Probabilistic Graphical Models for Climate Data Analysis

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Graphical Models

- Graphical models
 - Dependencies between (random) variables, avoid I.I.D. assumptions
 - Closer to reality, learning/inference is much more difficult
- Basic nomenclature
 - Node = Random Variable, Edge = Statistical Dependency
- Directed Graphs
 - A *directed* graph between random variables
 - Example: Bayesian networks, Hidden Markov Models
 - Joint distribution is a product of $P(\text{child}|\text{parents})$
- Undirected Graphs
 - An *undirected* graph between random variables
 - Example: Markov/Conditional random fields
 - Joint distribution in terms of potential functions

Example: Burglary Network

Latent Variable Models

- Bayesian network with hidden variables
 - Semantically more accurate, less parameters

Key Problems

- Structure Learning
 - Given: **Samples**
 - Problem: Learn the **Structure**
- Parameter Estimation
 - Given: **Samples** and **Structure**
 - Problem: Estimate Parameters
- Inference:
 - Given: **Structure**, **Parameters**, and some variables (part of a **Sample**)
 - Problem: Find other variables (part of a **Sample**)

Abrupt Changes in Climate

- Significant Droughts
 - Persistent over space and time
 - Catastrophic consequences
- Examples:
 - Late 1960s Sahel drought
 - 1930s North American Dust Bowl

CRU Dataset

- Monthly precipitation dataset from 1901-2006
- 0.5 latitude x 0.5 longitude
- Precipitation over land only (67420 locations)
- Convert to a yearly dataset to eliminate the monthly variance

Markov Random Fields (MRF)

- Undirected graph $G = (V, E)$
- Each node takes discrete values from $\mathcal{X} = \{1, \dots, k\}$
- Node potential $f_u(x_u)$
- Edge potential $f_{uv}(x_u, x_v)$
- Joint distribution over all variables

$$P(x) \propto \exp \left\{ \sum_{u \in V} f_u(x_u) + \sum_{(u,v) \in E} f_{uv}(x_u, x_v) \right\}$$

- MAP inference problem

$$x^* \in \operatorname{argmax}_{x \in \mathcal{X}^n} \left\{ \sum_{u \in V} f_u(x_u) + \sum_{(u,v) \in E} f_{uv}(x_u, x_v) \right\}$$

Drought Detection with MAP Inference

- Model dependencies using a 4-nearest neighbor grid
 - Replicate grid over time
 - Each node can be 0 (normal) or 1 (dry)
- Toy example:
 - $m = 3, n = 4, N = 12, T = 5$
 - Total # States: $2^{60} = 1.1529 \times 10^{18}$
- CRU data:
 - $m = 720, n = 360, N = 67,420, T = 106$
 - Total # States: $2^{7,146,520} > 10^{2,382,200}$
- Drought Detection
 - MAP inference: Integer program with millions of variables
 - Linear Programming (LP) relaxation
 - Post-process the MAP states to identify significant droughts

Pre-Processing

- Arid and semi-arid regions are more likely to have droughts
- Calculate the median precipitation for each location
- Cluster the medians
- Assign different percentiles for each cluster

Results: Droughts starting in 1920-30s

Results: Droughts starting in 1960-70s

Significant Droughts: 1901-2006

Land Variable Regression

- Land-Sea variable interactions
 - How do sea variables affect land variables?
 - Are proximal locations important?
 - Are there long range spatial dependencies (tele-connections)?
- NCEP/NCAR Reanalysis 1: monthly means for 1948-2010
 - Covariates: Temperature, Sea Level Pressure, Precipitation, Relative Humidity, Horizontal Wind Speed Vertical Wind Speed
 - Response: Temperature, Precipitation at 9 locations
- High-dimensional Regression
 - Dimension $p = Lm$, L locations, m variables/location
 - Two considerations:
 - Not all locations are relevant
 - Even if a location is relevant, not all variables are relevant

Land Regions for Prediction: Temp, Precip

Land regions chosen for prediction

Dataset and Assumptions

- NCEP/NCAR Reanalysis 1: monthly means for 1948-2010
 - Coarsened to $10^\circ \times 10^\circ$ spatial resolution
 - Anomaly series computed from data
 - Training Set : 1948-1997
 - Test Set : 1998-2007
- Monthly means
 - Consider dependencies over Space
 - Considered independent over Time

Ordinary Least Squares

- Naive method: Use *ordinary least squares* (OLS) :

$$\hat{\theta} := \operatorname{argmin}_{\theta \in \mathbb{R}^{mL}} \left\{ \frac{1}{n} \|y - X\theta\|_2^2 \right\}$$

- For $n < mL$, OLS estimate non-unique
- In general, y depends on all mL covariates : complex model

Sparse Regression

- Add sparsity inducing norm regularizer to OLS
 - Examples: L_1 norm, L_1/L_q mixed norm, *hierarchical* norms
- Sparse Regression
 - Simultaneous feature selection and regression
 - Statistically consistent in the “ $n \ll p$ ” regime
 - Avoiding the Curse of Dimensionality
- Sparse regularizer with two considerations
 - Not all locations are relevant
 - Even if a location is relevant, not all variables are relevant

Sparse Group Lasso (SGL)

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- High-dimensional Regression with “Sparse” “Group Sparsity”

$$\hat{\theta}_{SGL} = \operatorname{argmin}_{\theta \in \mathbb{R}^{mL}} \left\{ \frac{1}{n} \|y - X\theta\|_2^2 + \lambda_1 \|\theta\|_1 + \lambda_2 \|\theta\|_{1,\mathcal{G}} \right\}$$

$$\|\theta\|_1 = \sum_{i=1}^{mL} |\theta_i| \quad \|\theta\|_{1,\mathcal{G}} = \sum_{k=1}^L \|\theta_{G_k}\|_2$$

$\mathcal{G} = \{G_1, \dots, G_L\}$: groups of m variables at L locations

- Efficient Optimization
 - Primal: Proximal gradient method
 - Dual: Block-coordinate dual ascent

Consistency of Sparse Group Lasso

Theorem 1 *Let A be any subspace of $\mathbb{R}^p$ of dimension s_A . Let θ^* be the optimal (unknown) regression parameter, and let $r_{(1, \mathcal{G}_2, \alpha)}^{A^\perp}(\theta^*)$ be the sparse-group lasso norm of θ^* restricted to $A^\perp$, the orthogonal subspace of A . Then, if λ_n satisfies the lower bound in Lemma 1, with probability at least $(1 - \frac{2}{(pT)^k})$, we have*

$$\|\hat{\theta}_{\lambda_n} - \theta^*\|_2^2 \leq \frac{4\lambda_n^2}{k_{\mathcal{L}}^2} s_A + \frac{2\lambda_n}{k_{\mathcal{L}}} r_{(1, \mathcal{G}_2, \alpha)}^{A^\perp}(\theta^*),$$

where $\hat{\theta}_{\lambda_n}$ is the SGL estimator.

Corollary 1 *If the optimal parameter θ^* is in the subspace A , then*

$$\|\hat{\theta}_{\lambda_n} - \theta^*\|_2^2 \leq \frac{4\lambda_n^2}{k_{\mathcal{L}}^2} s_A = O\left(\frac{\log p}{n}\right).$$

Prediction RMSE

Variable	Region	SGL	Network Clusters [4]	OLS
Air Temperature	Brazil	0.198	0.534	0.348
	Peru	0.247	0.468	0.387
	West USA	0.270	0.767	0.402
	East USA	0.304	0.815	0.348
	W Europe	0.379	0.936	0.493
	Sahel	0.320	0.685	0.413
	S Africa	0.136	0.726	0.267
	India	0.205	0.649	0.300
	SE Asia	0.298	0.541	0.383
Precipitation	Brazil	0.261	0.509	0.413
	Peru	0.312	0.864	0.523
	West USA	0.451	0.605	0.549
	East USA	0.365	0.686	0.413
	W Europe	0.358	0.45	0.551
	Sahel	0.427	0.533	0.523
	S Africa	0.235	0.697	0.378
	India	0.146	0.672	0.264
	SE Asia	0.159	0.665	0.312

SGL Regression: Brazil

Relevant Covariates for Temperature in **Brazil**

SGL Regression: India

Relevant Covariates for Temperature in **India**

Statistical Networks

correlated points and precision path for years 1986–1987 (El Nino)

Challenges and Open Questions

- Abrupt Change Detection
 - Detection: preprocessing, multivariate analysis, efficiency
 - Attribution: multivariate, spatiotemporal, causal
 - Abrupt Changes vs Extremes (intensity, duration, frequency)
- Sparse Regression, Statistical Networks
 - Observable Setting
 - Computing the precision matrix, Multivariate extensions
 - Spatiotemporal dependencies, Oscillations
 - Partially observable setting
 - Problem formulation
 - Latent variable models
 - Tele-connections, Attributions

Conclusions

- Graphical Models
 - Works with dependencies between variables
 - Key problems: structure and parameter learning, inference
- Abrupt Change Detection
 - Inference in discrete MRF
 - High dimensional integer/linear program
 - Detection of significant droughts from 1901-2006
- Sparse Regression and Networks
 - Sparse group lasso for multivariate regression
 - Statistical networks based on dependencies
- Key challenges
 - Feynman's 3-step 'algorithm' for problem solving
 - Questions+Expertise+Data from Climate Scientists

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