Supplementary Materials for the manuscript entitled Fishing for Interactions: A Network Science Approach to Modeling Fish School Search

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S1 Analyzing the number of hubs in the interaction networks

Figure S1 shows three main messages:

- Figure S1 (A,D,G) presents that there is no clear relationship between the number of hubs and the best fitness value found in a simulation.
- Figure S1 (B,E,H) presents that the fish influence distribution - how much a fish i influenced the entire swarm on the final interaction network - is different for each fitness function. The distributions show different skewness.
- Figure S1 (C,F,I) presents that the number of hubs in the network over iterations varies depending on each function.

Supplementary Figure S1: First column shows the number of hubs found on the 30 simulations related to the respective best fitness value for each function: (A) Rastrigin, (D) Rosenbrock and (G) Schwefel. Second column shows the distribution of how much each fish influenced the swarm in the best experiment of each function: (B) Rastrigin, (E) Rosenbrock and (H) Schwefel. Third column shows the number of hubs found on the interaction network of each movement over iterations ($tw = k = 100$) from the best experiment of each function: (C) Rastrigin, (F) Rosenbrock and (I) Schwefel.

We observe that the number of hubs in the final interaction network does not reach values higher than 15% of the entire swarm and that over iterations the Schwefel function reaches almost 30% of the swarm as hubs. Moreover, we also observe that the number of hubs in Schwefel function is much higher than Rastrigin and Rosenbrock functions.

S2 Interaction network from the worst results

We show in the Figure S2 the interaction network and the weight distribution from the experiment for each function with the worst fitness value. The characteristics on the final networks of the worst experiment are different from the best experiment for each function (Figure 2), but I_{vol} still shows higher values of edge weights than instinctive movement (I_{inst}).

Supplementary Figure S2: First column shows the adjacency matrix of the cumulative network, I_{both} , from the worst results found by the functions: Rastrigin (A), Rosenbrock (C) and Schwefel (E). Second column shows the Weight Distribution (frequency) of the weights on the cumulative final networks for each movement type (I_{inst} , I_{vol} , I_{both}) from the worst results found by the functions: Rastrigin (B), Rosenbrock (D) and Schwefel (F).

S3 Comparisons between divergences and fitness function

To understand whether differences on the network were impacting the fitness gain directly, we compute the Pearson and Spearman Correlations between the delta fitness and delta divergences in Table S1. We extended the methodology applied by Santana et al. [1] comparing the fitness gain to the Portrait and Kullback divergences, and we also analyze both correlations: Pearson and Spearman. For the Pearson correlation, we identify if there is a linear relationship between

Supplementary Table S1: Pearson and Spearman Correlations between delta fitness (Δf) and delta divergence ($\Delta PD|\Delta KD$)

Correlation	Experiment	Comparison by	Metrics	Rastrigin	Rosenbrock	Schwefel
Pearson	Best one	Iterations	$(\Delta f, \Delta PD)$	+0.0821	-0.0007	-0.0919
Pearson	Best one	Iterations	$(\Delta f, \Delta KD)$	+0.3364	-0.0010	-0.0396
Spearman	Best one	Iterations	$(\Delta f, \Delta PD)$	+0.1360	-0.0377	-0.0612
Spearman	Best one	Iterations	$(\Delta f, \Delta KD)$	+0.0048	-0.0037	-0.0625

the two values, and for Spearman, we identify if two values change together with not necessarily with the same constant rates.

We observe that most correlations are closer to 0 than 1, indicating the absence of a relationship. This absence of correlation may be due to the unclear delay between changes over the network and fitness improvement. In future works, we aim to study this delay and relationship to understand if indeed there is no correlation between the delta fitness and delta divergences.

S4 Portrait Divergence of cumulative networks

Figure S3 shows the Portrait and Kullback divergences of the cumulative networks from iteration 0 until $t + tw$ where $tw = 25$. We observe that computing both divergences on the cumulative network from the iteration 0 until any iteration t hides the peculiarities of the network evolution, and we also observe a high similarity on the network structure.

Supplementary Figure S3: Portrait Divergence (PD) and Kullback Divergence (KD) of cumulative networks using the best result from each function.

S5 Comparing different FSS step values

The majority of the results varying the initial values of steps showed no extra impact on the network structure and flow of FSS. However, in this section, we show one exception found from Rastrigin function.

Figures S4 and S5 present the Portrait and Kullback Divergences from the 30 final cumulative networks in different simulations, and the best results of Rastrigin function using $tw = 100$, $k = 1$ and three different pair of steps values: (i) $s_{ind} = [10^{-1}, 10^{-3}]$ and $s_{vol} = 2 \cdot s_{ind}$; (ii) $s_{ind} = 0.1$ and $s_{vol} = 0.2$; (iii) $s_{ind} = 0.1$ and $s_{vol} = 0.4$. In (i) s_{ind} is linearly decreased by iteration from 10^{-1} to 10^{-3} .

Figure S4 presents the Portrait and Kullback Divergences of the final cumulative networks from the 30 simulations applied to Rastrigin function. We observe that there is an extra impact while changing the values of steps. The network flow of each simulation and function display a higher degree of heterogeneity. Moreover, we point out that when $s_{ind} = 0.1$ and $s_{vol} = 0.2, 0.4$, we observe a higher range of different (greenish) networks structures over the simulations.

Supplementary Figure S4: Using Portrait Divergence (PD) and Kullback Divergence (KD) to compare the interaction networks of the 30 simulations using different volitive and instinctive step values in Rastrigin function of three different ranges of steps (s_{ind} , s_{vol}) using $tw = 100$ and $k = 1$.

Supplementary Figure S5: Using Portrait Divergence (PD) and Kullback Divergence (KD) to compare the evolution of networks from the best experiment in Rastrigin function of three different ranges of steps (s_{ind} , s_{vol}) using $tw = 100$ and $k = 1$.

S6 Impact of fish weight values

We investigated the influence of fish weight values in the FSS using Portrait and Kullback divergences (Figure S6 and Figure S7). The initial values of each experiment are represented as follows:

$$w': W_i(0) = 500, \min_W = 100 \text{ and } \max_W = 1000;$$

$$w'': W_i(0) = 5000, \min_W = 100 \text{ and } \max_W = 10000;$$

$$w''': W_i(0) = 100, \min_W = 100 \text{ and } \max_W = 10000.$$

Even when we initialize FSS using different fish weight values, the structure of final cumulative interaction networks remains similar ($PD < 0.42$). Besides, we note that the simulations present higher KD values (> 0.5) indicating that there are differences in the influence flow. Thus, we conclude that there is no extra contribution to the network structure and flow changes when applying other weight values. Moreover, when we analyze the Portrait and Kullback divergences from the best result of Rastrigin function using $tw = 100$, $k = 1$ (Figure S7), we identify that the changes on PD and KD over iterations do not reflect on completely different network signatures.

Supplementary Figure S6: Portrait (PD) and Kullback (KD) Divergences comparing the 30 simulations using different initial values of the fish weight applied on the Rastrigin function.

Supplementary Figure S7: Using Portrait Divergence (PD) and Kullback Divergence (KD) to compare the evolution of networks from the best experiment in Rastrigin function of three different fish weight values using $tw = 100$ and $k = 1$.

S7 Comparing swarm-based algorithms

Figure S8 indicates that the pairwise comparison between the ABC, FSS, 2-DPSO and 5-DPSO shows values of PD and KD higher than 0.5. Therefore, the network evolution of the social interactions of four algorithms have distinctive signatures (structure and flow).

Supplementary Figure S8: Kullback and Portrait Divergence comparing the best simulation of each algorithm in Rastrigin function using $tw = 100$ and $k = 1$.

References

- [1] C. Santana, E. Keedwell, and R. Menezes. An approach to assess swarm intelligence algorithms based on complex networks. In *Proceedings of the 2020 Genetic and Evolutionary Computation Conference*, pages 31–39, 2020.