ϵ -shotgun: ϵ -greedy Batch Bayesian Optimisation

Supplementary Material

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1 INTRODUCTION

In this supplementary paper, we show the formulae of each of the synthetic functions used in this work and present the full batch optimisation results and convergence plots with all batch sizes $q \in \{2, 5, 10, 20\}$ for the synthetic, robot pushing and pipe shape optimisation problems.

2 SYNTHETIC FUNCTION DETAILS

In the following section we give the formulae of each of the 10 synthetic functions optimised in this work. Where functions have been modified from their original form, we label the original functions as $g(\mathbf{x})$ and minimised function as $f(\mathbf{x})$.

2.1 WangFreitas

$$g(x) = 2 \exp \left(-\frac{1}{2} \left(\frac{x-a}{\theta_1} \right)^2 \right) + 4 \exp \left(-\frac{1}{2} \left(\frac{x-b}{\theta_2} \right)^2 \right) \quad (1)$$

$$f(x) = -g(x), \quad (2)$$

where $a = 0.1$, $b = 0.9$, $\theta_1 = 0.1$ and $\theta_2 = 0.01$.

2.2 Branin

$$f(\mathbf{x}) = a(x_2 - bx_1^2 + cx_1 - r)^2 + s(1 - t) \cos(x_1) + s, \quad (3)$$

where $a = 1$, $b = \frac{5.1}{4\pi^2}$, $c = \frac{5}{\pi}$, $r = 6$, $s = 10$, $t = \frac{1}{8\pi}$ and x_i refers to the i -th element of vector $\mathbf{x}$.

2.3 BraninForrester

$$f(\mathbf{x}) = a(x_2 - bx_1^2 + cx_1 - r)^2 + s(1 - t) \cos(x_1) + s + 5x_1, \quad (4)$$

where $a = 1$, $b = \frac{5.1}{4\pi^2}$, $c = \frac{5}{\pi}$, $r = 6$, $s = 10$, and $t = \frac{1}{8\pi}$.

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2.4 Cosines

$$g(\mathbf{x}) = 1 - \sum_{i=1}^2 \left[(1.6x_i - 0.5)^2 - 0.3 \cos(3\pi(1.6x_i - 0.5)) \right] \quad (5)$$

$$f(\mathbf{x}) = -g(\mathbf{x}). \quad (6)$$

2.5 logGoldsteinPrice

$$g(\mathbf{x}) = (1 + (x_1 + x_2 + 1)^2 \times (19 - 14x_1 + 3x_1^2 - 14x_2 + 6x_1x_2 + 3x_2^2)) \times (30 + (2x_1 - 3x_2)^2) \quad (7)$$

$$\times (18 - 32x_1 + 12x_1^2 + 48x_2 - 36x_1x_2 + 27x_2^2) \\ f(\mathbf{x}) = \log(g(\mathbf{x})). \quad (8)$$

2.6 logSixHumpCamel

$$g(\mathbf{x}) = \left(4 - 2.1x_1^2 + \frac{x_1^4}{3} \right) x_1^2 + x_1x_2 + (-4 + 4x_2^2) x_2^2 \quad (9)$$

$$f(\mathbf{x}) = \log(g(\mathbf{x}) + a + b), \quad (10)$$

where $a = 1.0316$ and $b = 10^{-4}$. Note that because $g(\mathbf{x})$ has a minimum value of -1.0316 , we add a plus a small constant (b) to avoid taking the logarithm of a negative number; this does not change the function's landscape.

2.7 modHartman6

$$g(\mathbf{x}) = - \sum_{i=1}^4 \alpha_i \exp \left(- \sum_{j=1}^6 A_{ij} (x_j - P_{ij})^2 \right) \quad (11)$$

$$f(\mathbf{x}) = -\log(-g(\mathbf{x})) \quad (12)$$

where

$$\alpha = (1.0, 1.2, 3.0, 3.2)^T \quad (13)$$

$$\mathbf{A} = \begin{pmatrix} 10 & 3 & 17 & 3.50 & 1.7 & 8 \\ 0.05 & 10 & 17 & 0.1 & 8 & 14 \\ 3 & 3.5 & 1.7 & 10 & 17 & 8 \\ 17 & 8 & 0.05 & 10 & 0.1 & 14 \end{pmatrix} \quad (14)$$

$$\mathbf{P} = 10^{-4} \begin{pmatrix} 1312 & 1696 & 5569 & 124 & 8283 & 5886 \\ 2329 & 4135 & 8307 & 3736 & 1004 & 9991 \\ 2348 & 1451 & 3522 & 2883 & 3047 & 6650 \\ 4047 & 8828 & 8732 & 5743 & 1091 & 381 \end{pmatrix}. \quad (15)$$

2.8 logGSobol

$$g(\mathbf{x}) = \prod_{i=1}^D \frac{4x_i - 1}{2} \quad (16)$$

$$f(\mathbf{x}) = \log(g(\mathbf{x})), \quad (17)$$

where $a = 1$ and $D = 10$.

2.9 logRosenbrock

$$g(\mathbf{x}) = \sum_{i=1}^{D-1} \left[100 (x_{i+1} - x_i^2)^2 + (x_i - 1)^2 \right] \quad (18)$$

$$f(\mathbf{x}) = \log (g(\mathbf{x}) + 0.5), \quad (19)$$

where $D = 10$. Note, similarly to logSixHumpCamel, because $g(\mathbf{x})$ has a minimum value of 0, we add a value to ensure it is always positive.

2.10 logStyblinskiTang

$$g(\mathbf{x}) = \frac{1}{2} \sum_{i=1}^D (x_i^4 - 16x_i^2 + 5x_i) \quad (20)$$

$$f(\mathbf{x}) = \log (g(\mathbf{x}) + 40D), \quad (21)$$

where $D = 10$. Since $g(\mathbf{x})$ has a minimum value of $-39.16599D$, we add $40D$ to it to ensure it is always positive.

3 ADDITIONAL RESULTS

In the following we display the full set of results for the experimental evaluations carried out in this paper. In Section 3.1 we display tabulated results for the synthetic functions with batch sizes $q \in \{2, 5, 20\}$ and show convergence plots for all batch sizes. Similarly, in Sections 3.2 and 3.3 we display tabulated results and convergence plots for the robot pushing and PitzDaily real-world test problems.

3.1 Synthetic functions

Method	WangFreitas (1)		BraninForrester (2)		Branin (2)		Cosines (2)		logGoldsteinPrice (2)	
	Median	MAD	Median	MAD	Median	MAD	Median	MAD	Median	MAD
LP	2.00	2.15×10^{-8}	4.23×10^{-5}	5.66×10^{-5}	3.24×10^{-5}	4.30×10^{-5}	1.73×10^{-4}	2.44×10^{-4}	5.56×10^{-5}	7.75×10^{-5}
PLAyBOOK	2.00	3.73×10^{-9}	3.16×10^{-5}	4.28×10^{-5}	3.07×10^{-5}	4.06×10^{-5}	1.33×10^{-4}	1.76×10^{-4}	3.71×10^{-5}	3.73×10^{-5}
KB	2.00	7.68×10^{-9}	1.07×10^{-4}	1.47×10^{-4}	2.42×10^{-5}	2.99×10^{-5}	6.54×10^{-4}	6.64×10^{-4}	4.02×10^{-2}	4.70×10^{-2}
qEI	2.00	9.01×10^{-11}	2.47×10^{-7}	3.49×10^{-7}	3.26×10^{-6}	3.94×10^{-6}	1.09×10^{-5}	1.39×10^{-5}	9.33×10^{-5}	1.31×10^{-4}
TS	2.00	5.68×10^{-9}	1.14×10^{-4}	1.17×10^{-4}	9.04×10^{-5}	9.53×10^{-5}	2.69×10^{-4}	3.23×10^{-4}	2.04×10^{-4}	2.23×10^{-4}
ϵ S-RS (0.1)	2.00	5.66×10^{-8}	6.80×10^{-6}	1.00×10^{-5}	3.21×10^{-6}	3.90×10^{-6}	3.56×10^{-6}	5.11×10^{-6}	9.71×10^{-7}	1.42×10^{-6}
ϵ S-PF (0.1)	2.00	3.70×10^{-13}	2.07×10^{-6}	2.95×10^{-6}	2.43×10^{-6}	2.45×10^{-6}	4.26×10^{-6}	6.12×10^{-6}	9.39×10^{-8}	1.31×10^{-7}
ϵ S-0	2.00	2.52×10^{-9}	7.58×10^{-6}	1.09×10^{-5}	2.46×10^{-6}	3.01×10^{-6}	1.85×10^{-6}	2.37×10^{-6}	1.15×10^{-7}	1.66×10^{-7}
Method	logSixHumpCamel (2)		modHartman6 (6)		logGSobol (10)		logRosenbrock (10)		logStyblinskiTang (10)	
	Median	MAD	Median	MAD	Median	MAD	Median	MAD	Median	MAD
LP	1.29×10^{-1}	1.44×10^{-1}	5.90×10^{-4}	6.04×10^{-4}	7.07	1.84	5.96	1.69	2.17	4.35×10^{-1}
PLAyBOOK	1.12×10^{-1}	1.22×10^{-1}	5.22×10^{-4}	5.69×10^{-4}	8.01	1.84	5.44	1.42	2.11	2.55×10^{-1}
KB	4.86	1.07	4.53×10^{-3}	2.89×10^{-3}	7.64	1.21	4.55	1.42	2.08	3.34×10^{-1}
qEI	1.78×10^{-2}	1.92×10^{-2}	2.72×10^{-3}	2.27×10^{-3}	6.83	1.57	5.36	1.57	2.07	2.70×10^{-1}
TS	3.28×10^{-1}	3.05×10^{-1}	2.40×10^{-2}	1.04×10^{-2}	9.84	9.55×10^{-1}	8.08	3.93×10^{-1}	2.85	1.52×10^{-1}
ϵ S-RS (0.1)	7.64×10^{-5}	8.01×10^{-5}	4.73×10^{-4}	6.02×10^{-4}	9.43	3.01	3.62	1.01	1.81	3.65×10^{-1}
ϵ S-PF (0.1)	1.13×10^{-4}	1.26×10^{-4}	6.67×10^{-4}	8.69×10^{-4}	8.46	1.92	4.91	2.61	1.81	4.67×10^{-1}
ϵ S-0	1.61×10^{-4}	2.28×10^{-4}	8.30×10^{-4}	8.45×10^{-4}	9.24	2.10	3.68	1.04	1.81	3.64×10^{-1}

Table 1. Optimisation results with a batch size of $q = 2$. Median absolute distance from the optimum (left) and median absolute deviation from the median (MAD, right) after 100 batches (200 function evaluations) across the 51 runs. The method with the lowest median performance is shown in dark grey, with those with statistically equivalent performance shown in light grey.

Method	WangFreitas (1)		BraninForrester (2)		Branin (2)		Cosines (2)		logGoldsteinPrice (2)	
	Median	MAD	Median	MAD	Median	MAD	Median	MAD	Median	MAD
LP	2.00	9.13×10^{-10}	5.39×10^{-5}	7.43×10^{-5}	1.17×10^{-5}	1.49×10^{-5}	5.16×10^{-4}	7.04×10^{-4}	1.01×10^{-4}	1.16×10^{-4}
PLAyBOOK	2.00	3.72×10^{-10}	2.48×10^{-5}	3.66×10^{-5}	2.15×10^{-5}	3.02×10^{-5}	2.97×10^{-3}	4.18×10^{-3}	4.93×10^{-4}	6.12×10^{-4}
KB	2.00	2.96×10^{-9}	5.19×10^{-4}	7.32×10^{-4}	3.03×10^{-5}	2.65×10^{-5}	1.09×10^{-3}	1.31×10^{-3}	4.55×10^{-2}	5.87×10^{-2}
qEI	1.18×10^{-8}	1.69×10^{-8}	9.61×10^{-7}	9.95×10^{-7}	2.30×10^{-6}	1.90×10^{-6}	1.31×10^{-4}	1.41×10^{-4}	8.41×10^{-5}	1.00×10^{-4}
TS	2.00	1.32×10^{-8}	2.35×10^{-4}	2.52×10^{-4}	1.21×10^{-4}	1.11×10^{-4}	4.71×10^{-4}	5.93×10^{-4}	6.26×10^{-4}	5.77×10^{-4}
eS-RS (0.1)	2.00	6.66×10^{-11}	1.02×10^{-6}	1.47×10^{-6}	1.53×10^{-6}	1.66×10^{-6}	1.77×10^{-6}	2.28×10^{-6}	1.81×10^{-6}	2.61×10^{-6}
eS-PF (0.1)	2.00	3.59×10^{-13}	9.26×10^{-7}	1.35×10^{-6}	1.31×10^{-6}	1.23×10^{-6}	1.42×10^{-6}	1.52×10^{-6}	3.82×10^{-7}	5.65×10^{-7}
eS-0	2.00	4.09×10^{-11}	2.59×10^{-6}	3.80×10^{-6}	2.52×10^{-6}	2.73×10^{-6}	8.21×10^{-7}	1.01×10^{-6}	3.60×10^{-7}	5.09×10^{-7}
Method	logSixHumpCamel (2)		modHartman6 (6)		logGSobol (10)		logRosenbrock (10)		logStyblinskiTang (10)	
	Median	MAD	Median	MAD	Median	MAD	Median	MAD	Median	MAD
LP	1.58×10^{-1}	2.14×10^{-1}	8.77×10^{-4}	9.82×10^{-4}	8.16	2.07	5.84	1.14	2.18	3.77×10^{-1}
PLAyBOOK	1.55×10^{-1}	1.79×10^{-1}	7.03×10^{-4}	7.37×10^{-4}	8.53	2.05	5.69	1.05	2.26	3.15×10^{-1}
KB	4.69	1.14	6.10×10^{-3}	6.06×10^{-3}	7.10	1.62	4.68	1.67	2.09	3.39×10^{-1}
qEI	5.04×10^{-2}	5.54×10^{-2}	3.77×10^{-3}	2.73×10^{-3}	8.48	2.22	7.06	1.10	2.12	3.58×10^{-1}
TS	5.66×10^{-1}	4.71×10^{-1}	2.87×10^{-2}	1.04×10^{-2}	1.03×10^1	8.36×10^{-1}	8.11	4.52×10^{-1}	2.85	1.08×10^{-1}
eS-RS (0.1)	5.05×10^{-5}	5.35×10^{-5}	2.85×10^{-4}	2.85×10^{-4}	8.27	2.08	5.60	1.94	1.81	3.63×10^{-1}
eS-PF (0.1)	2.28×10^{-4}	3.17×10^{-4}	4.03×10^{-4}	5.00×10^{-4}	7.83	2.24	3.99	1.45	1.81	4.84×10^{-1}
eS-0	3.95×10^{-4}	5.73×10^{-4}	4.51×10^{-4}	5.05×10^{-4}	7.86	1.85	3.97	1.39	1.81	3.63×10^{-1}

Table 2. Optimisation results with a batch size of $q = 5$. Median absolute distance from the optimum (left) and median absolute deviation from the median (MAD, right) after 40 batches (200 function evaluations) across the 51 runs. The method with the lowest median performance is shown in dark grey, with those with statistically equivalent performance shown in light grey.

Method	WangFreitas (1)		BraninForrester (2)		Branin (2)		Cosines (2)		logGoldsteinPrice (2)	
	Median	MAD	Median	MAD	Median	MAD	Median	MAD	Median	MAD
LP	2.00	1.10×10^{-9}	2.80×10^{-6}	3.93×10^{-6}	3.81×10^{-6}	4.89×10^{-6}	2.25×10^{-2}	2.90×10^{-2}	1.90×10^{-3}	2.70×10^{-3}
PLAyBOOK	8.28×10^{-5}	1.23×10^{-4}	9.99×10^{-6}	1.45×10^{-5}	3.36×10^{-6}	3.97×10^{-6}	1.07×10^{-1}	1.58×10^{-1}	1.68×10^{-3}	2.44×10^{-3}
KB	8.58×10^{-7}	1.26×10^{-6}	2.96×10^{-7}	3.59×10^{-7}	1.14×10^{-6}	1.09×10^{-6}	2.83×10^{-5}	4.13×10^{-5}	2.27×10^{-3}	2.83×10^{-3}
qEI	1.79×10^{-7}	2.56×10^{-7}	6.18×10^{-5}	5.57×10^{-5}	4.19×10^{-5}	4.14×10^{-5}	1.21×10^{-3}	1.65×10^{-3}	3.57×10^{-4}	3.86×10^{-4}
TS	2.00	1.60×10^{-7}	9.09×10^{-4}	9.60×10^{-4}	6.50×10^{-4}	5.91×10^{-4}	2.48×10^{-3}	1.96×10^{-3}	2.96×10^{-3}	3.03×10^{-3}
eS-RS (0.1)	2.00	2.07×10^{-8}	8.87×10^{-7}	1.24×10^{-6}	1.73×10^{-6}	1.90×10^{-6}	7.06×10^{-7}	1.01×10^{-6}	3.02×10^{-6}	4.43×10^{-6}
eS-PF (0.1)	2.00	2.15×10^{-11}	1.49×10^{-6}	2.16×10^{-6}	2.24×10^{-6}	2.49×10^{-6}	1.14×10^{-6}	1.55×10^{-6}	7.42×10^{-7}	9.61×10^{-7}
eS-0	2.00	4.01×10^{-11}	7.46×10^{-7}	8.54×10^{-7}	1.69×10^{-6}	1.72×10^{-6}	6.06×10^{-7}	8.44×10^{-7}	9.03×10^{-7}	1.31×10^{-6}
Method	logSixHumpCamel (2)		modHartman6 (6)		logGSobol (10)		logRosenbrock (10)		logStyblinskiTang (10)	
	Median	MAD	Median	MAD	Median	MAD	Median	MAD	Median	MAD
LP	5.35×10^{-1}	5.93×10^{-1}	3.24×10^{-3}	4.31×10^{-3}	8.32	1.58	6.68	8.03×10^{-1}	2.13	3.72×10^{-1}
PLAyBOOK	6.35×10^{-1}	6.76×10^{-1}	2.40×10^{-2}	3.14×10^{-2}	1.00×10^1	2.37	6.29	1.32	2.25	2.92×10^{-1}
KB	1.65	1.36	2.51×10^{-4}	2.92×10^{-4}	8.17	1.83	5.49	1.56	2.11	3.74×10^{-1}
qEI	4.98×10^{-1}	3.87×10^{-1}	2.79×10^{-2}	1.73×10^{-2}	1.06×10^1	1.77	7.97	6.08×10^{-1}	2.34	2.06×10^{-1}
TS	1.19	9.82×10^{-1}	5.04×10^{-2}	1.47×10^{-2}	1.07×10^1	1.21	8.63	4.04×10^{-1}	2.84	1.47×10^{-1}
eS-RS (0.1)	4.44×10^{-3}	6.56×10^{-3}	2.08×10^{-4}	2.62×10^{-4}	8.71	2.45	5.34	1.26	1.89	2.93×10^{-1}
eS-PF (0.1)	1.04×10^{-2}	1.53×10^{-2}	1.57×10^{-4}	1.79×10^{-4}	8.16	3.04	5.60	1.20	2.05	3.48×10^{-1}
eS-0	2.55×10^{-2}	3.77×10^{-2}	1.87×10^{-4}	2.24×10^{-4}	8.90	2.94	5.00	1.35	2.05	3.45×10^{-1}

Table 3. Optimisation results with a batch size of $q = 20$. Median absolute distance from the optimum (left) and median absolute deviation from the median (MAD, right) after 10 batches (200 function evaluations) across the 51 runs. The method with the lowest median performance is shown in dark grey, with those with statistically equivalent performance shown in light grey.

Fig. 1. Illustrative convergence plots for the WangFreitas ($d = 1$), BraninForrester ($d = 2$), Branin ($d = 2$) and Cosines ($d = 2$) benchmark problems (rows) for four batch sizes $q \in \{5, 10, 20\}$ (columns). Each plot shows the median difference between the best function value seen and the true optimum, with shading representing the interquartile range across the 51 runs.

Fig. 2. Illustrative convergence plots for the $\log\text{GoldsteinPrice}$ ($d=1$), $\log\text{SixHumpCamel}$ ($d=2$), modHartman6 ($d=6$) and $\log\text{GSobol}$ ($d=10$) benchmark problems (rows) for four batch sizes $q \in \{2, 5, 10, 20\}$ (columns). Each plot shows the median difference between the best function value seen and the true optimum, with shading representing the interquartile range across the 51 runs.

Fig. 3. Illustrative convergence plots for the logRosenbrock ($d = 10$) and logStyblinskiTang ($d = 10$) benchmark problems (rows) for four batch sizes $q \in \{5, 10, 20\}$ (columns). Each plot shows the median difference between the best function value seen and the true optimum, with shading representing the interquartile range across the 51 runs.

3.2 Active Learning for Robot Pushing

Method	PUSH4 (q=2)		PUSH4 (q=5)		PUSH4 (q=10)		PUSH4 (q=20)	
	Median	MAD	Median	MAD	Median	MAD	Median	MAD
LP	1.78×10^{-1}	1.30×10^{-1}	1.84×10^{-1}	1.14×10^{-1}	1.71×10^{-1}	1.27×10^{-1}	2.32×10^{-1}	1.87×10^{-1}
PLAyBOOK	1.97×10^{-1}	1.19×10^{-1}	1.66×10^{-1}	1.48×10^{-1}	1.36×10^{-1}	1.19×10^{-1}	2.18×10^{-1}	2.49×10^{-1}
KB	1.50×10^{-1}	8.05×10^{-2}	1.52×10^{-1}	1.05×10^{-1}	1.97×10^{-1}	1.18×10^{-1}	1.53×10^{-1}	9.83×10^{-2}
qEI	1.33×10^{-1}	8.07×10^{-2}	2.00×10^{-1}	1.09×10^{-1}	2.29×10^{-1}	1.22×10^{-1}	2.54×10^{-1}	1.42×10^{-1}
TS	2.89×10^{-1}	1.77×10^{-1}	3.05×10^{-1}	2.08×10^{-1}	3.38×10^{-1}	2.03×10^{-1}	3.39×10^{-1}	1.70×10^{-1}
εS-RS (0.1)	1.02×10^{-2}	9.70×10^{-3}	3.11×10^{-2}	2.95×10^{-2}	5.53×10^{-2}	4.84×10^{-2}	1.55×10^{-1}	1.38×10^{-1}
εS-PF (0.1)	1.71×10^{-2}	1.27×10^{-2}	2.73×10^{-2}	2.73×10^{-2}	4.65×10^{-2}	4.36×10^{-2}	1.37×10^{-1}	1.31×10^{-1}
εS-0	1.05×10^{-2}	1.03×10^{-2}	2.30×10^{-2}	2.03×10^{-2}	5.44×10^{-2}	5.80×10^{-2}	1.02×10^{-1}	9.97×10^{-2}

Method	PUSH8 (q=2)		PUSH8 (q=5)		PUSH8 (q=10)		PUSH8 (q=20)	
	Median	MAD	Median	MAD	Median	MAD	Median	MAD
LP	2.28	1.43	2.06	1.46	2.32	1.55	2.71	1.86
PLAyBOOK	1.92	1.58	2.42	1.58	2.60	1.73	2.85	1.62
KB	2.46	1.31	2.38	1.91	2.05	1.44	2.17	1.51
qEI	1.82	1.41	2.25	1.65	2.24	1.20	1.97	1.45
TS	2.96	1.61	3.18	1.82	2.51	1.72	3.05	2.27
εS-RS (0.1)	1.50	2.08	1.82	1.27	2.26	2.00	1.85	1.58
εS-PF (0.1)	1.86	1.54	2.00	1.34	1.65	1.51	1.76	1.46
εS-0	1.90	1.86	2.00	1.75	2.05	1.73	2.24	2.02

Table 4. Optimisation results for the robot pushing problems with batch sizes $q \in \{2, 5, 10, 20\}$. Median absolute distance from the optimum (left) and median absolute deviation from the median (MAD, right) after 200 function evaluations across the 51 runs. The method with the lowest median performance is shown in dark grey, with those with statistically equivalent performance shown in light grey.

Fig. 4. Illustrative convergence plots for the `PUSH4` ($d = 4$) and `PUSH8` ($d = 8$) real-world robot pushing active learning functions (rows) for four batch sizes $q \in \{5, 10, 20\}$ (columns). Each plot shows the median difference between the best function value seen and the true optimum, with shading representing the interquartile range across the 51 runs.

3.3 Pipe Shape Optimisation

Method	PitzDaily (q=2)		PitzDaily (q=5)		PitzDaily (q=10)		PitzDaily (q=20)	
	Median	MAD	Median	MAD	Median	MAD	Median	MAD
LP	8.67×10^{-2}	3.38×10^{-3}	8.62×10^{-2}	3.74×10^{-3}	8.72×10^{-2}	3.98×10^{-3}	8.87×10^{-2}	7.02×10^{-3}
PLAyBOOK	8.65×10^{-2}	3.80×10^{-3}	8.60×10^{-2}	3.15×10^{-3}	8.79×10^{-2}	5.21×10^{-3}	9.03×10^{-2}	5.56×10^{-3}
KB	8.44×10^{-2}	1.58×10^{-3}	8.45×10^{-2}	2.23×10^{-3}	8.55×10^{-2}	2.08×10^{-3}	8.49×10^{-2}	2.44×10^{-3}
qEI	8.43×10^{-2}	1.49×10^{-3}	8.60×10^{-2}	3.72×10^{-3}	9.04×10^{-2}	4.88×10^{-3}	9.51×10^{-2}	4.94×10^{-3}
TS	9.44×10^{-2}	5.25×10^{-3}	9.31×10^{-2}	4.19×10^{-3}	9.32×10^{-2}	6.13×10^{-3}	9.30×10^{-2}	3.89×10^{-3}
ϵ S-RS (0.1)	8.40×10^{-2}	1.49×10^{-3}	8.49×10^{-2}	2.65×10^{-3}	8.58×10^{-2}	4.03×10^{-3}	8.55×10^{-2}	3.51×10^{-3}
ϵ S-PF (0.1)	8.40×10^{-2}	1.42×10^{-3}	8.46×10^{-2}	2.18×10^{-3}	8.57×10^{-2}	3.81×10^{-3}	8.49×10^{-2}	2.73×10^{-3}
ϵ S-0	8.79×10^{-2}	5.91×10^{-3}	8.83×10^{-2}	7.26×10^{-3}	8.81×10^{-2}	7.03×10^{-3}	8.84×10^{-2}	6.53×10^{-3}

Table 5. Optimisation results for the pipe shape optimisation problem with batch sizes $q \in \{2, 5, 10, 20\}$. Median absolute distance from the optimum (left) and median absolute deviation from the median (MAD, right) after 200 function evaluations across the 51 runs. The method with the lowest median performance is shown in dark grey, with those with statistically equivalent performance shown in light grey.

Fig. 5. Illustrative convergence plots for the PitzDaily ($d = 10$) real-world pipe shape optimisation problem for four batch sizes $q \in \{5, 10, 20\}$ (columns). Each plot shows the median difference between the best function value seen and the true optimum, with shading representing the interquartile range across the 51 runs.