Average-Position Coordination for Distributed Multi-User Networked Haptic Cooperation

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Proportional-derivative (PD) control is often used to coordinate the two copies of the virtual environment in distributed two-users networked haptic cooperation. However, a distributed PD controller designed for force interactions between two users may destabilize the haptic cooperation among multiple users because the effective coordination gain for each local copy of the virtual environment increases with the participant count. This paper proposes the average position (AP) strategy to upper bound the effective stiffness for the shared virtual object (SVO) coordination and, thus, to increase the stability of distributed multi-user haptic cooperation. The paper first motivates the AP strategy via continuous-time analysis of the autonomous dynamics of an SVO distributed among N users connected across a network with infinite bandwidth and no communication delay. We then investigate the effect of AP coordination on distributed multi-user haptic interactions over a network with limited bandwidth and constant and small communication delay via multi-rate stability and performance analyses of cooperative manipulations of an SVO by up to five operators. The paper shows that AP coordination: (1) has bounded effective coordination gain; (2) increases the stability region of distributed multi-user haptic cooperation compared to conventional PD coordination; and (3) renders less viscous SVO dynamics to operators than PD coordination. Three-users experimental manipulations of a shared virtual cube validate the analysis.

Keywords: networked haptic cooperation, distributed control, multi-rate control, average-position scheme

1. Introduction

The rapid growth of computer networks over the past decade has provided the impetus for adding the sense of touch to applications, such as cooperative industrial design (Glencross et al., 2007), surgical teletraining (Mitra & Niemeyer, 2007), telerehabilitation (Sugarman, Dayan, Weisel-Eichler, & Tiran, 2006), and multi-user on-line computer games (Kim, Han, Kim, Kim, & Song, 2007). In these applications, multiple distant users must interact with other users and/or with the shared virtual environment simultaneously.

Haptic sharing of virtual environments can be supported through centralized control and client-server communications (I. Lee & Choi, 2009; S. Lee & Kim, 2009), or through distributed control

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and peer-to-peer communications (Fotoohi, Sirouspour, & Capson, 2007; Huang & Lee, 2013). In the client-server architecture, users send their motion information to a central server that simulates the virtual environment and returns interaction forces to users. In the peer-to-peer paradigm, each user simulates a local copy of the virtual environment and sends their motion information with other peers. Peer-to-peer architectures are often preferred because they introduce only half of the communication delay of client-server architectures in the force feedback loop, and, as a result, they can support haptic cooperation in stiffer virtual environments. Therefore, this paper focuses on multi-user haptic cooperation with peer-to-peer communications.

Prior work on collaborative haptic systems can be described in the following categories: (1) communications approaches, which improve the haptic experience in the shared virtual environment through novel network or data transmission protocols (Eid, Cha, & El-Saddik, 2011; Marshall, Yap, & Yu, 2008; Schuwerk, Paggetti, Chaudhari, & Steinbach, 2014; Zhengzheng, Zhan, Xiang, & Wanjie, 2013); (2) control approaches, which provide stability and transparency guarantees through new methods to model and control the multiple simultaneous force interactions; and (3) experimental studies, which validate communication and control approaches through implementation on various testbeds (Basdogan, Ho, Srinivasan, & Slater, 2000; Buttolo, Oboe, & Hannaford, 1997; Gunn, Hutchins, & Adcock, 2005; Qin, Choi, Xu, Pang, & Heng, 2013; Sankaranarayanan & Hannaford, 2006). Because this paper addresses the coordination control of a distributed shared virtual object (SVO), it discusses prior control methods for multi-user haptic cooperation in more detail.

Many existing control approaches stabilize the multi-user haptic cooperation via passivity-based conditions on PD-type control gains. In a centralized setting, a state-space passivity condition in the form of linear matrix inequalities is employed to ensure the stability of multi-user haptic interactions (Bianchini, Orlandesi, & Prattichizzo, 2010). Virtual coupling parameters are computed to guarantee the stability of the cooperative haptic system for a connectivity known a priori of a centralized SVO. However, neither experimental validation nor performance analysis are presented. A modified passive-set-position-modulation algorithm (Lee & Huang, 2010) on a centralized haptic cooperation system was implemented (Ansari, Zareinejad, Rezaei, Baghestan, & Sarli, 2012). The passive-set-position-modulation approach requires local injection of damping into the SVO dynamics that may result to poor performance and impose sluggish motion of the SVOs. In a decentralized study, a distributed control framework for N -users haptic collaboration was proposed (Cheong, Niculescu, & Kim, 2009) that demanded ring-type network connectivity and a priori knowledge of the system dynamics and of the connectivity sequence. Huang and Lee (2013) introduced a discrete-time passivity framework to asymptotically synchronize N copies of a distributed SVO with second order discrete-time dynamics. They formulated the SVO synchronization as a discrete-time second-order consensus problem and employed discrete-time passive integrators (Lee & Huang, 2008) to design a passive peer-to-peer control architecture. Their passivity condition relies on local damping injection at each user location and thus makes the technique conservative for virtual environments where little local SVO damping is of interest. Passivity-based conditions on multi-rate multi-user haptic cooperation with wave domain communications were derived in a centralized setting (Yasrebi & Constantinescu, 2009) and were used in a distributed setting (Rakhsha & Constantinescu, 2012). Analysis and experiments in those works showed that the passive multi-rate wave communications could maintain multi-user haptic interaction stability regardless of the magnitude of fixed communication delays. Absolute stability conditions on PD-type control gains were presented (Khademian & Hashtrudi-Zaad, 2011; Li, Tavakoli, & Huang, 2014a) for trilateral haptic systems, and recently (Li, Tavakoli, & Huang, 2014b), for multi-user haptic systems.

Although PD control is often used to coordinate the two copies of the SVO in distributed two-user networked haptic cooperation, its direct application to haptic interaction among multiple users may lead to instability as the effective coordination gain for each local copy of the SVO increases

with the participant count. This paper proposes the average position (AP) strategy to upper bound the effective stiffness for the shared virtual object (SVO) coordination, and thus, to increase the stability of distributed multi-user haptic cooperation. The paper extends preliminary results (Rakhsha & Constantinescu, 2011) via analytical and experimental comparisons of the proposed AP and conventional PD schemes. Analysis focuses on multi-user haptic cooperation across a computer network with limited bandwidth and constant communication delays. To support fast sampling of the users' force feedback loops in the presence of slower network update rates, this paper integrates AP coordination into a dual-rate haptic control architecture. In the dual-rate architecture, users' local haptic rendering loops are sampled at the typical force control interval $T_c = 0.001$ s and receive synchronized updates from the other local feedback loops every $T_n = MT_c$, where M is a positive integer. This work employs lifting (Araki & Yamamoto, 1986; Fotoohi et al., 2007; Barbagli, Prattichizzo, & Salisbury, 2005) to model multi-user networked haptic cooperation with AP coordination and to investigate its stability and performance. This paper analyzes the autonomous dynamics of SVO distribution among N users over a network with infinite bandwidth and no communication delay; investigates the stability and performance of the AP scheme among up to five users over a network with limited bandwidth, small and constant communication delay; and validates the multi-rate stability and performance analyses via experiments in which three users manipulate a virtual cube together.

In the remainder of the paper, Section 2 motivates the proposed AP coordination of a distributed SVO before introducing it and shows that its effective stiffness is upper bounded by the coordination stiffness of two-users cooperation. Section 3 derives the closed-loop state-space dynamics of dual-rate, multi-user networked haptic interaction and investigates the stability of force feedback with AP coordination for up to five operators. Section 4 presents the frequency-domain performance of the AP coordination of networked haptic cooperation among up to five users. Section 5 validates that AP coordination increases the stability and the performance of distributed multi-user networked haptic cooperation through experiments in which three networked users manipulate a shared virtual cube together. Section 6 concludes the paper with suggestions for future work.

2. Average-position coordination

Distributed multi-user networked haptic cooperation is implemented by providing a local copy of the SVO to each peer user, coordinating all SVO copies through distributed coordination control, and rendering the dynamics and contacts of the local SVO to each user through haptic interaction control. Typically, virtual coupling is selected for haptic rendering and PD controllers are used to coordinate the distributed SVO copies to each other, as depicted in Fig. 1. The mass of the SVO is equally distributed among, and the damping of the SVO is inherited by, all of its copies. Such conventionally distributed PD coordination of the SVO is straightforward to implement, but its effective stiffness grows with the number of users. Therefore, controller gains designed for two-user haptic cooperation destabilize force interactions among multiple operators. This paper proposes the average position (AP) coordination strategy to upper bound the effective SVO coordination gain to the stiffness of two-users coordination.

The gain dependency of conventional PD coordination on the number of cooperating peers can be derived from the autonomous dynamics of SVO distribution across a network with infinite bandwidth and no communication delay. These dynamics are obtained starting from the dynamics of the i -th local copy of the distributed SVO:

$$\frac{m_O}{N} \ddot{x}_{O_i} + b_O \dot{x}_{O_i} = \sum_{j=1, j \neq i}^N (K_T (x_{O_j} - x_{O_i}) + B_T (\dot{x}_{O_j} - \dot{x}_{O_i})), \quad (1)$$

Figure 1. Conventional PD coordination of distributed haptic cooperation among N networked users.

where m_O and b_O are the mass and damping of the SVO, N is the number of users among which the SVO is distributed, K_T and B_T are the stiffness and damping gains of the coordinating PD controller, and x_{Ol} , $\dot{x}_{Ol}$, $\ddot{x}_{Ol}$ are the position, velocity and acceleration, respectively, of the l -th SVO copy, with $l = 1, \dots, N$. Together, the N local SVO dynamics in Eq. (1) yield the autonomous dynamics of the distributed SVO with PD coordination:

$$\begin{aligned} \frac{m_O}{N} \mathbf{I}_{N \times N} \ddot{\mathbf{x}}_O + ((b_O + NB_T) \mathbf{I}_{N \times N} - B_T \mathbf{1}_{N \times 1} \mathbf{1}_{1 \times N}) \dot{\mathbf{x}}_O \\ + (NK_T \mathbf{I}_{N \times N} - K_T \mathbf{1}_{N \times 1} \mathbf{1}_{1 \times N}) \mathbf{x}_O = \mathbf{0}. \end{aligned} \quad (2)$$

In Eq. (2), $\mathbf{x}_O$, $\dot{\mathbf{x}}_O$ and $\ddot{\mathbf{x}}_O$ are N -dimensional vectors that collect the positions, velocities and accelerations of all SVO copies, respectively. $\mathbf{I}_{N \times N}$ is the N -dimensional unity matrix and $\mathbf{1}_{r \times c}$ is an $r \times c$ matrix with all entries of 1. The stiffness matrix of the SVO's PD coordination in Eq. (2) is:

$$\mathbf{K}_{PD} = K_T (N \mathbf{I}_{N \times N} - \mathbf{1}_{N \times 1} \mathbf{1}_{1 \times N}), \quad (3)$$

and has one zero eigenvalue, which corresponds to the rigid body motion of the SVO, and one eigenvalue with geometric multiplicity $(N - 1)$, which is called effective coordination stiffness herein and grows with N . To bound the effective coordination stiffness, this paper proposes the AP coordination strategy.

In the AP scheme, each SVO copy is locally coordinated to the average position of all other remote SVO copies, as schematically depicted in Fig. 2 for the SVO copy of Peer i . In this figure, m_{HDi} and b_{HDi} are the mass and damping of the haptic device, $m_{Oi} = \frac{m_O}{N}$ and $b_{Oi} = b_O$ are the mass and damping of the SVO copy, K_{Ci} and B_{Ci} are the stiffness and damping of the virtual coupler, and K_T and B_T are the stiffness and damping of the distributed AP coordination controller, all at Peer i . The autonomous dynamics of the i -th local copy of the distributed SVO with AP

Figure 2. Average-position (AP) coordination of haptic cooperation among N users as applied at Peer i .

coordination are:

$$\begin{aligned} \frac{m_O}{N} \ddot{x}_{Oi} + b_O \dot{x}_{Oi} &= K_T \left(\frac{\sum_{j=1, j \neq i}^N x_{Oj}}{N-1} - x_{Oi} \right) \\ &+ B_T \left(\frac{\sum_{j=1, j \neq i}^N \dot{x}_{Oj}}{N-1} - \dot{x}_{Oi} \right), \end{aligned} \quad (4)$$

and, combined with the dynamics of the other local copies, yield the autonomous dynamics of the distributed SVO with AP coordination:

$$\begin{aligned} \frac{m_O}{N} \mathbf{I}_{N \times N} \ddot{\mathbf{x}}_O &= \left(\left(b_O + \frac{NB_T}{N-1} \right) \mathbf{I}_{N \times N} - \frac{B_T}{N-1} \mathbf{1}_{N \times 1} \mathbf{1}_{1 \times N} \right) \dot{\mathbf{x}}_O \\ &+ K_T \left(\frac{N}{N-1} \mathbf{I}_{N \times N} - \frac{1}{N-1} \mathbf{1}_{N \times 1} \mathbf{1}_{1 \times N} \right) \mathbf{x}_O. \end{aligned} \quad (5)$$

The stiffness matrix of AP coordination of the SVO is:

$$\mathbf{K}_{AP} = K_T \left(\frac{N}{N-1} \mathbf{I}_{N \times N} - \frac{1}{N-1} \mathbf{1}_{N \times 1} \mathbf{1}_{1 \times N} \right), \quad (6)$$

and has one zero eigenvalue, which corresponds to the rigid body motion of the distributed SVO, and one eigenvalue with geometric multiplicity $(N-1)$, which decreases strictly monotonically to K_T as N grows. Note that the distributed SVO dynamics are controlled by the eigenvalues of $(N/m_O)\mathbf{K}_{AP}$, and therefore, the AP scheme needs to be coupled with inversely proportional scaling of K_T to guarantee stable SVO distribution for arbitrary N . Although not employed in the work presented in this paper, such coupling is straightforward to implement.

According to Fig. 2, the dynamics of the distributed N -users networked haptic cooperation with AP coordination combine the following:

- the dynamics of the i -th haptic device:

$$m_{HDi} \ddot{x}_{HDi} + b_{HDi} \dot{x}_{HDi} = F_{hi} - F_{Ci}, \quad (7)$$

- with the dynamics of the i -th SVO copy:

$$m_{O_i}\ddot{x}_{O_i} + b_{O_i}\dot{x}_{O_i} = F_{C_i} - F_{T_i}, \quad (8)$$

for all $i = 1, N$. In Equations (7) and (8): F_{h_i} is the force applied by Peer i to their haptic interface; F_{C_i} is the virtual coupling force at Peer i :

$$F_{C_i} = K_{C_i}(x_{HD_i} - x_{O_i}) + B_{C_i}(\dot{x}_{HD_i} - \dot{x}_{O_i}); \quad (9)$$

and F_{T_i} is the force applied by the AP coordination controller on the i -th SVO copy:

$$F_{T_i} = K_T(x_{O_i} - x_{O_{i_d}}) + B_T(\dot{x}_{O_i} - \dot{x}_{O_{i_d}}), \quad (10)$$

with $x_{O_{i_d}}$ and $\dot{x}_{O_{i_d}}$ being the desired position and velocity of the i -th SVO copy, respectively. In the AP strategy, they are computed via the following equations:

$$x_{O_{i_d}}(t) = \frac{\sum_{j=1, j \neq i}^N x_{O_j}(t - T_d)}{N - 1}$$

and

$$\dot{x}_{O_{i_d}}(t) = \frac{\sum_{j=1, j \neq i}^N \dot{x}_{O_j}(t - T_d)}{N - 1},$$

where T_d is the communication delay of the network. In this paper, the network delay T_d is assumed constant, equal in all communication channels, and an integer multiple M of the network packet update interval T_n , $T_d = MT_n$.

3. Stability of dual-rate haptic cooperation among up to five users

Since the typical network update interval is longer than the sampling interval required for realistic haptic rendering, the distributed haptic cooperation among N networked users is a dual rate closed-loop system. Its stability is controlled by the eigenvalues of its multi-rate state transition matrix. This matrix is computed using the lifting approach introduced in (Araki & Yamamoto, 1986) and first applied to haptic cooperation in (Fotoohi et al., 2007). The derivations are succinctly overviewed in this section.

The open-loop continuous-time state-space dynamics of N -users networked haptic cooperation combine the dynamics of the users, the haptic interfaces, and the SVO copies, and then group system inputs and outputs into fast and slow sub-vectors (hereafter indicated with the c and n indices, respectively). The inputs comprise the virtual coupling forces, updated at the fast haptic rate (Eq. (9)), and the SVO coordination forces, with components updated both fast and slow (Eq. (10)), grouped into:

$$\mathbf{u}^\top = (\mathbf{u}_c^\top \quad \mathbf{u}_n^\top)^\top, \quad (11)$$

where:

$$\mathbf{u}_c^\top = (F_{C1} \quad F_{T1_c} \quad \dots \quad F_{CN} \quad F_{TN_c})^\top, \quad (12)$$

$$\mathbf{u}_n^\top = (F_{T1_n} \quad \dots \quad F_{TN_n})^\top, \quad (13)$$

$$F_{T1_c} = K_T x_{O1} + B_T \dot{x}_{O1}, \quad (14)$$

and,

$$F_{T1_n} = -K_T x_{O1_d} - B_T \dot{x}_{O1_d}. \quad (15)$$

The state vector comprises the states of all haptic interfaces and SVO copies:

$$\mathbf{x}^\top = (\mathbf{x}_{\text{peer}_1} \quad \dots \quad \mathbf{x}_{\text{peer}_i} \quad \dots \quad \mathbf{x}_{\text{peer}_N})^\top, \quad (16)$$

where:

$$\mathbf{x}_{\text{peer}_i}^\top = (x_{\text{HD}_i} \quad \dot{x}_{\text{HD}_i} \quad x_{\text{O}_i} \quad \dot{x}_{\text{O}_i})^\top; \quad i = 1, \dots, N. \quad (17)$$

The output vector is:

$$\mathbf{y}^\top = (\mathbf{y}_c^\top \quad \mathbf{y}_n^\top)^\top, \quad (18)$$

where:

$$\mathbf{y}_c^\top = \mathbf{x}^\top \quad (19)$$

and,

$$\mathbf{y}_n^\top = (\mathbf{y}_{\text{peer}_{1n}} \quad \dots \quad \mathbf{y}_{\text{peer}_{in}} \quad \dots \quad \mathbf{y}_{\text{peer}_{Nn}})^\top \quad (20)$$

with,

$$\mathbf{y}_{\text{peer}_{in}}^\top = (x_{\text{O}_{id}} \quad \dot{x}_{\text{O}_{id}})^\top; \quad i = 1, \dots, N. \quad (21)$$

Hence, the open-loop continuous-time state-space dynamics of N -user networked haptic cooperation with AP coordination are as follows:

$$\begin{aligned} \dot{\mathbf{x}}_{4N \times 1} &= \mathbf{A}_{4N \times 4N} \mathbf{x}_{4N \times 1} + \mathbf{B}_{4N \times 3N} \mathbf{u}_{3N \times 1}, \\ \mathbf{y}_{6N \times 1} &= \mathbf{C}_{6N \times 4N} \mathbf{x}_{4N \times 1} \end{aligned} \quad (22)$$

and their discretization is obtained by lifting (Araki & Yamamoto, 1986), in the form of:

$$\begin{aligned} \mathbf{x}_{\text{D}_{(M,4N) \times 1}}[k+1] &= \mathbf{A}_{\text{D}_{(M,4N) \times (M,4N)}} \mathbf{x}_{\text{D}_{(M,4N) \times 1}}[k] \\ &\quad + \mathbf{B}_{\text{D}_{(M,4N) \times ((2M+1),N)}} \mathbf{u}_{\text{D}_{((2M+1),N) \times 1}}[k] \\ \mathbf{y}_{\text{D}_{(2M+1),2N \times 1}}[k] &= \hat{\mathbf{C}}_{\text{D}_{((2M+1),2N) \times (M,4N)}} \mathbf{x}_{\text{D}_{(M,4N) \times 1}}[k] \\ &\quad + \hat{\mathbf{D}}_{\text{D}_{((2M+1),2N) \times ((2M+1),N)}} \mathbf{u}_{\text{D}_{((2M+1),N) \times 1}}[k] \end{aligned} \quad (23)$$

The derivation of $\mathbf{A}_D$, $\mathbf{B}_D$, $\hat{\mathbf{C}}_D$ and $\hat{\mathbf{D}}_D$ and the incorporation of communication delays, via augmenting the state with the delayed inputs, are detailed in (Fotoohi et al., 2007).

The stability of the dual-rate distributed N -users networked haptic cooperation hinges on the eigenvalues of the closed-loop state transition matrix:

$$\mathbf{A}_D^{cl} = \mathbf{A}_{D_{aug}} + \mathbf{B}_{D_{aug}} \mathbf{F}_D (\mathbf{I} - \mathbf{D}_{D_{aug}} \mathbf{F}_D)^{-1} \mathbf{C}_{D_{aug}},$$

where $\mathbf{A}_{D_{aug}}$, $\mathbf{B}_{D_{aug}}$, $\mathbf{C}_{D_{aug}}$ and $\mathbf{D}_{D_{aug}}$ are the state transition matrices obtained after augmentation with computational and communication delays, and the feedback matrix $\mathbf{F}_D$ includes the virtual coupling and SVO coordination forces. The $\mathbf{F}_D$ is computed by using the approach introduced in (Araki & Yamamoto, 1986). Namely, the N -users haptic cooperation is stable if and only if all eigenvalues of $\mathbf{A}_D^{cl}$ are inside the unit circle.

Stability regions

This section presents a numerical investigation of the stability regions of dual-rate haptic cooperation among up to five users, with AP and PD coordination. The following parameters are used in the following computations: $m_{\text{HD}_i} = 0.1$ kg, $b_{\text{HD}_i} = 5.0$ Ns/m; $m_O = 0.45$ kg, $b_O = 0.0$ Ns/m, $T_c = 0.001$ s, $T_n = 0.008$ s. The haptic device parameters are selected to approximate the mass

Figure 3. Stability region for haptic cooperation among up to five users with PD coordination, and for haptic cooperation among up to eight users with AP coordination ($B_T = 0$ Ns/m).

Figure 4. Stability region for haptic cooperation among up to five users with PD coordination, and for haptic cooperation among up to eight users with AP coordination ($B_T = 5$ Ns/m).

and viscous damping of the Novint devices and the SVO parameters are chosen heuristically. Fig. 3 depicts the stability regions for undamped control, i.e., $B_T = B_{ci} = 0$ Ns/m. Fig. 4 shows the stability regions for damped control with $B_T = B_{ci} = 5$ Ns/m.

Figs. 3 and 4 show that the proposed AP coordination maintains the stability region of haptic cooperation largely unaffected by the number of interacting users, regardless of damping in the coordination and haptic rendering controllers. This is in contrast to the PD coordination, where stability region decreases as the number of peers grows.

4. Performance evaluation

In distributed multi-user haptic cooperation, the position discrepancy among the SVO copies threatens the fidelity of the force interactions. For example, suppose that Peer i tries to move their local copy of the SVO while other users are not in touch with their local copies. If the coordination of the N distributed SVO copies is perfect, Peer i imposes the same motion on all SVO copies and feels the multiple SVO copies as rigidly attached to each other. However, for coordination with limited gains, Peer i feels the SVO copies as connected to each other through springs with finite stiffness and with slowly moving ends (due to the delayed updates received across the network). This section uses the admittance $G_i(z) = \frac{F_{h_i}(z)}{\dot{x}_i(z)}$ of the SVO copies to compare the dynamics of the distributed SVO rendered to users by AP and PD coordination to the dynamics of an “ideal” virtual object. The G_i -parameters are computed after lifting the dual-rate system to its unirate counterpart.

The frequency responses of a SVO distributed among up to five users and coordinated via the AP and PD schemes are depicted in Fig. 5. The parameter values selected for the analysis are as follows: $m_O = 0.45$ kg; $b_O = 0$ Ns/m; $K_T = 1000$ N/m; $B_T = 5$ Ns/m respectively; and $T_d = T_n = 0.008$ s in all communication links.

Figure 5. SVO admittance for haptic cooperation among up to five users, with AP and PD coordination, and with communication delay $T_d = T_n = 0.008$ s in all links.

The deviations from the ideal frequency response show that the distributed SVO has larger viscous damping than the “ideal” SVO, due to the low packet update rate, delays in the communication

channel, and the damping of the coordination (Fotoohi et al., 2007). Nonetheless, Fig. 5 confirms that: (1) a distributed SVO with AP coordination has admittance closer to the admittance of a pure mass than a distributed SVO with PD coordination, and (2) damping of the distributed SVO rendered to users increases with peer count. This is expected given that the number of communication links increases, as each link introduces additional damping in the cooperation.

5. Experimental validation

This section validates the analytical results in the sections above through experiments performed on a testbed that comprises three Falcon Novint haptic devices connected to three computers. The computers all run Windows XP on an Intel Core 2 Duo CPU at 2.67 Ghz with 2 GB RAM, and communicate over a local area network via the UDP protocol with a data transmission rate of 125 Hz. The virtual environment is simulated at 1 kHz via a C++ console application and includes a shared virtual cube constrained by a virtual enclosure to move along a single horizontal x -direction, and three virtual spheres representing the haptic devices. Given the proximity of the three computers, the actual network delay is negligible. Therefore, a Wide Area Network Emulator (WANem) is used to implement a desired network delay T_d , equal in all communication channels. Fig. 6 depicts the experimental testbed with three cooperating users.

Figure 6. Experimental testbed.

In all experiments, the parameter values are as follows: The mass and damping of the SVO are $m_O = 0.45$ kg; and $b_O = 0$ Ns/m, respectively; the coordination damping is $B_T = 5$ Ns/m; the virtual coupling stiffness and damping are $K_C = 3500$ N/m and $B_C = 5$ Ns/m, respectively; and the round-trip network delay is $T_d = 0.048$ s.

5.1 Stability tests

In the experiments carried out in this section, two of the three users cooperatively manipulate the SVO by moving it back and forth along the rectilinear enclosure. The third user is present in the shared virtual environment, but is not in contact with the SVO. The two users who manipulate the SVO are instructed to keep it moving, without concern for the resultant motion. Nonetheless,

because of the small workspace of the Novint haptic devices, the two users take frequent turns at pushing the SVO, and the resulting SVO motion appears sinusoidal.

The coordination stiffness K_T is selected to verify that AP coordination increases the stability region of distributed three-users haptic cooperation as compared to PD coordination. The empirically chosen numerical value is $K_T = 2100$ N/m. For K_T as specified, the experimental results show that haptic cooperation is unstable with PD coordination (Fig. 7a) and is stable with AP coordination (Fig. 7b). Hence, the results in Fig. 7 confirm that, compared to the PD scheme, the AP strategy can keep the distributed three-user haptic cooperation stable for stiffer coordination.

Figure 7. Experimental three-users haptic cooperation ($K_T = 2100$ N/m). Note that the coordination forces appear to grow unbounded faster than the SVO positions for PD coordination only because of the plot ranges. In turn, these ranges are chosen to permit the comparative evaluation of the PD and AP controllers' effects on the haptic interaction.

5.2 Perceived viscosity tests

This section contrasts the performance of AP coordination to the performance of conventional PD coordination through controlled experiments. The coordination stiffness is set to $K_T = 1000$ N/m, to ensure that the three-users haptic cooperation is stable both for the PD and for the AP coordination schemes. To guarantee the same initial conditions during successive experiments, users are replaced by controlled forces applied to the haptic devices via commands sent to the servo motors.

At the beginning of each experiment, Peer 1 is at the right end of the rectilinear virtual enclosure and pushes the SVO with a constant force $F_{h1} = 1.5$ N. Peers 2 and 3 are not in contact with the SVO. Thus, the SVO travels along the enclosure to the left during the experiment. Fig. 8 depicting the SVO position at the Peer 1 site shows the following: the SVO reaches a constant velocity when

pushed by the constant $F_{h1} = 1.5$ N, regardless whether AP or PD coordination is used (i.e., both the AP and PD controllers render a viscously damped SVO to users), and the SVO moves 20 mm in 1.22 s (its velocity saturates slower) under AP coordination, and in 1.514 s (its velocity saturates faster) under PD coordination, (i.e., the AP controller renders a less viscous SVO than the PD controller), as predicted analytically.

Figure 8. Experimental three-users haptic cooperation with communication delay $T_d = 6T_n = 0.048$ s.

6. Conclusions

This paper has proposed the average position (AP) coordination for distributed multi-user networked haptic cooperation. Through continuous-time analysis of the autonomous dynamics of an SVO distributed among N users, as well as through multirate analysis of the closed-loop state-space dynamics of up to five-users haptic cooperation, this paper has shown that the AP strategy bounds the coordination gain of the SVO, and thus, increases the stability region of distributed multi-user haptic cooperation as compared to conventional PD coordination. Using numerical analysis, this paper has illustrated that AP coordination is also beneficial to cooperation transparency. Namely, the AP strategy injects less damping in the local feedback loops and therefore renders the SVO dynamics more accurately than (and enhances the performance of the interaction compared to) conventional PD coordination. Experiments in which three users manipulate a shared virtual cube together have validated the analytical results.

Upcoming work will study the performance of AP coordination in virtual environments relevant to applications, such as multi-user haptic computer games for tele-rehabilitation and education. In these applications, haptic tasks will be defined in collaboration with therapists and/or educators to help cooperating users achieve specific therapeutic and/or educational goals. Future work will also investigate techniques for guaranteeing the stability of distributed haptic cooperation among an arbitrary number of users, across networks with varying communication delay and packet loss, and for SVO with coupled multi degrees-of-freedom dynamics.

Acknowledgements

This work was supported through an NSERC Discovery Grant.

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