of active research in that AUVs are affected by ocean flows. For example, underwater gliders that are one type of AUVs have low speed, i.e. 25cm/s -

35cm/s, which is comparable to or less than flow speed in the coastal ocean. Such low speed severely degrades the performance on the navigation of AUVs [18, 2].

Controlled Lagrangian particle tracking (CLPT), which is a new paradigm for the evaluation of ocean model accuracy, has been developed in [15, 16]. To apply CLPT, we simulate a vehicle motion model with incorporated flow models and controllers to generate the predicted trajectory of AUVs, then the predicted trajectory is compared with the measured true trajectory of AUVs. The difference between the predicted and true trajectories of AUVs is called the controlled Lagrangian prediction error (CLPE).

In this paper, we present an adaptive control law to decrease CLPE. CLPE can be increasing over time when we use feedback controllers designed in [15, 16]. This implies that the true trajectory is significantly deviated from the predicted trajectory. Hence, we need a new control strategy that enables the reduction of CLPE to improve the navigation performance.

Many researchers study various path-planning algorithms to address the control problem of AUVs against complex ocean flows: the A*-based algorithm [5], which generates minimum traveling time routes in ocean spatial variability; the fast marching based algorithm in a static flow field and an obstacle field [13]; a front tracking method [14], which finds optimal minimum time trajectories in a strong current field. The above research work generates the predicted trajectory of AUVs assuming that the flow field is known from a flow model.

However, true flows are unknown, and flow models have large error induced by missing physics and unknown boundary conditions [7, 6]. Many researchers have developed various algorithms to identify true flows: Lagrangian data assimilation [11, 3, 2], which update flow models using measurements such as depth averaged velocities; glider computerized tomography algorithms [17, 4], which map flows in a specific domain by motion integration error of the vehicles. The algorithms focus on updating flow models and mapping flows regardless of the performance of controllers in the ocean.

For the proposed adaptive control law in this paper, we design two controllers. One controller is designed for the predicted trajectory of AUVs. The controller consists of constant feedback gain, constant feedforward gain, and a flow canceling term that uses a parametric flow model with known parameters. On the other hand, the other controller is designed for controlling AUVs in the true flow field. In the controller, feedback gain, feedforward gain, and parameters

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of the flow canceling term are updated according to a designed adaptation law using CLPE. These controllers allow AUVs to follow the predicted trajectory while identifying the true flow field.

We organize this paper into the following sections: Section II describes a problem setup introducing the vehicle motion model, flow models, and the state feedback controllers with flow canceling. Section III presents an adaptive control law. Section IV presents the simulation results that show the verification of the theoretical analysis, and Section VII provides conclusions.

2. PROBLEM FORMULATION

We predict the trajectory of AUVs for their successful missions and safe operation. The predicted trajectory is generated by simulating AUVs. To deal with the control problem of AUVs following the predicted trajectory in a true flow field, we describe the vehicle motion model, flow models, state feedback controllers with the flow canceling strategy.

2.1 Vehicle motion model

Let $D \subset \mathbb{R}^2$ be a domain. Then let $\mathbf{F}(\mathbf{x}, t)$ be a deterministic ambient flow velocity, where $\mathbf{F}: D \times [0, \infty] \rightarrow \mathbb{R}^2$. Let $\mathbf{v}$ be a through-water velocity, or the controlled velocity of the AUV, where $\mathbf{v}: D \times [0, \infty] \rightarrow \mathbb{R}^2$.

$$\frac{d\mathbf{x}}{dt} = \mathbf{F}_R(\mathbf{x}, t) + \mathbf{v}(\mathbf{x}, t, \mathbf{z}_c(t)). \quad (1)$$

Subscript R for the flow $\mathbf{F}$ denotes a real ocean flow. $\mathbf{F}_R$ and $\mathbf{v}$ are locally Lipschitz in $\mathbf{x} = [x_1, x_2]^\top \in D$. $\mathbf{x}$ is the true position of the AUV. $\mathbf{z}_c(t) = [z_{1c}(t), z_{2c}(t)]^\top$ is the desired trajectory of the AUV. When we define $\mathbf{z}(t)$ as the predicted position of the vehicle, the predicted trajectory of the vehicle are generated from integrating

$$\frac{d\mathbf{z}}{dt} = \mathbf{F}_M(\mathbf{z}, t) + \mathbf{v}(\mathbf{z}, t, \mathbf{z}_c(t)), \quad (2)$$

where subscript M for the flow $\mathbf{F}$ is a modeled flow.

2.2 Flow models

Flow fields can be represented by spatial and temporal basis functions [12]. We consider that spatial and temporal basis functions are the combination of Gaussian radial basis function and tidal basis function. Let $\phi: D \times [0, \infty] \rightarrow \mathbb{R}^2$ be $[\phi_1(\mathbf{x}, t), \phi_2(\mathbf{x}, t)]^\top$.

$$\mathbf{F}_R(\mathbf{x}, t) = \theta \phi(\mathbf{x}, t) \quad (3)$$

$$\mathbf{F}_M(\mathbf{x}, t) = \alpha_M \phi(\mathbf{x}, t), \quad (4)$$

where $\theta = \text{diag}\{\theta_1, \theta_2\}$, and α_M is $\text{diag}\{\alpha_{M1}, \alpha_{M2}\}$. The combined function is

$$\phi_i(\mathbf{x}, t) = \exp\left(-\frac{\|\mathbf{x} - \mathbf{c}_i\|^2}{2\sigma_i^2}\right) \cos(\omega_i t), \quad (5)$$

where $\mathbf{c}_i$ is the centers, σ_i is the widths, ω_i tidal frequencies, and $i = 1, 2$ [12].

2.3 State feedback controllers with flow canceling

The flow canceling strategy is to cancel out flow velocity by controlling the heading angle of AUVs. [16] introduced flow canceling controllers by which AUVs are able to stay at or around a target station. With the flow canceling

strategy, we design two types of controllers as the combination of feedback and feedforward structures. Let $K_M = \text{diag}\{K_{M1}, K_{M2}\}$, and $\Gamma_M = \text{diag}\{\Gamma_{M1}, \Gamma_{M2}\}$ be diagonal matrices with known parameters. Let $\alpha_R = \text{diag}\{\alpha_{R1}, \alpha_{R2}\}$, $K_R = \text{diag}\{K_{R1}, K_{R2}\}$, and $\Gamma_R = \text{diag}\{\Gamma_{R1}, \Gamma_{R2}\}$ be diagonal matrices with time-varying parameters. Then,

$$\dot{\mathbf{v}}(\mathbf{z}, t, \mathbf{z}_c(t)) = -\alpha_M \phi(\mathbf{z}, t) - K_M \mathbf{z} + \Gamma_M \mathbf{z}_c(t) \quad (6)$$

$$\dot{\mathbf{v}}(\mathbf{x}, t, \mathbf{z}_c(t)) = -\alpha_R \phi(\mathbf{x}, t) - K_R \mathbf{x} + \Gamma_R \mathbf{z}_c(t). \quad (7)$$

The desired controller represented by equation (6) contains a flow canceling term, a positional feedback term, and a feedforward term with known fixed parameters for achieving given missions. By plugging equations (6) and (4) into equation (2), the closed loop dynamics of the desired controller is

$$\dot{\mathbf{z}} = -K_M \mathbf{z} + \Gamma_M \mathbf{z}_c(t). \quad (8)$$

The feedback term enables vehicles to go toward a target point, and the feedforward term changes the target point according to missions of the vehicles. Meanwhile, the true controller represented by equation (7) includes time-varying parameters despite the same structure of the desired controller. The closed loop dynamics of the true controller is

$$\dot{\mathbf{x}} = (\theta - \alpha_R) \phi(\mathbf{x}, t) - K_R \mathbf{x} + \Gamma_R \mathbf{z}_c(t) \quad (9)$$

when we plug equations (7) and (3) into equation (1). If $\mathbf{x}$ and $\mathbf{z}$ have the same initial condition and $\alpha_R = \theta$, $K_R = K_M$, and $\Gamma_R = \Gamma_M$, the predicted trajectory is identical to the true trajectory because of closed-loop dynamics. Our goal is to design an adaptive control law so that the closed loop dynamics of the true controller follows that of the desired controller.

2.4 Controlled Lagrangian prediction error

To control the AUV that follows the predicted trajectory in the true flow field, we first derive controlled Lagrangian prediction error (CLPE) dynamics that models how much the true trajectory is deviated from the prediction trajectory. By subtracting equation (8) from equation (9), CLPE dynamics is represented by

$$\begin{aligned} \dot{\mathbf{e}} &= \dot{\mathbf{x}} - \dot{\mathbf{z}} \\ &= (\theta - \alpha_R) \phi(\mathbf{x}, t) - K_R \mathbf{x} + \Gamma_R \mathbf{z}_c(t) + K_M \mathbf{z} - \Gamma_M \mathbf{z}_c(t) \\ &= (\theta - \alpha_R) \phi(\mathbf{x}, t) + (K_M - K_R) \mathbf{x} - K_M \mathbf{e} + (\Gamma_R - \Gamma_M) \mathbf{z}_c(t). \end{aligned} \quad (10)$$

If $\alpha_R = \theta$, $K_R = K_M$, and $\Gamma_R = \Gamma_M$, CLPE goes to zero as time goes by, which implies that the true trajectory follows the predicted trajectory. In addition, AUVs identifies the true flow field. Our goal is to design an adaptation law for updating parameters α_R , K_R , and Γ_R by using CLPE dynamics so that CLPE converges to zero.

3. AN ADAPTIVE CONTROL LAW

To design an adaptive control law for controlled Lagrangian particle tracking, we need some definitions and assumptions.

Definition 3.1 [9] Signal u is persistent exciting if there exist positive constants κ_1 , κ_2 , and T such that $\kappa_2 I > \int_t^{t+T} u(s) u^\top(s) ds$ and $\int_t^{t+T} u(s) u^\top(s) ds > \kappa_1 I \forall t$.

Assumption 3.1 $\mathbf{z}_c$ is a persistent exciting signal.

Assumption 3.2 K_M , or the feedback gain matrix of equation (8), is positive definite.

Remark 3.1 $\mathbf{z}_c$ is bounded because of persistent excitation.

Remark 3.2 The predicted trajectory of AUVs is usually generated from stable closed loop dynamics of equation (8). It requires gain matrix K_M is positive definite.

This section describes an adaptive control law for AUVs. Let $\bar{\alpha}_R = [\alpha_{R1}, \alpha_{R2}]^\top$, $\bar{K}_R = [K_{R1}, K_{R2}]^\top$, and $\bar{\Gamma}_R = [\Gamma_{R1}, \Gamma_{R2}]^\top$ be two dimensional vectors with time-varying parameters. Let $\bar{\theta} = [\theta_1, \theta_2]^\top$, $\bar{K}_M = [K_{M1}, K_{M2}]^\top$, and $\bar{\Gamma}_M = [\Gamma_{M1}, \Gamma_{M2}]^\top$ be two dimensional vectors with desired fixed parameters. Let $\tilde{\phi} = \text{diag}\{\phi_1, \phi_2\}$, $\tilde{\mathbf{x}} = \text{diag}\{x_1, x_2\}$, and $\tilde{\mathbf{z}}_c = \text{diag}\{z_{1c}, z_{2c}\}$ be diagonal matrices. Let γ be any positive constant. We design updating law for time-varying parameters $\bar{\alpha}_R$, $\bar{K}_R$, and $\bar{\Gamma}_R$ by the following equations.

$$\dot{\bar{\alpha}}_R = \gamma \tilde{\phi}^\top(\mathbf{x}, t) \mathbf{e} \quad (11)$$

$$\dot{\bar{K}}_R = \gamma \tilde{\mathbf{x}}^\top \mathbf{e} \quad (12)$$

$$\dot{\bar{\Gamma}}_R = -\gamma \tilde{\mathbf{z}}_c^\top \mathbf{e}. \quad (13)$$

Theorem 3.1 Under assumption 3.1 and 3.2, and using equations (11), (12), (13), CLPE converges to zero when time goes to infinity.

PROOF. Consider a candidate Lyapunov function:

$$V(\mathbf{e}, \bar{\alpha}_R, \bar{K}_R, \bar{\Gamma}_R) = \frac{1}{2} \left\{ \mathbf{e}^\top \mathbf{e} + \frac{1}{\gamma} (\bar{\theta} - \bar{\alpha}_R)^\top (\bar{\theta} - \bar{\alpha}_R) + \frac{1}{\gamma} (\bar{K}_M - \bar{K}_R)^\top (\bar{K}_M - \bar{K}_R) + \frac{1}{\gamma} (\bar{\Gamma}_R - \bar{\Gamma}_M)^\top (\bar{\Gamma}_R - \bar{\Gamma}_M) \right\}. \quad (14)$$

The derivative of V is

$$\begin{aligned} \dot{V} = & -\mathbf{e}^\top K_M \mathbf{e} + (\bar{\theta} - \bar{\alpha}_R)^\top \left(\tilde{\phi}(\mathbf{x}, t)^\top \mathbf{e} - \frac{1}{\gamma} \dot{\bar{\alpha}}_R \right) \\ & + (\bar{K}_M - \bar{K}_R)^\top \left(\tilde{\mathbf{x}}^\top \mathbf{e} - \frac{1}{\gamma} \dot{\bar{K}}_R \right) + (\bar{\Gamma}_R - \bar{\Gamma}_M)^\top \left(\tilde{\mathbf{z}}_c^\top \mathbf{e} + \frac{1}{\gamma} \dot{\bar{\Gamma}}_R \right). \end{aligned} \quad (15)$$

By assumption 3.2 and using equation (11), (12), and (13), $\dot{V} = -\mathbf{e}^\top K_M \mathbf{e} \leq 0$. $\dot{V}$ is negative semi-definite and this implies $\mathbf{e}$, $\bar{\alpha}_R$, $\bar{K}_R$, and $\bar{\Gamma}_R$ are bounded. In addition, $\dot{V} = -2\mathbf{e}^\top K_M \dot{\mathbf{e}} = -2\mathbf{e}^\top K_M \{(\bar{\theta} - \alpha_R) \phi(\mathbf{x}, t) + (K_M - K_R) \mathbf{x} - K_M \mathbf{e} + (\Gamma_R - \Gamma_M) \mathbf{z}_c\}$. By assumption 3.1, $\mathbf{z}_c$ is bounded, and $\mathbf{z}$ is bounded because equation (8) represents linear systems. $\mathbf{x}$ is bounded because $\mathbf{x} = \mathbf{e} + \mathbf{z}$. In addition, α_R , K_R , and Γ_R are bounded. Thus, $\dot{V}$ is bounded, and hence $\dot{V}$ is uniformly continuous. By Barbalat's lemma in [1, 10], $\mathbf{e} \rightarrow 0$ when $t \rightarrow \infty$. $\square$

Theorem 3.2 Under the same setting of theorem 3.1, $\bar{\alpha}_R$, $\bar{K}_R$, and $\bar{\Gamma}_R$ converges to $\bar{\theta}$, $\bar{K}_M$, and $\bar{\Gamma}_M$, respectively.

PROOF. In order to identify the true flow field from the proposed adaptive control law, we prove the convergence of parameters $\bar{\alpha}_R$, $\bar{K}_R$, and $\bar{\Gamma}_R$. Let η_1 , η_2 , and η_3 be $\bar{\theta} - \bar{\alpha}_R$, $\bar{K}_M - \bar{K}_R$, and $\bar{\Gamma}_R - \bar{\Gamma}_M$, respectively. We rewrite equation (10) using η_1 , η_2 , and η_3 as follows:

$$\dot{\mathbf{e}} = \tilde{\phi}(\mathbf{x}, t) \eta_1 + \tilde{\mathbf{x}} \eta_2 - K_M \mathbf{e} + \tilde{\mathbf{z}}_c \eta_3. \quad (16)$$

We augment $\mathbf{e}$, η_1 , η_2 , and η_3 to new state variable X . Then

$$\begin{aligned} \dot{X} &= A(t)X, \quad A(t) = \begin{bmatrix} -K_M & \tilde{\phi} & \tilde{\mathbf{x}} & \tilde{\mathbf{z}}_c \\ -\gamma \tilde{\phi} & 0 & 0 & 0 \\ -\gamma \tilde{\mathbf{x}} & 0 & 0 & 0 \\ -\gamma \tilde{\mathbf{z}}_c & 0 & 0 & 0 \end{bmatrix} \\ Y &= CX, \quad C = \begin{bmatrix} I & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}. \end{aligned} \quad (17)$$

Our goal is to show the origin of $\dot{X} = A(t)X$ is uniformly asymptotically stable, which implies that $\bar{\alpha}_R = \bar{\theta}$, $\bar{K}_R = \bar{K}_M$, and $\bar{\Gamma}_R = \bar{\Gamma}_M$. By theorem 3.4.8 in [8], a necessary and sufficient condition is that there exist a symmetric matrix P such that $c_1 I \leq P \leq c_2 I$ and $A^\top P + PA + \dot{P} + \nu C^\top C \leq 0$ are satisfied $\forall t$ and some constant $\nu > 0$, where $c_1 > 0$, and $c_2 > 0$ and C is such that (C, A) is a uniformly completely

observable. Let $P = \begin{bmatrix} \frac{1}{2} K_M^{-1} & 0 & 0 & 0 \\ 0 & \frac{1}{2\gamma} K_M^{-1} & 0 & 0 \\ 0 & 0 & \frac{1}{2\gamma} K_M^{-1} & 0 \\ 0 & 0 & 0 & \frac{1}{2\gamma} K_M^{-1} \end{bmatrix}$.

Let V' be $X^\top P X$. Then,

$$\dot{V}' = X^\top (A^\top P + P^\top A + \dot{P}) X \leq -\nu X^\top C^\top C X = -\nu \|Y\|^2, \quad (18)$$

where $\dot{P} = 0$.

Now we will prove (C, A) is a uniformly completely observable. Because it is hard to prove the observability of time varying system matrix A , we will instead show $(C, A + LC)$ is uniformly completely observable with some bounded matrix L , called output injection by lemma 4.8.1 in [8]. Let

$$L = \begin{bmatrix} K_M & 0 & 0 & 0 \\ \gamma \tilde{\phi} & 0 & 0 & 0 \\ \gamma \tilde{\mathbf{x}} & 0 & 0 & 0 \\ \gamma \tilde{\mathbf{z}}_c & 0 & 0 & 0 \end{bmatrix}. \quad \text{Since } \mathbf{x} \text{ and } \mathbf{z}_c \text{ are bounded, and } \tilde{\phi} \text{ is a sinusoidal function with exponential magnitude, } L \text{ is}$$

bounded. Then, $A + LC = \begin{bmatrix} 0 & \tilde{\phi} & \tilde{\mathbf{x}} & \tilde{\mathbf{z}}_c \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$. Thus,

$$\begin{aligned} \dot{X} &= AX = (A + LC)X - LY \\ Y &= CX. \end{aligned} \quad (19)$$

Let $\eta = [\eta_1, \eta_2, \eta_3]^\top$, and $w = [\tilde{\phi}, \tilde{\mathbf{x}}, \tilde{\mathbf{z}}_c]^\top$. We have the following equation corresponding to equation (19).

$$\begin{aligned} \dot{\mathbf{e}} &= -K_M \mathbf{e} + w^\top \eta \\ \dot{\eta} &= 0 \\ Y &= \mathbf{e}. \end{aligned} \quad (20)$$

Because $\tilde{\mathbf{z}}_c$ is persistent exciting, $\tilde{\mathbf{x}}$ in steady-state is persistent exciting by lemma 4.8.3 in [8]. $\tilde{\phi}$ is a sinusoidal function. Hence, w is persistent exciting. Let $\Psi(\zeta) = \int_t^\tau \exp^{-K_M(\tau-\zeta)} w(\zeta) d\zeta$. By lemma 4.8.3 in [8], $\Psi(\zeta)$ satisfies persistent exciting conditions because of a stable, minimum phase, proper rational transfer function ($sI_{2 \times 2} + K_M$). Therefore, there exists constant $\rho_1, \rho_2, T_0 > 0$ such that $\rho_2 I \geq \frac{1}{T_0} \int_t^{t+T_0} \Psi(\zeta) \Psi^\top(\zeta) d\zeta \geq \rho_1 I \quad \forall t \geq 0$. By applying lemma 4.8.4 in [8] to the system of equation (20), $(C, A + LC)$ is uniformly completely observable; hence, the system of equation (17) is uniformly com-

pletely observable. Therefore, $\bar{\alpha}_R$, $\bar{K}_R$, and $\bar{\Gamma}_R$ converge to $\bar{\theta}$, $\bar{K}_M$, and $\bar{\Gamma}_M$, respectively. $\square$

Although the basis functions well capture the spatial variability of true flows in a specific region, the functions still include deterministic errors induced by the variability out of the region. To address the robustness of the proposed adaptive algorithm, we show the boundedness of CLPE when the true flow model has deterministic disturbances such as unstructured uncertainties. We assume $\mathbf{F}_R(\mathbf{x}, t) = \theta\phi(\mathbf{x}, t) + \Delta$, where $\|\Delta\|$ is bounded by $\Delta_{\max}$. Then,

$$\dot{\mathbf{e}} = (\theta - \alpha_R)\phi(\mathbf{x}, t) + (K_M - K_R)\mathbf{x} - K_M\mathbf{e} + (\Gamma_R - \Gamma_M)\mathbf{z}_c + \Delta. \quad (21)$$

Theorem 3.3 Under the same setting of theorem 3.1, the bound of CLPE is

$$\|\mathbf{e}\| \leq \frac{1}{\beta} \|\Delta\| \quad (22)$$

where the positive constant $\beta < \lambda_{\min}(K_M)$.

PROOF. Let V be the Lyapunov function represented by equation (14). By using equation (21), the derivate of V is

$$\begin{aligned} \dot{V} = & -\mathbf{e}^T K_M \mathbf{e} + (\bar{\theta} - \bar{\alpha}_R)^T \left(\tilde{\phi}(\mathbf{x}, t)^T \mathbf{e} - \frac{1}{\gamma} \dot{\tilde{\alpha}}_R \right) \\ & + (\bar{K}_M - \bar{K}_R)^T \left(\tilde{\mathbf{x}}^T \mathbf{e} - \frac{1}{\gamma} \dot{\tilde{K}}_R \right) + (\bar{\Gamma}_R - \bar{\Gamma}_M)^T \left(\tilde{\mathbf{z}}_c^T \mathbf{e} + \frac{1}{\gamma} \dot{\tilde{\Gamma}}_R \right) \\ & + \mathbf{e}^T \Delta. \end{aligned} \quad (23)$$

Then, we plug the adaptive law represented by equations (11), (12), and (13) into equation (23). Then,

$$\begin{aligned} \dot{V} = & -\mathbf{e}^T K_M \mathbf{e} + \mathbf{e}^T \Delta \\ \leq & -\lambda_{\min}(K_M) \mathbf{e}^T \mathbf{e} + \mathbf{e}^T \Delta \\ \leq & -\lambda_{\min}(K_M) \|\mathbf{e}\|^2 + \|\mathbf{e}\| \|\Delta\| \\ \leq & -(\lambda_{\min}(K_M) - \beta) \|\mathbf{e}\|^2 + \|\mathbf{e}\| \|\Delta\| - \beta \|\mathbf{e}\|^2 \end{aligned} \quad (24)$$

When $\|\mathbf{e}\| \geq \frac{1}{\beta} \|\Delta\|$ given positive constant $\beta < \lambda_{\min}(K)$, $\dot{V} \leq -(\lambda_{\min}(K_M) - \beta) \|\mathbf{e}\|^2$, which means $\dot{V}$ is negative definite. Thus, CLPE is bounded by $\|\mathbf{e}\| \leq \frac{1}{\beta} \|\Delta\|$. $\square$

4. SIMULATION RESULTS

In this section, we describe simulation results for the proposed control law of the AUV. For flow models, we select $\theta = \text{diag}\{1, 0.5\}$ for the parameters of true flows, and $\alpha_M = \text{diag}\{0.1, 0.1\}$ for the parameters of modeled flows. The Gaussian radial basis functions includes that $\mathbf{c}_i$ and σ_i are $[0, 0]^T$ and 5, respectively, where $i = 1, 2$. Tidal frequencies ω_1 and ω_2 are 12.42 hours, feedback gain of desired controller, K_M is $\text{diag}\{1, 1\}$. Γ_M is the same as K_M . The initial conditions of all parameters in the adaptation law are zeros. Adaptation speed, or γ is designed as 0.5. We consider a desired circle trajectory with radius r . Let $\mathbf{z}_c = [r \cos \Theta, r \sin \Theta]^T$ be a sinusoidal signal. For the simulation of the proposed control law, we select the desired trajectory with $r = 3$, and $\Theta = \frac{\pi}{5} \lfloor \frac{t}{20} \rfloor$. We call one cycle when the AUV travels ten waypoints. Figure 1 represents waypoints and predicted trajectories of AUVs.

When we see Figure 2, CLPE goes to zero over 20 intervals (400 seconds), which is about two cycles. In Figure 3, the parameters of modeled flow converge to the parameters

Figure 1: The predicted trajectory and waypoints: Solid lines show the predicted trajectories of AUVs. AUVs start to go to the waypoint (3,0) from the origin, and AUVs keep moving to another waypoints counter clockwise direction sequentially. Arrows represent true flows at the initial time. Arrows are changing according to time because of tidal basis function.

of true flow, individually. Feedback and feedforward gains converge to the gains of desired controller with the similar trend of flow model parameters. These results support our theoretical analysis in the previous section.

Figure 2: CLPE: Interval represents time interval of 20 seconds. 10 intervals means one cycle. Over two cycle, CLPE error is significantly reduced, and CLPE converges to zero.

5. CONCLUSION

This paper contributes to establishing an adaptive law for controlled Lagrangian particle tracking. We develop the adaptive law under the paradigm of controlled Lagrangian particle tracking (CLPT). When the controlled velocity of AUVs has no constraint, we derive the adaptation law proving that controlled Lagrangian prediction error (CLPE) con-

Figure 3: Parameters: The parameters of modeled flow converge to those of true flow

verges to zero, and the parameters of the flow models converge to true flow parameters when time goes to infinity. For the robustness of the proposed algorithm, we show that CLPE is ultimately bounded when true flows are uncertain deterministically. Future work is to develop an adaptive control law when localization service is infrequently available.

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