Optimal Multi-Unit Mechanisms with Private Demands

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We study a pricing problem that is motivated by the following examples. A cloud computing platform such as Amazon EC2 sells virtual machines to clients, each of who needs a different number of virtual machine hours. Similarly, cloud storage providers such as Dropbox have customers that require different amounts of storage. Software companies such as Microsoft sell software subscriptions that can have different levels of service. The levels could be the number of different documents you are allowed to create, or the number of hours you are allowed to use the software. Companies like Google and Microsoft sell API calls to artificial intelligence software such as face recognition, to other software developers. Video and mobile games are increasingly designed in such a way that one can pay for better access to certain features. Spotify and iTunes sell music subscription, and different people listen to different number of songs in a month. Cellphone service providers like AT&T and Verizon offer cellular phone call minutes and data. People have widely varying amounts of data consumption.

Pricing is an important component in all these examples. The aim of this paper is to understand how to price such goods, for a monopolist seller who aims to maximize her revenue. The following common features (to a first degree of approximation) of these examples are crucial for our model.

- The marginal cost of offering a higher level of service is essentially a constant.
- The valuations for the different levels of service are roughly linear, subject to a cap.

Based on this, we consider the following problem. There is a single good with multiple units of it for sale. Equivalently, there is a single service with various levels of service. The marginal cost to the seller for procuring another unit of the good is a constant. There is a population of buyers, each of who has a *linear* valuation for consuming a number of units of the good, subject to a *cap* (which we refer to as *demand*). The type/private information of a buyer is determined by the per-unit valuation and the demand. Buyers try to maximize their utility which is quasi linear, i.e., the valuation minus their payment. The question is what is the revenue maximizing pricing scheme.

Additional Key Words and Phrases: Revenue Maximization; Multidimensional Auctions

The standard approach in mechanism design [5] is Bayesian: assume that the types are drawn from a given distribution and find/characterize the incentive compatible (IC) mechanism that maximizes the expected revenue when the buyer types are drawn from this distribution. The optimization is over randomized mechanisms, which in our case corresponds to pricing lotteries.

It appears that the optimal mechanism is usually randomized for small examples with discrete support. This phenomenon is quite common. While Myerson [5] showed that the optimal mechanism for single dimensional types is deterministic under quite a general assumption about the prior distribution called regularity, even slight multi dimensional generalizations end up in randomized mechanisms as optimal [3]. However, practical considerations force a seller to stick to deterministic

The full version of this paper can be found in [2].

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mechanisms for the most part. Moreover, the optimal randomized mechanism sometimes doesn't even have a finite description [1]. Hence it is important to understand the structure of the optimal deterministic mechanism. In this paper we offer two insights in this regard.

Our first contribution is to identify a natural condition that guarantees that the optimal (randomized) mechanism is deterministic. The condition we need is called *decreasing marginal revenue* (DMR). Regularity requires that the *virtual value function* is monotone, or equivalently, that the revenue function is concave in the *quantile space*. DMR requires that *in the value space*, the marginal revenue is decreasing or equivalently that revenue function is concave. In other words, a probability distribution with CDF F is DMR, if the function $v(1 - F(v))$, the expected revenue of posting a price v , is concave. The condition we need for the optimal pricing to be deterministic is that the marginal distributions for v , conditioned on a given demand, are all DMR.

Our first result implies that the optimal pricing scheme is a price vector, which offers each number of units for a given price. **Our second contribution is to show that the revenue as a function of the price vector is concave**, under the same assumption of DMR. This implies that the optimal prices can be found efficiently using the ellipsoid or other cutting plane methods [4]. Note that DMR is a condition on the marginal distributions of values, and does not immediately imply concavity as a function of the vector of prices.

As a corollary, we obtain that under the DMR assumption, there is an efficient *dynamic pricing* scheme, defined as follows. Consider a repeated setting where in each round $\tau \in \{1, 2, \dots, T\}$, the seller posts a price vector $\mathbf{p}^\tau$, a buyer is drawn from a fixed distribution, and buys her utility maximizing bundle. The seller does not know the distribution of buyer types, and has to only use the purchase information in previous rounds to set the price. The goal is to approach the optimal revenue as T goes to infinity. Given that the distribution satisfies the DMR assumption, our result on the concavity of the revenue curve implies that the average revenue per round converges to the optimal revenue at the rate of $\frac{\text{poly}(n)}{\sqrt{T}}$, where n is the number of units. These bounds are quite strong, since the best known bounds for the dynamic pricing problem in general scale *exponentially* in n . The concavity of the revenue function is an assumption often made to escape this curse of dimensionality [6]; we show that this assumption can be weakened, from the concavity of the multidimensional function to that of the 1 dimensional revenue functions for each d .

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