ON SATURATED PARTITIONS

Abstract

Stephan-Ylan Solomon

Let L be a language over a vocabulary V and let denote by $E(V)$ the set of all equivalence relations (partitions) on V . If $\rho \in E(V)$ and $x \in V$ then we shall denote by $\rho(x)$ the cell of ρ containing the element x .

Definition 1. A partition $\rho \in E(V)$ is said to be saturated if for every marked ρ -structure $\rho(x_1) \rho(x_2) \dots \rho(x_n)$ and for every i ($1 \leq i \leq n$) there exist such elements x'_j ($j=1, \dots, i-1, i+1, \dots, n$) that $x'_j \in \rho(x_j)$ and $x'_1 \dots x'_{i-1} x_i x'_{i+1} \dots x'_n \in L$.

Our purpose is to find (in the case of a finite vocabulary) the greatest saturated partition Z of the language.

Definition 2. Let $\rho \in E(V)$ and $x, y \in V$. We shall say that x ρ -dominates y ($x \xrightarrow{\rho} y$) if for every string $x_1 \dots x_n \in L$ where $x_1 = x$ there exist such elements x'_j ($j=1, \dots, i-1, i+1, \dots, n$) that $x'_j \in \rho(x_j)$ and $x'_1 \dots x'_{i-1} y x'_{i+1} \dots x'_n \in L$.

We can introduce now the partition ρ^* , called the asterisk of the partition ρ : $x \rho^* y$ if both $x \xrightarrow{\rho} y$ and $y \xrightarrow{\rho} x$ hold true.

Theorem 1. ρ is a saturated partition if and only if ρ is finer than ρ^* .

The connection between the asterisk and the derivative of a partition is given by:

Theorem 2. In order that $\rho' = \rho^*$ it is necessary and sufficient that ρ be saturated.

By using the notation: $\rho^{o*} = \rho$ and $\rho^{(n)*} = (\rho^{(n-1)*})^*$ we have :

Theorem 3. There exists a natural number n so that $Z = \rho^{n*}$ (where ∞ is the improper partition of V).