A survey of cluster validity indices for automatic data clustering using differential evolution

SUPPLEMENTARY MATERIAL

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1 CLUSTER VALIDITY INDICES

This section describes several cluster validity indices (CVIs), which have been used specifically as fitness functions in metaheuristic clustering algorithms to address the automatic clustering problem.

1.1 Definitions

Some basic definitions are given before presenting the CVIs:

- A *pattern* (or *object*) $\mathbf{x}$ is a single data item represented by a vector of measurements $\mathbf{x} = [x_1, x_2, \dots, x_D]^T$, where $x_i \in \mathbb{R}$ is a *feature* (or *attribute*) and D denotes the *dimensionality*.
- A *dataset* is denoted by $\mathbf{X} = \{\mathbf{x}_1, \mathbf{x}_2, \dots, \mathbf{x}_N\} \in \mathbb{R}^D$, where N is the total number of patterns in the dataset.
- A *clustering*, denoted by $\mathbf{C} = \{\mathbf{c}_k \mid k = 1, \dots, K\}$, refers to a set of mutually disjoint clusters that partition $\mathbf{X}$ into K groups.
- The *number of objects* in a cluster $\mathbf{c}_k$ is denoted by $n_k = |\mathbf{c}_k|$.
- The *centroid of a cluster* (or *prototype*) is expressed as $\bar{\mathbf{c}}_k = 1/n_k \sum_{\mathbf{x}_i \in \mathbf{c}_k} \mathbf{x}_i$, whereas the *centroid of a dataset* $\mathbf{X}$ is denoted by $\bar{\mathbf{X}} = 1/N \sum_{\mathbf{x}_i \in \mathbf{X}} \mathbf{x}_i$.
- A *distance measure* is a metric (or quasi-metric) used to quantify the proximity between two patterns. Probably the most common distance metric is the Euclidean distance, expressed as $d_e(\mathbf{x}, \mathbf{y}) = \sqrt{\sum_{i=1}^D (x_i - y_i)^2}$.
- The *point symmetry distance* [4] between the object $\mathbf{x}_i$ and the cluster $\mathbf{c}_k$ is defined by

$$d_{ps}(\mathbf{x}_i, \mathbf{c}_k) = \frac{1}{k_n} \min_{\mathbf{x}_j \in \mathbf{c}_k} (k_n) \{d_e(2\bar{\mathbf{c}}_k - \mathbf{x}_i, \mathbf{x}_j)\}. \quad (1)$$

The point $2\bar{\mathbf{c}}_k - \mathbf{x}_i$ is called the symmetrical (reflected) point of $\mathbf{x}_i$ with respect to the centroid of $\mathbf{c}_k$ and $k_n = 2$ are its unique nearest neighbors. The function $\min$ can be seen as an adaptation

of the min function, where $\min(n)$ computes the sum of the n lowest arguments.

1.2 Cluster validity indices

A CVI defines a relation between intracluster cohesion (within-group scatter) and intercluster separation (between-group scatter) to estimate the quality of a clustering solution [25]. The 22 CVIs evaluated in this study are described in the next. An acronym is defined to identify each CVI, followed by an up arrow ($\uparrow$) or a down arrow ($\downarrow$) to indicate whether the index is to be maximized or minimized. A general description is provided for every CVI in terms of criteria of cluster cohesion and separation.

- The Dunn index ($\mathbf{DI}_{\uparrow}$) [14]. This is a ratio-type index in which the cohesion is quantified by the maximum cluster diameter and the separation by the nearest neighbor distance. It is defined by

$$DI(\mathbf{C}) = \frac{\min_{\mathbf{c}_k \in \mathbf{C}} \left\{ \min_{\mathbf{c}_r \in \mathbf{C} \setminus \mathbf{c}_k} \{\delta(\mathbf{c}_k, \mathbf{c}_r)\} \right\}}{\max_{\mathbf{c}_k \in \mathbf{C}} \{\Delta(\mathbf{c}_k)\}}, \quad (2)$$

where

$$\begin{aligned} \delta(\mathbf{c}_k, \mathbf{c}_r) &= \min_{\mathbf{x}_i \in \mathbf{c}_k, \mathbf{x}_j \in \mathbf{c}_r} \{d_e(\mathbf{x}_i, \mathbf{x}_j)\}, \\ \Delta(\mathbf{c}_k) &= \max_{\mathbf{x}_i, \mathbf{x}_j \in \mathbf{c}_k} \{d_e(\mathbf{x}_i, \mathbf{x}_j)\}. \end{aligned}$$

- The Calinski-Harabasz index ($\mathbf{CH}_{\uparrow}$) [8]. This is a ratio-type index in which the cohesion is quantified by the sum of the distances of the patterns to their respective centroids, and the separation is measured by the sum of the distances from the centroids to the global prototype. It is defined by

$$CH(\mathbf{C}) = \frac{N - K}{K - 1} \times \frac{\sum_{\mathbf{c}_k \in \mathbf{C}} n_k d_e^2(\bar{\mathbf{c}}_k, \bar{\mathbf{X}})}{\sum_{\mathbf{c}_k \in \mathbf{C}} \sum_{\mathbf{x}_i \in \mathbf{c}_k} d_e^2(\mathbf{x}_i, \bar{\mathbf{c}}_k)}. \quad (3)$$

- The C-Index ($\mathbf{CI}_{\downarrow}$) [22]. This is a normalized index in which the cohesion is quantified by the sum of the distances between all the patterns in the same cluster, and the separation is based on the sum of the smallest and largest distances between all the patterns in the dataset. It is computed by

$$CI(\mathbf{C}) = \frac{S(\mathbf{C}) - S_{\min}(\mathbf{C})}{S_{\max}(\mathbf{C}) - S_{\min}(\mathbf{C})}, \quad (4)$$

where

$$\begin{aligned} S(\mathbf{C}) &= \sum_{\mathbf{c}_k \in \mathbf{C}} \sum_{\mathbf{x}_i, \mathbf{x}_j \in \mathbf{c}_k} d_e(\mathbf{x}_i, \mathbf{x}_j), \\ S_{\min}(\mathbf{C}) &= \min_{n_w} \sum_{\mathbf{x}_i, \mathbf{x}_j \in \mathbf{X}} \{d_e(\mathbf{x}_i, \mathbf{x}_j)\}, \\ S_{\max}(\mathbf{C}) &= \max_{n_w} \sum_{\mathbf{x}_i, \mathbf{x}_j \in \mathbf{X}} \{d_e(\mathbf{x}_i, \mathbf{x}_j)\}. \end{aligned}$$

and where n_w is the number of pairs of objects in a partition that are in the same cluster: $n_w = \sum_{\mathbf{c}_k \in \mathbf{C}} \binom{n_k}{2}$.

- The Davies-Bouldin index ($\mathbf{DB}_{\downarrow}$) [11]. In this index, the cohesion is quantified by the mean distance of the objects to their

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respective centroids, and the separation quantifies the distance between centroids. It is expressed by

$$DB(C) = \frac{1}{K} \max_{c_k \in C} \max_{c_r \in C \setminus c_k} \left\{ \frac{S(c_k) + S(c_r)}{d_e(\bar{c}_k, \bar{c}_r)} \right\}, \quad (5)$$

where

$$S(c_k) = \frac{1}{n_k} \sum_{x_i \in c_k} d_e(x_i, \bar{c}_k).$$

- The Silhouette index (**Si**_↓) [30]. This index is a normalized summation-type index in which the cohesion is measured by the sum of the distances between all the points in the same cluster, and the separation is based on the nearest neighbor distance between points in different groups. It is defined by

$$SI(C) = \frac{1}{N} \sum_{c_k \in C} \sum_{x_i \in c_k} \frac{b(x_i, c_k) - a(x_i, c_k)}{\max\{b(x_i, c_k), a(x_i, c_k)\}}, \quad (6)$$

where

$$a(x_i, c_k) = \frac{1}{n_k - 1} \sum_{x_j \in c_k} d_e(x_i, x_j),$$

$$b(x_i, c_k) = \min_{c_r \in C \setminus c_k} \left\{ \frac{1}{n_r} \sum_{x_j \in c_r} d_e(x_i, x_j) \right\}.$$

- The Xie–Beni index (**XB**_↓) [38]. This is the ratio of the total variation to the minimum separation of the clusters and is defined by

$$XB(C) = \frac{\sum_{c_k \in C} \sum_{x_i \in c_k} d_e^2(x_i, \bar{c}_k)}{N \min_{c_k \in C} \min_{c_r \in C \setminus c_k} \{d_e^2(\bar{c}_k, \bar{c}_r)\}}. \quad (7)$$

- The generalized Dunn indices¹ (**gD31**_↑, **gD41**_↑, **gD51**_↑, **gD33**_↑, **gD43**_↑, and **gD53**_↑) [6]. These indices are modifications of the original Dunn index, and include a combination of three variations of δ (a separation criterion)

$$\delta_3(c_k, c_r) = \frac{1}{n_k \times n_r} \sum_{x_i \in c_k} \sum_{x_j \in c_r} d_e(x_i, x_j),$$

$$\delta_4(c_k, c_r) = d_e(\bar{c}_k, \bar{c}_r),$$

$$\delta_5(c_k, c_r) = \frac{1}{n_k + n_r} \left[\sum_{x_i \in c_k} d_e(x_i, \bar{c}_k) + \sum_{x_j \in c_r} d_e(x_j, \bar{c}_r) \right] \quad (8)$$

and two variations of Δ (a cohesion criterion)

$$\Delta_1(c_k) = \max_{x_i, x_j \in c_k} \{d_e(x_i, x_j)\},$$

$$\Delta_3(c_k) = \frac{2}{n_k} \sum_{x_i \in c_k} d_e(x_i, \bar{c}_k).$$

- The S_Dbw index (**SDbw**_↓) [18]. This is a ratio-type index that is based on the Euclidean norm $\|x\| = (x^T x)^{1/2}$, the standard deviation of the dataset $\sigma(X) = 1/|X| \sum_{x_i \in X} (x_i - \bar{X})^2$, and the standard deviation of the partition $\text{stdev}(C) = 1/K \sqrt{\sum_{c_k \in C} \|\sigma(c_k)\|}$. It is defined by

$$\begin{aligned} \text{Scat}(C) &= \frac{1}{K} \sum_{c_k \in C} \frac{\|\sigma(c_k)\|}{\|\sigma(X)\|}, \\ \text{Dbw}(C) &= \frac{1}{K(K-1)} \sum_{c_k \in C} \sum_{c_r \in C \setminus c_k} \frac{\text{den}(c_k, c_r)}{\max\{\text{den}(c_k), \text{den}(c_r)\}}, \\ \text{SDbw}(C) &= \text{Scat}(C) + \text{Dbw}(C), \end{aligned} \quad (9)$$

where

$$\begin{aligned} \text{den}(c_k) &= \sum_{x_i \in c_k} f(x_i, \bar{c}_k), \\ \text{den}(c_k, c_r) &= \sum_{x_i \in c_k \cup c_r} f\left(x_i, \frac{\bar{c}_k + \bar{c}_r}{2}\right), \end{aligned}$$

and

$$f(x_i, \bar{c}_k) = \begin{cases} 0 & \text{if } d_e(x_i, \bar{c}_k) > \text{stdev}(C) \\ 1 & \text{otherwise} \end{cases}.$$

- The CS index (**CS**_↓) [9]. This is a ratio-type index that quantifies the cohesion by using the cluster diameters. The measure of separation is based on the nearest neighbor distance between prototypes. It is computed by

$$CS(C) = \frac{\sum_{c_k \in C} \Delta(c_k)}{\sum_{c_k \in C} \min_{c_r \in C \setminus c_k} \{d_e(\bar{c}_k, \bar{c}_r)\}}, \quad (10)$$

where

$$\Delta(c_k) = \frac{1}{n_k} \sum_{x_i \in c_k} \max_{x_j \in c_k} \{d_e(x_i, x_j)\}.$$

- The Modified Davies–Bouldin index (**mDB**_↓) [25]. This variation of the DB index estimates the separation by the sum of the minimum distances between prototypes. It is defined by

$$\text{mDB}(C) = \frac{1}{K} \sum_{c_k \in C} \frac{\max_{c_r \in C \setminus c_k} \{S(c_k) + S(c_r)\}}{\min_{c_r \in C \setminus c_k} \{d_e(\bar{c}_k, \bar{c}_r)\}}. \quad (11)$$

- The Score Function index (**SF**_↑) [33]. This is a summation-type index in which the separation is measured by the sum of the distances from each centroid to the global prototype, and the cohesion is quantified by the mean distance of the objects to their respective centroids. It is computed by

$$\text{SF}(C) = 1 - \frac{1}{e^{\text{bcd}(C) - \text{wcd}(C)}}, \quad (12)$$

where

$$\text{wcd}(C) = \sum_{c_k \in C} \frac{1}{n_k} \sum_{x_i \in c_k} d_e(x_i, \bar{c}_k),$$

$$\text{bcd}(C) = \frac{\sum_{c_k \in C} n_k d_e(\bar{c}_k, \bar{X})}{N \times K}.$$

- The PBM index (**PBM**_↑) [26]. This index is defined as the product of cohesion and separation measures. The former is the sum of all pattern distances in a cluster to their respective centroids, whereas the latter is quantified by the maximum distance between the centroids. The PBM index is given by

$$\text{PBM}(C) = \left[\frac{\sum_{x_i \in X} d_e(x_i, \bar{X})}{\sum_{c_k \in C} \sum_{x_i \in c_k} d_e(x_i, \bar{c}_k)} \times \mathcal{D}(C) \right]^2, \quad (13)$$

where

$$\mathcal{D}(C) = \max_{c_k, c_r \in C} \{d_e(\bar{c}_k, \bar{c}_r)\}.$$

- The Symmetry index (**Sym**_↑) [4]. This ratio-type index is an adaptation of the PBM index [26]: the cohesion is quantified by the sum of the point symmetry distances in the same cluster, and the separation is quantified by the maximum Euclidean distance between the centroids. This index is defined by

$$\text{Sym}(C) = \frac{\max_{c_k, c_r \in C} \{d_e(\bar{c}_k, \bar{c}_r)\}}{\sum_{c_k \in C} \sum_{x_i \in c_k} d_{ps}(x_i, c_k)}. \quad (14)$$

- Point Symmetry-Distance indices² (**SDB**_↓ and **SDI**_↑) [31]. These are based on the PS distance and modify the cohesion criterion of the Davies–Bouldin and Dunn indices. The SDB index is

¹Bedeck and Pal proposed 18 variants, but we have selected those that were analyzed by Arbelaitz *et al.* [2], since these showed the best results.

²Saha and Bandyopadhyay presented nine indices but we have selected the two most representative.

computed the same way as the DB, but the computation of S is refined as follows:

$$S(c_k) = \frac{1}{n_k} \sum_{x_i \in c_k} d_{ps}(x_i, \bar{c}_k). \quad (15)$$

On the other hand, SDI is a modification of DI, where the cohesion criterion Δ is defined by

$$\Delta(c_k) = \max_{x_i \in c_k} \{d_{ps}(x_i, \bar{c}_k)\}. \quad (16)$$

- The COP index ($COP_{\downarrow}$) [16]. This is a ratio-type index in which the cohesion is quantified by the sum of the distances of the patterns to their respective centroids, and the separation is based on the furthest neighbor distance between patterns. It is defined by

$$COP(C) = \frac{1}{N} \sum_{c_k \in C} n_k \frac{\frac{1}{n_k} \sum_{x_i \in c_k} d_e(x_i, \bar{c}_k)}{\min_{x_i \notin c_k} \max_{x_j \in c_k} d_e(x_i, x_j)}. \quad (17)$$

- The SV index ($SV_{\uparrow}$) [42]. In this ratio-type index, the measure of cohesion is based on the distances from the border patterns to their respective centroids, and the measure of separation is based on the nearest neighbor distance between prototypes. It is computed by

$$SV(C) = \frac{\sum_{c_k \in C} \min_{c_r \in C \setminus c_k} \{d_e(\bar{c}_k, \bar{c}_r)\}}{\sum_{c_k \in C} \left(\frac{10}{n_k}\right) \max_{x_i \in c_k} \left(\frac{n_k}{10}\right) \{d_e(x_i, \bar{c}_k)\}}. \quad (18)$$

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