Optimal Recombination and Adaptive Restarts Improve GA Performance on the Asymmetric TSP

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ABSTRACT

We propose a new genetic algorithm (GA) with optimal recombination and adaptive restarts for the asymmetric travelling salesman problem (ATSP). The optimal recombination problem (ORP) is solved in a crossover operator based on a new exact algorithm that solves the ATSP on cubic digraphs. We use an adaptive restart rule based on the Schnabel census estimate. A computational experiment on the known benchmark instances shows that the proposed algorithm yields results competitive to those of state-of-the-art algorithms for the ATSP and confirms that the ORP may be used successfully in genetic algorithms.

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CCS CONCEPTS

• **Theory of computation** → **Routing and network design problems**; • **Computing methodologies** → *Randomized search*; **Genetic algorithms**.

KEYWORDS

Optimal recombination, Local search, Restart rule, Computational experiment

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1 SUMMARY OF OUR RESULTS

Genetic algorithms (GAs) are optimization methods inspired by natural biological evolution. A GA starts with an initial population, and goes from the current population to the next one applying selection, crossover (recombination) and mutation. The performance of a GA depends on the choice of the *crossover* operator, where the

components of parent solutions are combined to build new solutions. A supplementary problem that emerges in some crossover operators is called the *Optimal Recombination Problem* (ORP) [1, 6]. Given two feasible parent solutions, the ORP consists in finding the best possible offspring, where an allele of each gene is copied from one of the alleles of this gene in the parent solutions. Such recombination operators are called (strictly) gene transmitting [10]. Crossover operators solving the ORP exactly are usually called *optimized crossover* operators.

The first GA using ORP for the Maximum Independent Set problem appeared in the paper of Agarwal, Orlin and Tai [1]. Further results, e.g. [3, 13, 15], added more experimental support to expediency of solving optimal recombination problems in crossover operators (in particular for permutation problems).

The Travelling Salesman Problem (TSP) is one of the classical combinatorial optimization problems [8]. Given a complete digraph G with the set of vertices $V = \{v_1, \dots, v_n\}$, the set of arcs $A = \{(v_i, v_j) : v_i, v_j \in V, i \neq j\}$ and arc weights (lengths) $c(v_i, v_j) \geq 0$ of each arc $(v_i, v_j) \in A$, the TSP asks for a Hamiltonian circuit (a tour) with the minimum total length. If $c(v_i, v_j) \neq c(v_j, v_i)$ for at least one pair v_i, v_j then the TSP is called the Asymmetric Travelling Salesman Problem (ATSP).

We propose an ORP-based GA with adaptive restarts for the ATSP. The initial population is built using the greedy constructive heuristic of W. Zhang [16] (first solves an assignment problem, then applies the Karp patching) and the well known *arbitrary insertion* method [15]. The GA is based on the steady state replacement. A new mutation operator making random jumps in 3-opt or 4-opt neighborhoods is applied to parent solutions before recombination.

We use vectors of adjacencies for encoding tours of the ATSP. In these vectors, the immediate predecessor is indicated for each vertex. In the case of adjacency-based representation, the ORP aims at finding a travelling salesman's tour of minimum length among the circuits composed only of the arcs that are present in the parents, and contain all arcs common to both parents. The ORP for the ATSP is NP-hard [6], and may be considered as *Forced ATSP* on a digraph with bounded vertex degree, where the forced arcs correspond to common parent arcs. Then the ORP can be solved in $O(2^{d/4})$ time, where $d \leq 2n$ is the number of arcs which are present in one of the parents only. Here we use a special adaptation of the recursive algorithm of D. Eppstein[5], originally proposed for listing all Hamiltonian cycles in a graph of degree three.

In our GA, individuals of the initial and the final populations are improved by the 3-opt local search with the well-known “don't look bits” and “first improving move” strategies. Moreover, the optimized crossover may be considered as a best-improving move in

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the neighborhood defined by *two* (parent) solutions. The balance between different local search operators (3-opt on the initial and the final population and the optimized crossover during the evolution stage) is based on a GA restarting rule. In this paper, we consider two restarting rules. The first approach uses the classic restart rule, where the GA gets restarted as soon as the current iteration number becomes twice the iteration number when the best incumbent was found (denoted GA_{CL}). The second approach [7] is based on the Schnabel census method from biometrics, coupled with the maximum likelihood principle. This rule restarts the algorithm when the population diversity becomes so small that new solutions are unlikely to be found. The GA with this rule is denoted GA_{SC} .

We compare our ORP-based GA with various state-of-the-art approaches known for the ATSP:

- Memetic algorithm with Strategic Arc Crossover (SAX) and Recursive Arc Insertion (RAI) for local improvements ($MA_{SAX/RAI}$) of Buriol et al. [2],
- GA with Generalized Asymmetric Partition Crossover (GAPX) and 3-opt local optimization (GA_{GAPX}) of Tinós et al. [13],
- GA with Edge Assembly Crossover (EAX) and 3-opt local optimization of Nagata and Soler [9].

The listed GAs use adjacency-based crossover operators and apply a local search on GA iterations. The GAPX crossover from [13] produces a greedy approximate solution to the ORP.

An experimental evaluation was performed on 27 asymmetric instances from the TSPLIB library (series ftv, ft, and rbg, and individual problems br17, p43, ry48p, kro124p) [11]. We arranged the experiment so that both our GA using the restart rule based on the Schnabel census method GA_{SC} and an algorithm from the literature were given approximately equivalent CPU resource (scaling factors for the CPU-time limit are chosen on the basis a rough comparison of computers by performance table [4] and the CPU clock rates). We compared the frequencies of finding an optimum.

$MA_{SAX/RAI}$ [2] is one of the most well-known heuristics for the ATSP. Initially we compare GA_{SC} with $MA_{SAX/RAI}$. On each instance, GA_{SC} finds optima in at least 99% of the runs, and $MA_{SAX/RAI}$ in 84% of the runs on average. In 14 out of 27 instances, GA_{SC} finds an optimum more often than $MA_{SAX/RAI}$ (in all 14 cases the difference between the frequencies is statistically significant at level $p \leq 0.05$). Both algorithms demonstrate 100% frequency of obtaining an optimum on 13 problems.

GA_{GAPX} [13] presents 100% frequency of obtaining an optimum on all of 19 considered asymmetric instances from TSPLIB, given very large CPU time. GA_{SC} also showed 100% frequency of success under the CPU resource similar to [13].

GA_{EAX} [9] is one of the fastest metaheuristics, known for the ATSP. The average frequency of obtaining an optimum for GA_{SC} is 0.972, while that of GA_{EAX} is 0.953. In 11 out of 27 instances, GA_{SC} finds an optimum more often than GA_{EAX} (in 3 cases among these the difference between the frequencies is statistically significant). GA_{EAX} outperforms G_{SC} on four instances, but the difference between the frequencies of finding an optimum is not statistically significant. We also tested GA_{SC} on 128 instances (n is ranged from 64 to 315) from [12] and compared with GA_{EAX} .

Effects of Restarting Rules and Optimal Recombination. Preliminary computational experiments showed that by restarting our GA, we

avoid localization of the search and restore the population diversity, which leads to better results than in the case of no restarts, given the same amount of time.

We also compared two versions of our algorithm with restarts under the same conditions as in the experiment with GA_{EAX} . The GA_{SC} demonstrated better results than the GA_{CL} on 12 instances, and the difference between the frequencies was statistically significant in 5 cases. GA_{CL} slightly outperforms GA_{SC} on 4 instances, but the difference was statistically significant only in one case.

The optimized crossover operator, which solves the ORP exactly, may be considered as a deterministic “direct descendant” of the Directed Edge Crossover (DEC) [14]. The computational evaluation clearly shows an advantage of the ORP-based crossover over its randomized prototype DEC in our GA.

Conclusion

The proposed genetic algorithm for the ATSP uses the steady-state population management strategy, adjacency-based solutions representation, optimal recombination, local search and a restarting rule based on Schnabel census. The results of computational experiments show that the proposed GA yields results competitive or superior to those of other well-known algorithms.

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