Interactive evolutionary multiple objective optimization algorithm using a fast calculation of holistic acceptabilities e-Appendix (supplementary material)

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In the e-Appendix, we discuss the material which was not included in the main paper for the sake of conciseness. Specifically:

- (1) we discuss the results attained by iMOEA-HA coupled with different selection procedures SEL (Section 1);
- (2) we present the results attained by iMOEA-HA parameterized with different queue limits K (Section 2);
- (3) we compare the execution times of EMOSOR and iMOEA-HA coupled with different queue limits K (Section 3);
- (4) we compare different algorithms applied to the WFG1 benchmark problem (Section 4).

The experimental results provided in this e-Appendix follow the evaluation strategy discussed in the main paper.

1 COMPARISON OF DIFFERENT VARIANTS OF IMOE-HA COUPLED WITH VARIOUS SELECTION PROCEDURES

In this section, we verify the performance of iMOEA-HA coupled with two selection procedures SEL :

- $SEL = TUR$: Parent solutions are selected using a tournament of size 2. During the selection process, a solution included in a more favorable non-dominated front is promoted. In case of a tie, a solution with a lesser $HA-INV$ score is preferred.
- $SEL = RMP$: The mating pool is restricted by randomly selecting parent solutions from a subset of non-dominated solutions assigned to a random queue.

We ran the algorithms for WFG1-9 and $M = 2, \dots, 5$. Furthermore, the analysis was performed for different queue limits $K \in \{5, 10, 15, 20\}$. The results of this comparative study are provided in Tables 1 ($K = 5$), 2 ($K = 10$), 3 ($K = 15$), and 4 ($K = 20$).

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The results indicate that, in general, both variants of iMOEA-HA perform similarly. However, apart from some individual scenarios, iMOEA-HA restricting the mating pool ($SEL = RMP$) reported better results for WFG1-2, different numbers of objectives and queue sizes. For instance, for WFG1 and $K = 10$ or 15 , iMOEA-HA with $SEL = RMP$ outperformed its $SEL = TUR$ counterpart for all numbers of objectives. When it comes to WFG2, the former method proved a better performer when coupled with $K \in \{5, 10, 15, 20\}$ and run for $M = 2$. Note that WFG1-2 problems have particularly challenging shapes of PFs. Therefore, we conclude that restricting the mating pool increased the chances for generating better offspring in such complex fitness landscapes.

2 COMPARISON OF DIFFERENT VARIANTS OF IMOE-HA PARAMETERIZED WITH VARIOUS QUEUE LIMITS

This section compares the results attained by iMOEA-HA when coupled with different queue sizes K . We consider $K \in \{5, 10, 15, 20\}$, similarly to the main paper. The results of this extensive experiment are provided in Table 5. They reveal a general tendency that the performance of iMOEA-HA improves with the increase of K . For instance, for WFG1 with $M = 5$, the algorithm attained the average ranks $\bar{R}$ of 3.08, 2.51, 2.33, and 2.08 for the considered subsequent values of K . The reason is that the greater K is, the more accurate are the approximations of $HA-INV$ scores used to rank solutions in the population. Hence, the algorithm performs better in predicting the true DM's aspirations, converging the population towards more relevant solutions in the PF.

3 COMPARISON OF EXECUTION TIMES OF EMOSOR AND IMOE-HA COUPLED WITH DIFFERENT QUEUE LIMITS

In this section, we compare the execution times of EMOSOR and iMOEA-HA run for different queue limits K (see Table 6). As discussed in the main paper, iMOEA-HA required less time to perform the simulations. Even for the extreme case, i.e., $K = 20$, iMOEA-HA performed, on average, 2.34 faster than EMOSOR. Furthermore, the difference between the computational times of iMOEA-HA and its competitor increases for a greater number of objectives. A more favorable computational complexity of the proposed algorithm derives from imposing a limit for the number of the most preferred ranks considered when calculating the $HA-INV$ scores and using better-suited sorting procedures.

Table 1: Average ARSD and respective ranks $\bar{R}$ attained by iMOEA-HA with $SEL \in \{TUR, RMP\}$ and $K = 5$, applied to WFG1–9 problems with $M = 2$ –5 objectives.

	SEL	$M = 2$			$M = 3$			$M = 4$			$M = 5$		
		Mean	StD	$\bar{R}$	Mean	StD	$\bar{R}$	Mean	StD	$\bar{R}$	Mean	StD	$\bar{R}$
WFG1	<i>TUR</i>	2.143	18.585	1.51	1.252	8.896	1.78	1.590	5.684	1.69	2.697	4.363	1.46
	<i>RMP</i>	1.780	15.422	1.49	0.937	6.621	1.22	1.501	5.269	1.31	2.518	3.892	1.54
WFG2	<i>TUR</i>	0.639	2.434	1.76	2.219	6.615	1.68	3.174	12.726	1.52	7.228	25.564	1.47
	<i>RMP</i>	0.383	2.294	1.24	2.024	8.376	1.32	2.653	6.685	1.48	6.138	16.087	1.53
WFG3	<i>TUR</i>	0.298	2.714	1.72	0.123	0.876	1.51	0.105	0.145	1.41	0.198	0.255	1.48
	<i>RMP</i>	0.601	5.700	1.28	0.157	1.036	1.49	0.106	0.156	1.59	0.190	0.177	1.52
WFG4	<i>TUR</i>	8.261	82.302	1.49	0.528	3.258	1.51	1.248	3.590	1.48	3.057	6.958	1.53
	<i>RMP</i>	0.232	1.761	1.51	0.993	8.420	1.49	1.244	2.601	1.52	2.944	5.601	1.47
WFG5	<i>TUR</i>	5.747	52.899	1.48	1.283	9.926	1.53	2.145	9.106	1.62	3.823	9.082	1.51
	<i>RMP</i>	2.830	25.780	1.52	2.173	18.686	1.47	1.749	6.532	1.38	3.996	13.665	1.49
WFG6	<i>TUR</i>	2.069	17.953	1.61	1.085	7.756	1.46	1.617	3.902	1.58	2.822	4.660	1.53
	<i>RMP</i>	0.918	8.501	1.39	0.878	5.719	1.54	2.750	13.900	1.42	3.047	6.989	1.47
WFG7	<i>TUR</i>	0.059	0.512	1.45	0.301	1.003	1.52	1.116	3.281	1.46	2.812	7.128	1.47
	<i>RMP</i>	0.138	1.300	1.55	0.811	6.098	1.48	1.710	6.539	1.54	3.433	8.894	1.53
WFG8	<i>TUR</i>	12.001	107.595	1.51	1.888	12.983	1.58	2.359	9.412	1.68	5.011	19.609	1.54
	<i>RMP</i>	1.394	12.027	1.49	2.962	26.051	1.42	1.454	4.949	1.32	4.278	12.458	1.46
WFG9	<i>TUR</i>	2.997	21.673	1.26	1.034	6.871	1.45	2.339	9.419	1.45	4.090	9.776	1.48
	<i>RMP</i>	1.991	16.818	1.74	1.841	13.174	1.55	2.264	9.001	1.55	4.221	12.490	1.52

Table 2: Average ARSD and respective ranks $\bar{R}$ attained by iMOEA-HA with $SEL \in \{TUR, RMP\}$ and $K = 10$, applied to WFG1–9 problems with $M = 2$ –5 objectives.

	SEL	$M = 2$			$M = 3$			$M = 4$			$M = 5$		
		Mean	StD	$\bar{R}$	Mean	StD	$\bar{R}$	Mean	StD	$\bar{R}$	Mean	StD	$\bar{R}$
WFG1	<i>TUR</i>	2.419	21.101	1.67	1.383	10.450	1.68	1.593	4.833	1.67	2.354	2.831	1.61
	<i>RMP</i>	1.285	11.022	1.33	0.815	5.624	1.32	0.970	2.248	1.33	2.294	3.860	1.39
WFG2	<i>TUR</i>	0.963	6.250	1.68	1.811	5.772	1.58	2.797	7.621	1.59	6.432	29.759	1.50
	<i>RMP</i>	0.451	2.828	1.32	2.040	8.961	1.42	1.853	4.484	1.41	4.449	10.297	1.50
WFG3	<i>TUR</i>	0.297	2.548	1.69	0.201	1.547	1.64	0.085	0.128	1.55	0.186	0.244	1.53
	<i>RMP</i>	0.752	7.329	1.31	0.133	1.054	1.36	0.079	0.091	1.45	0.176	0.179	1.47
WFG4	<i>TUR</i>	0.105	0.841	1.53	0.388	2.529	1.46	0.952	2.167	1.59	2.534	9.371	1.53
	<i>RMP</i>	0.047	0.408	1.47	0.353	2.100	1.54	0.985	2.016	1.41	1.835	3.587	1.47
WFG5	<i>TUR</i>	3.210	25.789	1.62	1.177	8.686	1.55	2.837	16.682	1.52	3.653	11.314	1.43
	<i>RMP</i>	2.940	26.517	1.38	1.146	8.456	1.45	1.911	6.489	1.48	3.461	8.332	1.57
WFG6	<i>TUR</i>	4.127	38.425	1.50	0.843	6.058	1.53	1.314	4.754	1.53	2.600	6.721	1.50
	<i>RMP</i>	0.377	3.232	1.50	0.679	5.067	1.47	1.380	5.999	1.47	2.457	5.048	1.50
WFG7	<i>TUR</i>	0.019	0.156	1.53	0.246	0.639	1.55	0.932	2.158	1.51	3.506	14.174	1.54
	<i>RMP</i>	1.139	11.349	1.47	0.227	0.827	1.45	0.927	2.410	1.49	1.940	3.492	1.46
WFG8	<i>TUR</i>	2.398	13.576	1.51	1.332	10.343	1.51	1.408	4.466	1.46	3.703	10.324	1.59
	<i>RMP</i>	1.744	15.377	1.49	0.230	0.362	1.49	1.523	5.744	1.54	2.579	5.898	1.41
WFG9	<i>TUR</i>	11.684	106.913	1.47	1.044	5.176	1.56	2.104	8.050	1.53	4.700	14.737	1.51
	<i>RMP</i>	2.529	21.645	1.53	2.184	18.882	1.44	3.255	20.156	1.47	4.194	11.928	1.49

4 PERFORMANCE OF DIFFERENT ALGORITHMS APPLIED TO THE WFG1 TEST PROBLEM

This section compares EMOSOR with iMOEA-HA using different selection procedures when applied to the WFG1 test problem. The queue limit was set to $K = 15$ for iMOEA-HA. The results of this

experiment are provided in Table 7. They reveal that using the restricting mating pool ($SEL = RMP$) in iMOEA-HA significantly improved the algorithm's performance compared to the remaining methods. Precisely, iMOEA-HA employing the tournament selection ($SEL = TUR$) attained the worst average ARSD-based ranks ranging from 2.14 to 2.23. Therefore, it performed worse than EMOSOR, for which the respective ranks range from 1.86

Table 3: Average ARSD and respective ranks $\bar{R}$ attained by iMOEA-HA with $SEL \in \{TUR, RMP\}$ and $K = 15$, applied to WFG1–9 problems with $M = 2$ –5 objectives.

	SEL	$M = 2$			$M = 3$			$M = 4$			$M = 5$		
		Mean	StD	$\bar{R}$	Mean	StD	$\bar{R}$	Mean	StD	$\bar{R}$	Mean	StD	$\bar{R}$
WFG1	<i>TUR</i>	1.483	12.600	1.67	1.278	9.399	1.75	1.396	4.863	1.65	2.413	3.658	1.66
	<i>RMP</i>	1.532	13.366	1.33	0.751	4.996	1.25	0.984	2.229	1.35	1.763	3.676	1.34
WFG2	<i>TUR</i>	0.836	3.800	1.67	1.124	3.178	1.64	1.173	2.109	1.47	7.263	30.693	1.44
	<i>RMP</i>	0.362	2.101	1.33	0.797	2.371	1.36	1.840	4.605	1.53	4.750	10.018	1.56
WFG3	<i>TUR</i>	0.276	2.480	1.51	0.148	0.966	1.58	0.069	0.085	1.53	0.181	0.204	1.54
	<i>RMP</i>	0.304	2.846	1.49	0.124	0.967	1.42	0.073	0.097	1.47	0.167	0.170	1.46
WFG4	<i>TUR</i>	2.836	28.251	1.50	0.371	2.048	1.53	0.710	1.147	1.58	2.169	5.710	1.44
	<i>RMP</i>	0.265	2.597	1.50	0.196	0.946	1.47	0.831	2.402	1.42	2.153	4.565	1.56
WFG5	<i>TUR</i>	3.311	30.610	1.52	1.116	8.247	1.42	1.857	6.012	1.49	2.499	5.413	1.46
	<i>RMP</i>	2.659	24.168	1.48	1.284	9.847	1.58	1.728	6.661	1.51	3.860	15.540	1.54
WFG6	<i>TUR</i>	1.129	10.237	1.61	0.911	7.205	1.43	1.213	4.240	1.54	2.376	7.831	1.51
	<i>RMP</i>	0.793	7.355	1.39	0.720	5.458	1.57	1.033	3.256	1.46	2.154	5.275	1.49
WFG7	<i>TUR</i>	0.032	0.291	1.54	0.485	3.720	1.46	0.655	2.312	1.43	2.371	6.684	1.53
	<i>RMP</i>	0.004	0.011	1.46	0.189	0.683	1.54	1.025	4.547	1.57	2.711	14.009	1.47
WFG8	<i>TUR</i>	1.970	14.462	1.50	6.505	61.815	1.40	1.422	5.780	1.50	3.266	9.228	1.54
	<i>RMP</i>	1.324	11.668	1.50	0.398	1.328	1.60	1.319	5.035	1.50	4.287	14.921	1.46
WFG9	<i>TUR</i>	3.899	32.327	1.52	1.005	6.168	1.49	1.676	7.136	1.46	3.330	8.487	1.46
	<i>RMP</i>	2.599	23.345	1.48	0.958	6.461	1.51	2.193	9.124	1.54	3.515	10.770	1.54

Table 4: Average ARSD and respective ranks $\bar{R}$ attained by iMOEA-HA with $SEL \in \{TUR, RMP\}$ and $K = 20$, applied to WFG1–9 problems with $M = 2$ –5 objectives.

	SEL	$M = 2$			$M = 3$			$M = 4$			$M = 5$		
		Mean	StD	$\bar{R}$	Mean	StD	$\bar{R}$	Mean	StD	$\bar{R}$	Mean	StD	$\bar{R}$
WFG1	<i>TUR</i>	1.785	15.825	1.54	0.930	6.227	1.79	1.217	3.473	1.66	2.311	3.650	1.67
	<i>RMP</i>	1.467	12.697	1.46	0.661	4.341	1.21	0.866	2.116	1.34	1.828	3.797	1.33
WFG2	<i>TUR</i>	0.844	4.093	1.63	1.026	3.093	1.53	2.136	5.430	1.55	3.820	10.285	1.47
	<i>RMP</i>	0.342	1.704	1.37	2.085	8.382	1.47	2.051	5.085	1.45	5.656	14.964	1.53
WFG3	<i>TUR</i>	0.196	1.507	1.64	0.064	0.280	1.51	0.071	0.099	1.54	0.171	0.193	1.56
	<i>RMP</i>	0.084	0.634	1.36	0.087	0.491	1.49	0.061	0.073	1.46	0.163	0.170	1.44
WFG4	<i>TUR</i>	0.440	4.350	1.47	0.233	0.947	1.50	1.190	6.152	1.50	1.921	4.347	1.48
	<i>RMP</i>	0.048	0.303	1.53	0.157	0.360	1.50	1.013	4.015	1.50	1.691	3.208	1.52
WFG5	<i>TUR</i>	2.645	24.168	1.56	1.097	8.465	1.51	1.478	6.297	1.51	2.639	5.601	1.43
	<i>RMP</i>	2.642	24.168	1.44	1.151	9.146	1.49	1.631	5.748	1.49	3.095	7.295	1.57
WFG6	<i>TUR</i>	1.921	17.791	1.44	1.021	8.487	1.53	1.107	4.866	1.57	1.873	3.885	1.50
	<i>RMP</i>	0.567	5.150	1.56	1.024	8.976	1.47	0.897	3.901	1.43	2.194	4.950	1.50
WFG7	<i>TUR</i>	0.031	0.198	1.56	0.204	0.657	1.53	1.059	3.520	1.56	1.495	2.883	1.47
	<i>RMP</i>	0.054	0.510	1.44	0.235	0.718	1.47	0.643	1.273	1.44	1.902	6.944	1.53
WFG8	<i>TUR</i>	1.356	11.647	1.59	0.389	1.263	1.53	1.221	4.822	1.48	2.913	8.542	1.60
	<i>RMP</i>	1.317	11.636	1.41	0.282	0.939	1.47	1.306	4.784	1.52	2.510	7.503	1.40
WFG9	<i>TUR</i>	3.028	24.484	1.37	1.338	11.113	1.49	1.964	8.906	1.51	4.067	15.200	1.52
	<i>RMP</i>	1.708	14.249	1.63	1.355	10.864	1.51	2.282	13.293	1.49	3.444	9.938	1.48

to 2.11. However, switching from the tournament selection to a

restricted mating pool helped handle the bias in WFG1, resulting in the best-attained average ranks ranging from 1.57 to 1.83.

Table 5: Average ARSD and respective ranks $\bar{R}$ attained by iMOEA-HA with $K \in \{5, 10, 15, 20\}$, applied to WFG1–9 problems with $M = 2$ –5 objectives.

	K	$M = 2$			$M = 3$			$M = 4$			$M = 5$		
		Mean	StD	$\bar{R}$	Mean	StD	$\bar{R}$	Mean	StD	$\bar{R}$	Mean	StD	$\bar{R}$
WFG1	5	1.780	15.422	2.90	0.937	6.621	2.71	1.501	5.269	2.84	2.518	3.892	3.08
	10	1.285	11.022	2.43	0.815	5.624	2.62	0.970	2.248	2.52	2.294	3.860	2.60
	15	1.532	13.366	2.36	0.751	4.996	2.44	0.984	2.229	2.37	1.763	3.676	2.23
	20	1.467	12.697	2.31	0.661	4.341	2.23	0.866	2.116	2.27	1.828	3.797	2.09
WFG2	5	0.383	2.294	2.35	2.024	8.376	2.73	2.653	6.685	2.76	6.138	16.087	2.63
	10	0.451	2.828	2.53	2.040	8.961	2.66	1.853	4.484	2.56	4.449	10.297	2.55
	15	0.362	2.101	2.61	0.797	2.371	2.09	1.840	4.605	2.33	4.750	10.018	2.43
	20	0.342	1.704	2.51	2.085	8.382	2.52	2.051	5.085	2.35	5.656	14.964	2.39
WFG3	5	0.601	5.700	2.21	0.157	1.036	2.77	0.106	0.156	2.69	0.190	0.177	2.73
	10	0.752	7.329	2.61	0.133	1.054	2.37	0.079	0.091	2.52	0.176	0.179	2.46
	15	0.304	2.846	2.73	0.124	0.967	2.36	0.073	0.097	2.36	0.167	0.170	2.51
	20	0.084	0.634	2.45	0.087	0.491	2.50	0.061	0.073	2.43	0.163	0.170	2.30
WFG4	5	0.232	1.761	2.95	0.993	8.420	2.75	1.244	2.601	2.78	2.944	5.601	2.83
	10	0.047	0.408	2.48	0.353	2.100	2.72	0.985	2.016	2.49	1.835	3.587	2.38
	15	0.265	2.597	2.29	0.196	0.946	2.33	0.831	2.402	2.26	2.153	4.565	2.43
	20	0.048	0.303	2.28	0.157	0.360	2.20	1.013	4.015	2.47	1.691	3.208	2.36
WFG5	5	2.830	25.780	2.91	2.173	18.686	2.62	1.749	6.532	2.65	3.996	13.665	2.67
	10	2.940	26.517	2.45	1.146	8.456	2.53	1.911	6.489	2.62	3.461	8.332	2.59
	15	2.659	24.168	2.48	1.284	9.847	2.51	1.728	6.661	2.48	3.860	15.540	2.36
	20	2.642	24.168	2.16	1.151	9.146	2.34	1.631	5.748	2.25	3.095	7.295	2.38
WFG6	5	0.918	8.501	2.64	0.878	5.719	2.91	2.750	13.900	2.81	3.047	6.989	2.89
	10	0.377	3.232	2.50	0.679	5.067	2.24	1.380	5.999	2.56	2.457	5.048	2.57
	15	0.793	7.355	2.34	0.720	5.458	2.60	1.033	3.256	2.35	2.154	5.275	2.35
	20	0.567	5.150	2.52	1.024	8.976	2.25	0.897	3.901	2.28	2.194	4.950	2.19
WFG7	5	0.138	1.300	2.90	0.811	6.098	2.59	1.710	6.539	2.86	3.433	8.894	2.79
	10	1.139	11.349	2.42	0.227	0.827	2.50	0.927	2.410	2.51	1.940	3.492	2.65
	15	0.004	0.011	2.40	0.189	0.683	2.50	1.025	4.547	2.44	2.711	14.009	2.34
	20	0.054	0.510	2.28	0.235	0.718	2.41	0.643	1.273	2.19	1.902	6.944	2.22
WFG8	5	1.394	12.027	2.65	2.962	26.051	2.61	1.454	4.949	2.63	4.278	12.458	3.07
	10	1.744	15.377	2.58	0.230	0.362	2.62	1.523	5.744	2.63	2.579	5.898	2.49
	15	1.324	11.668	2.51	0.398	1.328	2.56	1.319	5.035	2.46	4.287	14.921	2.31
	20	1.317	11.636	2.26	0.282	0.939	2.21	1.306	4.784	2.28	2.510	7.503	2.13
WFG9	5	1.991	16.818	3.00	1.841	13.174	2.91	2.264	9.001	2.77	4.221	12.490	2.87
	10	2.529	21.645	2.53	2.184	18.882	2.46	3.255	20.156	2.48	4.194	11.928	2.63
	15	2.599	23.345	2.36	0.958	6.461	2.35	2.193	9.124	2.42	3.515	10.770	2.37
	20	1.708	14.249	2.11	1.355	10.864	2.28	2.282	13.293	2.33	3.444	9.938	2.13

Table 6: Average execution times of EMOSOR and iMOEA-HA with $K \in \{5, 10, 15, 20\}$, applied to WFG1–9 problems with $M = 2$ –5 objectives.

	Method	$M = 2$		$M = 3$		$M = 4$		$M = 5$	
		Mean	StD	Mean	StD	Mean	StD	Mean	StD
WFG1	EMOSOR	2.185	0.247	8.67	0.153	20.313	0.81	37.508	0.223
	iMOEA-HA with $K = 5$	0.616	0.067	1.98	0.106	4.175	0.10	7.302	0.091
	iMOEA-HA with $K = 10$	0.853	0.066	2.83	0.142	5.931	0.26	9.892	0.268
	iMOEA-HA with $K = 15$	1.151	0.087	3.99	0.159	7.845	0.30	13.227	0.231
	iMOEA-HA with $K = 20$	1.481	0.117	5.38	0.255	10.007	0.57	16.807	0.555
WFG2	EMOSOR	2.269	0.131	8.71	0.399	19.730	0.26	46.617	16.739
	iMOEA-HA with $K = 5$	0.467	0.069	1.82	0.085	4.064	0.13	6.437	0.566
	iMOEA-HA with $K = 10$	0.649	0.074	2.13	0.274	5.219	0.69	9.601	0.552
	iMOEA-HA with $K = 15$	0.858	0.107	3.72	0.210	7.420	0.89	11.380	1.837
	iMOEA-HA with $K = 20$	1.049	0.150	4.41	0.637	9.806	0.77	12.965	2.548
WFG3	EMOSOR	1.080	0.050	4.42	0.154	10.069	0.40	18.679	0.196
	iMOEA-HA with $K = 5$	0.320	0.041	1.01	0.024	2.195	0.07	3.897	0.098
	iMOEA-HA with $K = 10$	0.413	0.043	1.45	0.032	3.118	0.08	5.326	0.062
	iMOEA-HA with $K = 15$	0.562	0.063	2.01	0.048	4.133	0.11	6.986	0.173
	iMOEA-HA with $K = 20$	0.720	0.031	2.61	0.084	5.408	0.12	8.963	0.181
WFG4	EMOSOR	1.094	0.056	4.34	0.173	9.904	0.10	18.820	0.159
	iMOEA-HA with $K = 5$	0.310	0.021	1.07	0.069	2.249	0.09	3.914	0.091
	iMOEA-HA with $K = 10$	0.431	0.018	1.52	0.044	3.141	0.13	5.322	0.209
	iMOEA-HA with $K = 15$	0.591	0.063	2.05	0.076	4.146	0.13	6.872	0.214
	iMOEA-HA with $K = 20$	0.737	0.034	2.63	0.147	5.261	0.30	8.792	0.207
WFG5	EMOSOR	1.041	0.040	4.47	0.253	9.858	0.10	18.780	0.156
	iMOEA-HA with $K = 5$	0.337	0.037	1.06	0.054	2.294	0.05	3.858	0.081
	iMOEA-HA with $K = 10$	0.447	0.034	1.50	0.068	3.100	0.08	5.212	0.122
	iMOEA-HA with $K = 15$	0.603	0.044	2.09	0.076	4.126	0.15	6.746	0.175
	iMOEA-HA with $K = 20$	0.775	0.068	2.69	0.148	5.161	0.34	8.683	0.220
WFG6	EMOSOR	1.122	0.074	4.30	0.038	9.882	0.07	18.845	0.205
	iMOEA-HA with $K = 5$	0.322	0.025	1.04	0.037	2.202	0.12	3.764	0.133
	iMOEA-HA with $K = 10$	0.430	0.013	1.47	0.076	2.973	0.16	5.180	0.154
	iMOEA-HA with $K = 15$	0.548	0.060	1.93	0.162	4.043	0.25	6.693	0.219
	iMOEA-HA with $K = 20$	0.719	0.028	2.53	0.122	5.204	0.24	8.525	0.387
WFG7	EMOSOR	1.047	0.033	4.30	0.134	9.896	0.12	18.807	0.119
	iMOEA-HA with $K = 5$	0.313	0.017	1.09	0.069	2.292	0.06	3.997	0.091
	iMOEA-HA with $K = 10$	0.459	0.044	1.54	0.057	3.139	0.16	5.389	0.122
	iMOEA-HA with $K = 15$	0.582	0.018	2.08	0.057	4.199	0.20	6.971	0.137
	iMOEA-HA with $K = 20$	0.743	0.021	2.69	0.141	5.390	0.20	8.922	0.152
WFG8	EMOSOR	1.121	0.059	4.38	0.066	10.035	0.09	19.014	0.156
	iMOEA-HA with $K = 5$	0.399	0.153	1.11	0.092	2.313	0.09	4.141	0.154
	iMOEA-HA with $K = 10$	0.471	0.046	1.58	0.105	3.239	0.16	5.647	0.159
	iMOEA-HA with $K = 15$	0.607	0.061	2.11	0.115	4.395	0.29	7.221	0.423
	iMOEA-HA with $K = 20$	0.746	0.079	2.69	0.175	5.592	0.44	9.402	0.217
WFG9	EMOSOR	1.152	0.039	4.44	0.085	10.221	0.14	19.293	0.182
	iMOEA-HA with $K = 5$	0.409	0.036	1.26	0.051	2.623	0.10	4.392	0.132
	iMOEA-HA with $K = 10$	0.537	0.053	1.72	0.124	3.460	0.13	5.881	0.230
	iMOEA-HA with $K = 15$	0.676	0.025	2.21	0.162	4.447	0.30	7.278	0.470
	iMOEA-HA with $K = 20$	0.819	0.089	2.90	0.120	5.571	0.43	9.034	0.547

Table 7: Average ARSD and respective ranks $\bar{R}$ attained by EMOSOR and iMOEA-HA with $SEL \in \{TUR, RMP\}$ ($K = 15$), applied to WFG1 problem with $M = 2$ –5 objectives.

Method	$M = 2$			$M = 3$			$M = 4$			$M = 5$		
	Mean	StD	$\bar{R}$	Mean	StD	$\bar{R}$	Mean	StD	$\bar{R}$	Mean	StD	$\bar{R}$
EMOSOR	1.161	9.702	1.86	2.061	17.386	2.11	1.366	5.079	2.11	2.131	3.753	1.97
iMOEA-HA with $SEL = TUR$	1.483	12.600	2.31	1.278	9.399	2.32	1.396	4.863	2.14	2.413	3.658	2.23
iMOEA-HA with $SEL = RMP$	1.532	13.366	1.83	0.751	4.996	1.57	0.984	2.229	1.75	1.763	3.676	1.80