Switching and Data Injection Attacks on Stochastic Cyber-Physical Systems: Modeling, Resilient Estimation, and Attack Mitigation

SZE ZHENG YONG, Arizona State University

MINGHUI ZHU, Pennsylvania State University

EMILIO FRAZZOLI, Swiss Federal Institute of Technology (ETH)

In this article, we consider the problem of attack-resilient state estimation, that is, to reliably estimate the true system states despite two classes of attacks: (i) attacks on the switching mechanisms and (ii) false data injection attacks on actuator and sensor signals, in the presence of stochastic process and measurement noise signals. We model the systems under attack as hidden mode stochastic switched linear systems with unknown inputs and propose the use of a multiple-model inference algorithm to tackle these security issues. Moreover, we characterize fundamental limitations to resilient estimation (e.g., upper bound on the number of tolerable signal attacks) and discuss the topics of attack detection, identification, and mitigation under this framework. Simulation examples of switching and false data injection attacks on a benchmark system and an IEEE 68-bus test system show the efficacy of our approach to recover resilient (i.e., asymptotically unbiased) state estimates as well as to identify and mitigate the attacks.

CCS Concepts: • **Security and privacy** → **Intrusion detection systems**; *Intrusion/anomaly detection and malware mitigation*; • **Mathematics of computing** → *Maximum likelihood estimation*; • **General and reference** → *Estimation*;

Additional Key Words and Phrases: Resilient systems, switching attacks, false data injection attacks

ACM Reference format:

Sze Zheng Yong, Minghui Zhu, and Emilio Frazzoli. 2018. Switching and Data Injection Attacks on Stochastic Cyber-Physical Systems: Modeling, Resilient Estimation, and Attack Mitigation. *ACM Trans. Cyber-Phys. Syst.* 2, 2, Article 9 (June 2018), 26 pages.

<https://doi.org/10.1145/3204439>

1 INTRODUCTION

Cyber-physical systems (CPS) are systems in which computational and communication elements collaborate to control physical entities, and, for networked CPS, the Internet of Things (IoT) interlinks these physical and cyber worlds in a continuous and close interaction. The cyber-physical

This article was supported by the National Science Foundation, Grant No. 1239182. M. Zhu is partially supported by ARO W911NF-13-1-0421 (MURI) and NSF CNS-1505664.

Authors' addresses: S. Z. Yong, 551 E Tyler Mall, ERC 305, School for Engineering of Matter, Transport and Energy, Arizona State University, Tempe, AZ 85287, USA; email: szyong@asu.edu; M. Zhu, 229 Electrical Engineering West, School of Electrical Engineering and Computer Science, Pennsylvania State University, University Park, PA 16802, USA; email: muz16@psu.edu; E. Frazzoli, ETH Zürich, Institute for Dynamic Systems and Control, ML K 32.1, Sonneggstrasse 3, 8006 Zürich, Switzerland; email: efrazzoli@ethz.ch.

Permission to make digital or hard copies of all or part of this work for personal or classroom use is granted without fee provided that copies are not made or distributed for profit or commercial advantage and that copies bear this notice and the full citation on the first page. Copyrights for components of this work owned by others than ACM must be honored. Abstracting with credit is permitted. To copy otherwise, or republish, to post on servers or to redistribute to lists, requires prior specific permission and/or a fee. Request permissions from permissions@acm.org.

© 2018 ACM 2378-962X/2018/06-ART9 \$15.00

<https://doi.org/10.1145/3204439>

coupling introduces new functions to control systems and improves their performance. However, these systems are also exposed to new cyber vulnerabilities. Such systems, which include the power grid, autonomous vehicles, medical devices, and so on, are usually *safety-critical* and if compromised or malfunctioning, can cause serious harm to the controlled physical entities and the people operating or utilizing them. Recent incidents of attacks on CPS, e.g., the Maroochy water breach, the StuxNet computer worm, and various industrial security incidents (Cárdenas et al. (2008a) and Farwell and Rohozinski (2011)), highlight a need for CPS and IoT security, and for new designs of resilient estimation and control; thus, this problem has garnered considerable interest in recent years (see, e.g., Humayd et al. (2017) for a survey).

Literature Review. Much of the early research focus has been on the characterization of undetectable attacks and on attack detection and identification techniques, which range from the use of data timestamps in Zhu and Martínez (2013) to anomaly detection methods using residuals (e.g., Mo and Sinopoli (2010), Weimer et al. (2012), and Kwon et al. (2013)) with empirically chosen thresholds to tradeoff between false alarms and probability of anomaly/attack detection. On the other hand, attack mitigation is typically considered from two perspectives—preventive and reactive (Cómbita et al. (2015))—where preventive mitigation identifies and removes system vulnerabilities to prevent exploitation (e.g., Dan and Sandberg (2010)) while reactive attack mitigation initiates countermeasures after detecting an attack and is mainly studied using game theory (e.g., Ma et al. (2013), Zhu and Martínez (2011), and Zhu and Basar (2015)).

However, the ability to reliably estimate the true system states despite attacks (i.e., resilient estimates) is just as desirable, if not more than attack detection or the resulting attack mitigation because the availability of resilient state estimates would, among others, allow for continued operation with the same controllers as in the case without attacks or for pricing/prediction based on the real unbiased state information despite attacks. This problem has been studied for both static systems (e.g., Liu et al. (2011), Kosut et al. (2011) and Liang et al. (2017), and references therein) as well as dynamic systems, as are considered in this article.

For deterministic linear dynamic systems under actuator and sensor signal attacks (e.g., via false data injection (Cárdenas et al. (2008b), Mo and Sinopoli (2010), and Pasqualetti et al. (2013)), the resilient state estimation problem has been mapped onto an ℓ_0 optimization problem that is NP-hard (Pasqualetti et al. (2013) and Fawzi et al. (2014)); thus, a relaxation to a convex problem is considered in Fawzi et al. (2014). Further extensions (Pajic et al. (2014, 2015)) compute the worst-case bound on the state estimate error in the presence of additive noise errors with known bounds, while Yong et al. (2016a) consider the resilient state estimation problem that is robust to bounded multiplicative and additive modeling and noise errors. However, these approaches do not apply in the presence of additive stochastic (unbounded) noise signals, which is one of the security issues we consider in this article. On the other hand, systems with stochastic noise signals but only sensor attacks are considered in Mishra et al. (2015).

In addition, attacks that exploit the switching vulnerability of CPS and IoT or that alter its network topology have been recently identified as a serious CPS security concern. Some instances of such vulnerability are attacks on the circuit breakers of a smart grid (Liu et al. (2013)) or on the logic mode (e.g., failsafe mode) of a traffic infrastructure (Ghena et al. (2014)), on the meter/sensor data network topology (Kim and Tong (2013)) and on the power system network topology (Weimer et al. (2012)). However, to the best of our knowledge, no resilient state estimators for dynamic systems have been developed to deal with this new class of attacks.

Our techniques are based on simultaneous input and state estimation (see, e.g., Gillijns and De Moor (2007a), Gillijns and De Moor (2007b), Yong et al. (2016c), and Yong et al. (2017)), where data injection attacks can be modeled as unknown inputs of dynamical systems. Of particular importance to our approach are the stability and optimality properties as well as their relationship to

strong detectability (Yong et al. (2016c)). Inspired by the multiple-model (MM) approach (see, e.g., Bar-Shalom et al. (2004) and Mazor et al. (1998) and references therein), our previous work (Yong et al. (2016b)) introduced an inference algorithm that estimates hidden modes, unknown inputs, and states simultaneously, which we now propose as the key tool to achieve resilient estimation.

Contributions. In this article, we introduce a resilient state estimation algorithm that outputs reliable estimates of the true system states despite false data injection attacks and switching attacks. To our best knowledge, our resilient estimation algorithm is the first that addresses switching attacks as well as the first that successfully deals with both actuator and sensor attacks in the presence of stochastic noise signals.

Our approach is built upon a general purpose inference algorithm developed in our previous work (Yong et al. (2016b)) for hidden mode stochastic switched linear systems with unknown inputs. The first novelty of the present article lies in the modeling of switching and false data injection attacks on CPS in the presence of stochastic noise signals as an instance of this system class. In doing so, we show that unbiased state estimates (i.e., resilient state estimates) can be asymptotically recovered with the algorithm in Yong et al. (2016b). Second, we characterize fundamental limitations to resilient estimation that is useful for preventative mitigation, such as the upper bound on the number of correctable/tolerable attacks, and consider the subject of attack detection. In addition, we provide sufficient conditions for designing unidentifiable attacks (from the attacker's perspective) and also sufficient conditions to obtain resilient state estimates even when the attacks are not identified (from the system operator/defender's perspective). Finally, we design an attack-mitigating and stabilizing feedback controller that contributes to the literature on non-game-theoretic reactive attack mitigation.

A preliminary version of this article was presented in Yong et al. (2015), and this article expands on those results and includes new sections on attack detection and identification, as well as attack mitigation.

Paper Organization. Section 2 provides a motivating example of switching and data injection attacks on a multi-area power system. In Section 3, we describe the modeling of switching and false data injection attacks on CPS, and state our assumptions/models of the system and attacker. Section 4.1 reviews the multiple-model algorithm and its nice properties from Yong et al. (2016b) and provides an interpretation of the general purpose algorithm in the context of resilient state estimation. The rest of Section 4 is dedicated to the novel study of fundamental limitations to attack resilience. Next, we focus on attack detection and identification in Section 5, and provide some sufficient conditions as guidelines for system operators/designers, while we design an attack-mitigating feedback controller in Section 6. Section 7 then demonstrates the effectiveness of our proposed approach on a benchmark system and an IEEE 68-bus test system. Finally, we conclude with some remarks in Section 8.

2 MOTIVATING EXAMPLE

To motivate the problem of resilient state estimation of stochastic CPS under switching and false data injection attacks, let us consider an example of a power system with three control areas, each consisting of generators and loads, with transmission/tie-lines providing interconnections between areas (see Figure 1).

A malicious agent is assumed to have access to circuit breakers that control the tie-lines (similar to Liu et al. (2013)), and is thus able to sever the connection between control areas. Depending on the topology of the tie-line interconnection graph, such attacks may correspond to a *node/vertex/bus attack* (disconnection of a control area from all others) or a *link/edge/line attack* (disabling of a specific tie-line between two control areas), i.e., the power flow across the tie lines

Fig. 1. A three-area power system with radial topology (corresponding to *node/bus attack*).

is altered. Moreover, we assume that the system dynamics and state measurements are subject to random noise and attacks via additive false data injection in the actuator and sensor signals.

The goal of resilient state estimation is thus to obtain unbiased state estimates despite switching attacks, i.e., attacks on switches/circuit breakers, and data injection attacks on actuators and sensors.

3 PROBLEM FORMULATION

3.1 Attack Modeling

We consider two different classes of possibly time-varying attacks on CPS:

Data Injection Attacks. Attacks on actuator and sensor signals via manipulation or injection with “false” signals of unknown *magnitude* and *location* (i.e., subset of attacked actuators or sensors). In other words, signal attacks consist of both *signal magnitude attacks* and *signal location attacks*. *Examples:* Denial of service, deceptive attacks via data injection (Cárdenas et al. (2008b) and Pasqualetti et al. (2013)). *Switching Attacks.* Attacks on the switching mechanisms that change the system’s *mode* of operation, or on the sensor data or interconnection network *topology*, which we will also refer to as *mode attacks*. *Examples:* Attack on circuit breakers (Liu et al. (2013)), power network topology (Weimer et al. (2012)), sensor data network (Kim and Tong (2013)), and logic switch of a traffic infrastructure (Ghena et al. (2014)).

3.1.1 Data Injection Attacks. For clarity of exposition, we assume for the moment that there is only one mode of operation, and that the linear system dynamics is not perturbed by stochastic noise signals:

$$x_{k+1} = A_k x_k + B_k(u_k + d_k^a), \quad y_k = C_k x_k + D_k(u_k + d_k^a) + d_k^s,$$

where $x_k \in \mathbb{R}^n$ is the continuous state, $y_k \in \mathbb{R}^\ell$ is the sensor output, $u_k \in \mathbb{R}^m$ is the known input, $d_k^a \in \mathbb{R}^m$ and $d_k^s \in \mathbb{R}^\ell$ are attack signals that are injected into the actuators and sensors, respectively. The attack signals are sparse, i.e., if sensor $i \in \{1, \dots, \ell\}$ is not attacked, then necessarily $d_k^{s,(i)} = 0$ for all time steps k ; otherwise $d_k^{s,(i)}$ can take any value. Since we do not know which sensor is attacked, we refer to this uncertainty as the *signal location attack*, and the arbitrary values that $d_k^{s,(i)}$ can take as the *signal magnitude attack*. The same observation holds for the attacks on actuators d_k^a .

If, in addition, we have knowledge of which actuators and sensors are vulnerable to data injection attacks, we will incorporate this information using $\bar{G}_k$ and $\bar{H}_k$ to result in the following system dynamics:

$$x_{k+1} = A_k x_k + B_k u_k + \bar{G}_k d_k^a, \quad y_k = C_k x_k + D_k u_k + \bar{D}_k d_k^a + \bar{H}_k d_k^s.$$

Fig. 2. Illustration of a switched linear system with unknown inputs as a hybrid automaton with two modes, q and q' , where the system dynamics in each mode is linear.

If no such knowledge is available, $\bar{G}_k = B_k$, $\bar{D}_k = D_k$, and $\bar{H}_k = I$. Moreover, in some cases, the actuator and sensor attack signals are known to be mixed and cannot be separated. To take this into consideration, we represent the potentially “mixed” attack signals with d_k and introduce corresponding G_k and H_k matrices to obtain

$$x_{k+1} = A_k x_k + B_k u_k + G_k d_k, \quad y_k = C_k x_k + D_k u_k + H_k d_k.$$

In the absence of mixed attack signals, $d_k = [(d_k^a)^\top (d_k^s)^\top]^\top$, $G_k = [\bar{G}_k \ 0]$, and $H_k = [\bar{D}_k \ \bar{H}_k]$. The description of these matrices will be made more precise in Section 3.2.

3.1.2 Switching Attacks. On the other hand, a system may have multiple modes of operation, denoted by the set $\mathcal{Q}^m$ of cardinality $t_m \triangleq |\mathcal{Q}^m|$, either through the presence of switching mechanisms or different configurations/topologies of the sensor data or interconnection network, i.e., each mode $q_k \in \mathcal{Q}^m$ has its corresponding set of system matrices, $\{A_k^{q_k}, B_k^{q_k}, C_k^{q_k}, D_k^{q_k}, G_k^{q_k}, H_k^{q_k}\}$. A *switching attack* or *mode attack* then refers to the ability of an attacker to choose and change the mode of operation q_k without the knowledge of the system operator/defender.

Attacker Model Assumptions. We do not constrain the malicious *signal magnitude attack* to be a signal of any type (random or strategic) nor to follow any model, thus no prior “useful” knowledge of the dynamics of d_k is available (uncorrelated with $\{d_\ell\}$ for all $k \neq \ell$, $\{w_\ell\}$ and $\{v_\ell\}$ for all ℓ).

3.2 System Description

In this section, we take the perspective of a system operator/defender, i.e., as one with the goal of obtaining resilient/reliable state estimates. Thus, our techniques include the modeling of the system in a way that facilitates the design of a resilient state estimation algorithm. Since we now assume that the system is perturbed by random, unbounded process, and measurement noise signals, we model the switching and false data injection attacks on a noisy dynamic system using a *hidden mode switched linear discrete-time stochastic system with unknown inputs* (i.e., a dynamical system with multiple modes of operation where the system dynamics in each mode is linear and stochastic, and the mode and some inputs are not known/measured; cf. Figure 2):

$$\begin{aligned} (x_{k+1}, q_k) &= (A_k^{q_k} x_k + B_k^{q_k} u_k^{q_k} + G_k^{q_k} d_k^{q_k} + w_k^{q_k}, q_k), & x_k &\in \mathcal{C}_{q_k}, \\ (x_k, q_k)^+ &= (x_k, \delta^{q_k}(x_k)), & x_k &\in \mathcal{D}_{q_k}, \\ y_k &= C_k^{q_k} x_k + D_k^{q_k} u_k^{q_k} + H_k^{q_k} d_k^{q_k} + v_k^{q_k}, \end{aligned} \quad (1)$$

where $x_k \in \mathbb{R}^n$ is the continuous system state and $q_k \in \mathcal{Q} = \{1, 2, \dots, \mathfrak{N}\}$ is the hidden discrete state or *mode*, which a malicious attacker has access to, while $\mathcal{C}_{q_k}$ and $\mathcal{D}_{q_k}$ are flow and jump sets, and $\delta^{q_k}(x_k)$ is the mode transition function. For more details on the hybrid systems formalism, see Goebel et al. (2009). For each mode q_k , $u_k^{q_k} \in U_{q_k} \subset \mathbb{R}^m$ is the known input, $d_k^{q_k} \in \mathbb{R}^p$ the unknown

input or *attack signal* and $y_k \in \mathbb{R}^l$ the output, whereas the corresponding process noise $w_k^{q_k} \in \mathbb{R}^n$ and measurement noise $v_k^{q_k} \in \mathbb{R}^l$ are mutually uncorrelated, zero-mean Gaussian white random signals with known covariance matrices, $Q_k^{q_k} = \mathbb{E}[w_k^{q_k} w_k^{q_k \top}] \geq 0$ and $R_k^{q_k} = \mathbb{E}[v_k^{q_k} v_k^{q_k \top}] > 0$, respectively. Moreover, x_0 is independent of $v_k^{q_k}$ and $w_k^{q_k}$ for all k .

Our stochastic CPS model in Equation (1) is capable of capturing the unknowns or uncertainties introduced by the switching and data injection attacks to the system of interest that are both *categorical* and *continuous*. The hidden mode allows us to model the categorical nature of the switching and data injection attacks (*mode attack* and *signal location attack*), whereas the unknown input captures the continuous nature of the *signal magnitude attacks*.

At any particular time k , the stochastic CPS is in precisely one of its modes, which is not measured, hence *hidden*. The following remark motivates the consideration of more modes than those corresponding to switching/mode attacks given by Q^m .

Remark 3.1. Suppose again for simplicity that there is only one mode of operation, i.e., Q^m is a singleton. Then, in the ideal scenario for the system operator/defender that the system (A_k, G_k, C_k, H_k) is strongly detectable, unbiased estimates of states x_k can be obtained and the attack signal d_k can also be identified (Yong et al. (2016c)). Unfortunately, this property does not hold in general. In fact, Theorem 4.3 will reveal that we need a small number of vulnerable actuators and sensors to enable resilient state estimation. Thus, we will exploit the sparse nature of the false data injection attacks, and consider more models/modes in a set Q^d , each with fewer vulnerable actuators and sensors to make sure that strong detectability holds.

Thus, the modes we consider in the model set $Q \triangleq Q^m \times Q^d$ (as described below), whose cardinality will be characterized in Theorem 4.4, include:

- (i) the modes of operation, Q^m , that attacked switching mechanisms (e.g., circuit breakers, relays) operate via access to the jump set $\mathcal{D}_{q_k}$ and the mode transition function $\delta^{q_k}(\cdot)$, or the possible interconnection network topologies that dictate the system matrices, $A_k^{q_k}$ and $B_k^{q_k}$, and the sensor data network topologies, $C_k^{q_k}$ and $D_k^{q_k}$, that an attacker can choose (*mode attack*), as well as
- (ii) the different hypotheses for each mode, Q^d , about which actuators and sensors are attacked or not attacked, represented by $G_k^{q_k}$ and $H_k^{q_k}$ (*signal location attack*).

More precisely, we assume that $G_k^{q_k} \triangleq \mathcal{G}_k I_G^{q_k}$ and $H_k^{q_k} \triangleq \mathcal{H}_k I_H^{q_k}$ for some input matrices $\mathcal{G}_k \in \mathbb{R}^{n \times t_a}$ and $\mathcal{H}_k \in \mathbb{R}^{\ell \times t_s}$, where t_a and t_s are the number of actuator and sensor signals that are *vulnerable*, respectively. Note that $p_a^{q_k} \leq t_a \leq m$ and $p_s^{q_k} \leq t_s \leq \ell$, i.e., the number of *attacked* actuator signals $p_a^{q_k}$ under mode/hypothesis q_k cannot exceed the number of *vulnerable* actuators and in turn cannot exceed the total number of actuators m . The same holds for $p_s^{q_k}$ *attacked* sensors from t_s *vulnerable* sensors out of ℓ measurements. Moreover, we assume that the maximum total number of attacks is $p \triangleq p_a^{q_k} + p_s^{q_k} \leq p^*$, where p^* is the maximum number of asymptotically correctable signal attacks (cf. Theorem 4.3 for its characterization).

On the other hand, $I_G^{q_k} \in \mathbb{R}^{t_a \times p}$ and $I_H^{q_k} \in \mathbb{R}^{t_s \times p}$ are index matrices such that $d_k^{a, q_k} \triangleq I_G^{q_k} d_k$ and $d_k^{s, q_k} \triangleq I_H^{q_k} d_k$ represent the subvectors of $d_k \in \mathbb{R}^p$ representing *signal magnitude attacks* on the actuators and sensors, respectively. These matrices provide a means of incorporating information about the way the attacks affect the system, e.g., when the same attack is injected to an actuator and a sensor, or if some signals are *not* attacked, according to a particular hypothesis/mode q_k about the signal attack location. It is noteworthy that our approach specifies which actuators and sensors are *not attacked*, in contrast to the approach in Mishra et al. (2015), which removes *attacked* sensor measurements and is not applicable for actuator attacks.

The following are some examples for choosing $\mathcal{G}_k$, $\mathcal{H}_k$, $\mathcal{I}_G^{q_k}$, and $\mathcal{I}_H^{q_k}$ to encode additional information about the nature/structure of data injection attacks.

Example 3.2. For a two-state system with two vulnerable actuators and one vulnerable sensor, if the same attack signal is injected into the first actuator and the sensor under the hypothesis corresponding to mode q_k , then $\mathcal{G}_k = I_2$, $\mathcal{H}_k = 1$, $\mathcal{I}_G^{q_k} = I_2$, and $\mathcal{I}_H^{q_k} = [1 \ 0]$. In this case, we obtain $G_k^{q_k} = I_2$ and $H_k^{q_k} = [1 \ 0]$.

Example 3.3. For a three-state system with three actuators and two sensors, if the first actuator and the second sensor are not vulnerable and there are three attacks according to the hypothesis corresponding to mode q_k , then $\mathcal{G}_k = \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}$, $\mathcal{H}_k = [1]$, $\mathcal{I}_G^{q_k} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$ and $\mathcal{I}_H^{q_k} = [0 \ 0 \ 1]$. In this case, we have $G_k^{q_k} = \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$ and $H_k^{q_k} = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$.

System Assumptions. We require that the system is *strongly detectable*¹ in each mode. In fact, strong detectability is *necessary* for each mode to asymptotically correct the unknown attack signals (also necessary for deterministic systems (Sundaram and Hadjicostis (2007, Theorem 6))). Note also that strongly detectable systems need not be stable (cf. example in the proof of Theorem 4.3), but rather that the strongly undetectable modes of such systems are stable.

Knowledge of the System Operator/Defender. The matrices $A_k^{q_k}$, $B_k^{q_k}$, $G_k^{q_k}$, $C_k^{q_k}$, $D_k^{q_k}$, and $H_k^{q_k}$ are known, as well as the system assumption of strong detectability in each mode. Moreover, the only knowledge of the defender concerning the malicious attacker is about (i) the upper bound on the number of actuators/sensors that can be attacked, p , and (ii) the switching mechanisms/topologies that may be compromised. The upper bound p in the former assumption allows the defender, in the worst case, to enumerate all possible combinations of $G_k^{q_k}$ and $H_k^{q_k}$. On the other hand, the latter assumption allows the defender to consider all possible topologies/modes of operations.

Alternatively, the above assumptions on the system and attackers can be viewed as recommendations or guidelines for system designers/operators to secure their systems as a preventative attack mitigation measure. For instance, the requirement of strong detectability allows system designers to determine which actuators or sensors need to be safeguarded to guarantee resilient estimation.

3.3 Security Problem Statement

With the above characterization, the resilient state estimation problem is identical to the mode, state, and input estimation problem, where the unknown inputs represent the unknown signal magnitude attacks and each mode/model represents an *attack mode* (resulting from the unknown mode attacks and unknown signal attack locations). The *objective* of this article is as follows:

PROBLEM 1. *Given a stochastic cyber-physical system described by Equation (1),*

- (1) *Develop a resilient estimator that asymptotically recovers unbiased estimates of the system state and attack signal irrespective of the location or magnitude of attacks on its actuators and sensors as well as switching mechanism/topology (mode) attacks.*
- (2) *Characterize fundamental limitations associated to the inference algorithm we develop, specifically the maximum number of asymptotically correctable signal attacks and the maximum number of required models with our MM approach.*

¹A linear system is *strongly detectable* if $y_k = 0 \ \forall k \geq 0$ implies $x_k \rightarrow 0$ as $k \rightarrow \infty$ for all initial states x_0 and input sequences $\{d_i\}_{i \in \mathbb{N}}$ (see Yong et al. (2016c, Section 3.2) for necessary and sufficient conditions for this property).

Fig. 3. Multiple-model framework for hidden mode, input, and state estimation. Each model consists of a mode-matched filter that uses known inputs u and outputs y to estimate states $\hat{x}$ and unknown inputs $\hat{d}$, in addition to generalized innovations ν that, in turn, determine the most probable mode $\hat{q} = \arg \max_{j \in \{1, \dots, N\}} \mu^j$.

- (3) Study the conditions under which attacks can be detected or noticed (attack detection) and under which the attack strategy can be identified (attack identification) using the resilient state estimator we develop.
- (4) Design tools for attack mitigation via feedback control with attack rejection.

4 RESILIENT STATE ESTIMATION

To achieve resilient state estimation against switching attacks in the presence of stochastic process and measurement noise signals, we have shown in the previous section that the system under switching and false data injection attacks is representable as a hidden mode switched linear system with unknown inputs given in Equation (1). Since we do not know the true model (i.e., the attack strategy corresponding to the true *mode attack* and *signal location attack*), combinations of possible attack strategies need to be considered, and, as such, the MM estimation approach is a natural choice for solving this problem. Thus, we propose the use of the general purpose MM algorithm that we previously designed and applied to intention-aware vehicles (Yong et al. (2016b)) as our resilient state estimation algorithm for solving Problem 1.1.

We will begin with a brief summary of the MM inference algorithm and its nice properties (Yong et al. (2016b)). Then, we consider Problem 1.2 and characterize some fundamental limitations to resilient estimation in Section 4.2.

4.1 Resilient State Estimation Algorithm and Properties

4.1.1 Multiple-Model State and Input Filtering Algorithm. The MM approach we take is inspired by the MM filtering algorithms for hidden mode hybrid systems with *known* inputs (e.g., Bar-Shalom et al. (2004) and Mazor et al. (1998) and references therein), that have been widely applied for target tracking. Our MM framework (see Figure 3) consists of the parallel implementation of a bank of *input and state filters* (Yong et al. (2016c)) with each model corresponding to a system

Fig. 4. Illustration of the exchange of signals/information between components of a static multiple-model estimator with two mode-matched input and state filters.

mode (i.e., of *mode-matched filters that simultaneously estimate states and unknown inputs from sensor measurements and known inputs for each mode*). The objective of the MM approach is then to decide which model/mode is the best representation of the current system mode as well as to estimate the state and unknown input of the system based on this decision.

In this subsection, we provide an *abbreviated review* of the MM approach for simultaneous mode, state and unknown input estimation given in Yong et al. (2016b). Two variants of the MM inference algorithm—static and dynamic—were proposed in that work. The latter provides a possibility of incorporating prior knowledge about the switching strategy of the attack. However, we assume no such knowledge about the malicious agent; thus we consider only the static variant (cf. Algorithm 2 and Figure 4), which consists of: (i) a bank of mode-matched filters, and (ii) a likelihood-based approach for computing model probability.

Mode-Matched Filters. The bank of filters is comprised of $\mathfrak{N}$ simultaneous state and input filters, one for each mode, based on the optimal recursive filter developed in Yong et al. (2016c) (superscript q_k omitted to increase readability; cf. Algorithm 1).

Unknown Input Estimation:

$$\begin{aligned} \hat{d}_{1,k} &= M_{1,k}(z_{1,k} - C_{1,k}\hat{x}_{k|k} - D_{1,k}u_k), \\ \hat{d}_{2,k-1} &= M_{2,k}(z_{2,k} - C_{2,k}\hat{x}_{k|k-1} - D_{2,k}u_k), \\ \hat{d}_{k-1} &= V_{1,k-1}\hat{d}_{1,k-1} + V_{2,k-1}\hat{d}_{2,k-1}, \end{aligned} \quad (2)$$

Time Update:

$$\begin{aligned} \hat{x}_{k|k-1} &= A_{k-1}\hat{x}_{k-1|k-1} + B_{k-1}u_{k-1} + G_{1,k-1}\hat{d}_{1,k-1}, \\ \hat{x}_{k|k}^* &= \hat{x}_{k|k-1} + G_{2,k-1}\hat{d}_{2,k-1}, \end{aligned} \quad (3)$$

Measurement Update:

$$\hat{x}_{k|k} = \hat{x}_{k|k}^* + \tilde{L}_k(z_{2,k} - C_{2,k}\hat{x}_{k|k}^* - D_{2,k}u_k), \quad (4)$$

where $\hat{x}_{k-1|k-1}$, $\hat{d}_{1,k-1}$, $\hat{d}_{2,k-1}$ and $\hat{d}_{k-1}$ denote the optimal estimates of x_{k-1} , $d_{1,k-1}$, $d_{2,k-1}$ and d_{k-1} . The remaining notations are best understood in the context of the system transformation described in Appendix A.1. Due to space constraints, the filter derivation as well as necessary and sufficient conditions for filter stability and optimality are omitted; the reader is referred to Yong et al. (2016c) for details.

Mode Probability Computation. To compute the probability of each mode, the MM approach exploits the whiteness property (Yong et al. (2016b, Theorem 1)) of the *generalized innovation* sequence, v_k , defined as

$$v_k \triangleq \tilde{\Gamma}_k(z_{2,k} - C_{2,k}\hat{x}_{k|k}^* - D_{2,k}u_k), \quad (5)$$

i.e., $v_k \sim \mathcal{N}(0, S_k)$ (a multivariate normal distribution) with covariance $S_k \triangleq \mathbb{E}[v_k v_k^\top] = \tilde{\Gamma}_k \tilde{R}_{2,k}^* \tilde{\Gamma}_k^\top$ and where $\tilde{\Gamma}_k$ is chosen such that S_k is invertible and $\tilde{R}_{2,k}^*$ is given in Algorithm 1. In the context of resilient estimation, the generalized innovation represents the residual signal where the effects of false data injection attacks have been removed. Then, using this “attack-free” generalized innovation, we define the *likelihood function* for each mode q at time k conditioned on all prior measurements Z^{k-1} :

$$\mathcal{L}(q_k | z_{2,k}) \triangleq \mathcal{N}(v_k^{q_k}; 0, S_k^{q_k}) = \frac{\exp(-\frac{1}{2} v_k^{q_k \top} (S_k^{q_k})^{-1} v_k^{q_k})}{\sqrt{|2\pi S_k^{q_k}|}}. \quad (6)$$

Then, using Bayes’ rule, the posterior probability μ_k^j for each mode j is recursively computed from the prior probability μ_{k-1}^j using

$$\mu_k^j = P(q_k = j | z_{1,k}, z_{2,k}, Z^{k-1}) = \frac{\mathcal{N}(v_k^j; 0, S_k^j) \mu_{k-1}^j}{\sum_{i=1}^{\mathfrak{N}} \mathcal{N}(v_k^i; 0, S_k^i) \mu_{k-1}^i}. \quad (7)$$

Note that a heuristic lower bound on all mode probabilities is imposed such that the modes are kept alive in case of a switch in the attacker’s strategy. Finally, based on the posterior mode probabilities, the most likely mode at each time k , $\hat{q}_k$, and the associated state and input estimates and covariances, $\hat{x}_{k|k}$, $\hat{d}_k$, $P_{k|k}^x$, and P_k^d , are determined:

$$\hat{q}_k = j^* = \arg \max_{j \in \{1, \dots, \mathfrak{N}\}} \mu_k^j, \quad \hat{x}_{k|k} = \hat{x}_{k|k}^{j^*}, \quad \hat{d}_k = \hat{d}_{k|k}^{j^*}, \quad P_{k|k}^x = P_{k|k}^{x, j^*}, \quad P_k^d = P_k^{d, j^*}. \quad (8)$$

4.1.2 Properties of the Resilient State Estimator. Our previous work (Yong et al. (2016b)) shows that the resilient state estimator has nice asymptotic properties, namely (i) *mean consistency*, i.e., the geometric mean of the mode probability for the true model $* \in \mathcal{Q}$ asymptotically converges to 1 for all initial mode probabilities (Yong et al. (2016b, Theorem 8)) and (ii) *asymptotic optimality*, i.e., the state and input estimates in Equation (8) converge on average to optimal state and input estimates in the minimum variance unbiased sense (Yong et al. (2016b, Corollary 13)).

4.2 Fundamental Limitations of Attack-Resilient Estimation

Next, we consider Problem 1.2 and characterize fundamental limitations of the attack-resilient estimation problem and of our multiple mode filtering approach, which constitutes one of the main results in this article. First, assuming for the moment that there is only one mode of operation (no switching attacks), we will upper bound the number of asymptotically correctable signal attacks/errors (i.e., signal attacks whose effects can be asymptotically negated or cancelled such that unbiased state estimates are still available). Then, we provide the maximum number of models that is required by our MM approach to obtain resilient estimates despite attacks.

4.2.1 Number of Asymptotically Correctable Signal Attacks. More formally, we introduce the following definition for only data injection attacks, which in itself is an interesting research problem in the CPS security community.

Definition 4.1 (Asymptotically/Exponentially Correctable Signal Attacks). We say that p actuators and sensors signal attacks are asymptotically/exponentially correctable, if for any initial state

$x_0 \in \mathbb{R}^n$ and signal attack sequence $\{d_j\}_{j \in \mathbb{N}}$ in $\mathbb{R}^p$, we have an estimator such that the estimate bias asymptotically/exponentially tends to zero, i.e., $\mathbb{E}[\hat{x}_k - x_k] \rightarrow 0$ (and $\mathbb{E}[\hat{d}_{k-1} - d_{k-1}] \rightarrow 0$) as $k \rightarrow \infty$.

Remark 4.2. Note the distinction in the definitions of asymptotically/exponentially correctable signal attacks in Definition 4.1 and of correctable signal attacks in Fawzi et al. (2014, Definition 1). Their definition implies finite-time estimation and is related to strong observability (Fawzi et al. (2014)). Due to the new challenges of further considering stochastic noise signals and mode switching, we adopt the weaker notion of asymptotic estimation, which only requires a “weaker” condition of strong detectability (implied by strong observability (Yong et al. (2016c))). This is mainly for the sake of theoretical analysis. Simulation results demonstrate that our algorithm has practically finite-time convergence.

ALGORITHM 1: OPT-FILTER finds the optimal state and input estimates for mode q_k

Input: $q_k, \hat{x}_{k-1|k-1}^{q_k}, \hat{d}_{1,k-1}^{q_k}, P_{k-1|k-1}^{x,q_k}, P_{1,k-1}^{x,d,q_k}, P_{1,k-1}^{d,q_k}$ [superscript q_k omitted in the following]

▷ Estimation of $d_{2,k-1}$ and d_{k-1}

$$\hat{A}_{k-1} = A_{k-1} - G_{1,k-1}M_{1,k-1}C_{1,k-1};$$

$$\hat{Q}_{k-1} = G_{1,k-1}M_{1,k-1}R_{1,k-1}M_{1,k-1}^\top G_{1,k-1}^\top + Q_{k-1};$$

$$\tilde{P}_k = \hat{A}_{k-1}P_{k-1|k-1}^x \hat{A}_{k-1}^\top + \hat{Q}_{k-1};$$

$$\tilde{R}_{2,k} = C_{2,k}\tilde{P}_k C_{2,k}^\top + R_{2,k};$$

$$P_{2,k-1}^d = (G_{2,k-1}^\top C_{2,k}^\top \tilde{R}_{2,k}^{-1} C_{2,k} G_{2,k-1})^{-1};$$

$$M_{2,k} = P_{2,k-1}^d G_{2,k-1}^\top C_{2,k}^\top \tilde{R}_{2,k}^{-1};$$

$$\hat{x}_{k|k-1} = A_{k-1}\hat{x}_{k-1|k-1} + B_{k-1}u_{k-1} + G_{1,k-1}\hat{d}_{1,k-1};$$

$$\hat{d}_{2,k-1} = M_{2,k}(z_{2,k} - C_{2,k}\hat{x}_{k|k-1} - D_{2,k}u_k);$$

$$\hat{d}_{k-1} = V_{1,k-1}\hat{d}_{1,k-1} + V_{2,k-1}\hat{d}_{2,k-1};$$

$$P_{12,k-1}^d = M_{1,k-1}C_{1,k-1}P_{k-1|k-1}^x A_{k-1}^\top C_{2,k}^\top M_{2,k}^\top - P_{1,k-1}^d G_{1,k-1}^\top C_{2,k}^\top M_{2,k}^\top;$$

$$P_{k-1}^d = V_{k-1} \begin{bmatrix} P_{1,k-1}^d & P_{12,k-1}^d \\ P_{12,k-1}^{d\top} & P_{2,k-1}^d \end{bmatrix} V_{k-1}^\top;$$

▷ Time update

$$\hat{x}_{k|k}^\star = \hat{x}_{k|k-1} + G_{2,k-1}\hat{d}_{2,k-1};$$

$$P_{k|k}^{\star x} = G_{2,k-1}M_{2,k}R_{2,k}M_{2,k}^\top G_{2,k-1}^\top + (I - G_{2,k-1}M_{2,k}C_{2,k})\tilde{P}_k(I - G_{2,k-1}M_{2,k}C_{2,k})^\top;$$

$$\tilde{R}_{2,k}^\star = C_{2,k}P_{k|k}^{\star x} C_{2,k}^\top + R_{2,k} - C_{2,k}G_{2,k-1}M_{2,k}R_{2,k} - R_{2,k}M_{2,k}^\top G_{2,k-1}^\top C_{2,k};$$

▷ Measurement update

$$\tilde{P}_k = P_{k|k}^{\star x} C_{2,k}^\top - G_{2,k-1}M_{2,k}R_{2,k};$$

$$\tilde{L}_k = \tilde{P}_k \tilde{R}_{2,k}^{\star\top};$$

$$\hat{x}_{k|k}^\star = \hat{x}_{k|k}^\star + \tilde{L}_k(z_{2,k} - C_{2,k}\hat{x}_{k|k}^\star - D_{2,k}u_k);$$

$$P_{k|k}^x = \tilde{L}_k R_{2,k}^\star \tilde{L}_k^\top - \tilde{L}_k \tilde{P}_k^\top - \tilde{P}_k \tilde{L}_k^\top;$$

▷ Estimation of $d_{1,k}$

$$\tilde{R}_{1,k} = C_{1,k}P_{k|k}^x C_{1,k}^\top + R_{1,k};$$

$$M_{1,k} = \Sigma_k^{-1};$$

$$P_{1,k}^d = M_{1,k}\tilde{R}_{1,k}M_{1,k};$$

$$\hat{d}_{1,k} = M_{1,k}(z_{1,k} - C_{1,k}\hat{x}_{k|k}^\star - D_{1,k}u_k);$$

return $\tilde{R}_{2,k}^{\star,q_k}, \hat{x}_{k|k}^{\star,q_k}$

ALGORITHM 2: RESILIENT STATE ESTIMATOR (STATIC-MM-ESTIMATOR) finds resilient state estimates corresponding to most likely mode

Input: $\forall j \in \{1, 2, \dots, \mathfrak{R}\}: \hat{x}_{0|0}^j; \mu_0^j;$

$$\hat{d}_{1,0}^j = (\Sigma_0^j)^{-1}(z_{1,0}^j - C_{1,0}^j \hat{x}_{0|0}^j - D_{1,0}^j u_0); P_{1,0}^{d,j} = (\Sigma_0^j)^{-1}(C_{1,0}^j P_{0|0}^{x,j} C_{1,0}^{j\top} + R_{1,0}^j)(\Sigma_0^j)^{-1};$$

for $k = 1$ **to** N **do**

for $j = 1$ **to** $\mathfrak{R}$ **do**

$\triangleright$ Mode-Matched Filtering

 Run OPT-FILTER($j, \hat{x}_{k-1|k-1}^j, \hat{d}_{1,k-1}^j, P_{k-1|k-1}^{x,j}, P_{1,k-1}^{d,j}$);

$$\bar{v}_k^j \triangleq z_{2,k}^j - C_{2,k}^j \hat{x}_{k|k}^{*,j} - D_{2,k}^j u_k;$$

$$\mathcal{L}(j|z_{2,k}^j) = \frac{1}{(2\pi)^{p_R^j/2} |\tilde{R}_{2,k}^{j,*}|^{1/2}} \exp\left(-\frac{\bar{v}_k^{j\top} \tilde{R}_{2,k}^{j,*} \bar{v}_k^j}{2}\right);$$

end

for $j = 1$ **to** $\mathfrak{R}$ **do**

$\triangleright$ Mode Probability Update (small $\epsilon > 0$)

$$\bar{\mu}_k^j = \max\{\mathcal{L}(j|z_{2,k}^j) \mu_{k-1}^j, \epsilon\};$$

end

for $j = 1$ **to** $\mathfrak{R}$ **do**

$\triangleright$ Mode Probability Update (normalization)

$$\mu_k^j = \frac{\bar{\mu}_k^j}{\sum_{\ell=1}^{\mathfrak{R}} \bar{\mu}_k^\ell};$$

$\triangleright$ Output

 Compute Equation (8);

end

end

return $\hat{x}_{k|k}, P_{k|k}^x$

To derive an estimation-theoretic upper bound on the maximum number of signal attacks that can be asymptotically corrected, we assume that the true model or mode ($q_k = *$) is known. Thus, the resilient state estimation problem is identical to the state and input estimation problem in Yong et al. (2016c), where the unknown inputs represent the attacks on the actuator and sensor signals. It has been shown in Yong et al. (2016c) that unbiased states (and also unknown inputs) can be obtained asymptotically (exponentially) only if the system is strongly detectable (cf. Yong et al. (2016c) for more details, e.g., regarding filter stability and existence). With this in mind, the upper bound on the maximum number of signal attacks that can be asymptotically (exponentially) corrected is given as follows:

THEOREM 4.3 (MAXIMUM CORRECTABLE DATA INJECTION ATTACKS). *The maximum number of asymptotically (exponentially fast) correctable actuators and sensors signal attacks, p^* , for system Equation (1) is equal to the number of sensors, l , i.e., $p^* \leq l$ and the upper bound is achievable.*

PROOF. A necessary and sufficient condition for strong detectability (with the true model $q_k = *$) is given in Yong et al. (2016c) as

$$\text{rk} \begin{bmatrix} zI - A^* & -G^* \\ C^* & H^* \end{bmatrix} = n + p^*, \quad \forall z \in \mathbb{C}, |z| \geq 1. \quad (9)$$

Since the above system matrix has only $n + l$ rows, it follows that its rank is at most $n + l$. Thus, from the necessary condition for Equation (9), we obtain $n + p^* \leq n + l \Rightarrow p^* \leq l$. We show that the upper bound is achievable using the example of the discrete-time equivalent model (with time step $\Delta t = 0.1$ s) of the smart grid case study in Liu et al. (2013), where in both circuit breaker modes, $A = \begin{bmatrix} 0.9520 & 0.0936 \\ -0.9358 & 0.8584 \end{bmatrix}$ and $G = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$. If the first state is measured but compromised (e.g., $C = \begin{bmatrix} 1 & 0 \end{bmatrix}$ and $H = 1 \Rightarrow p^* = l$), it can be verified that the system is strongly detectable, i.e., with two invariant zeros at $\{0.9945 \pm 0.0311j\}$ that are strictly in the unit circle in the complex plane. Similarly, it can be verified that the unstable system with matrices $A = \begin{bmatrix} 1.5 & 1 \\ 0 & 0.1 \end{bmatrix}$, $G = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$, $C = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ and $H = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$ (i.e., with $p^* = l$) has an invariant zero at $\{0.1\}$ and is hence strongly detectable. Thus, in both cases, the optimal filter in Yong et al. (2016c) can be applied and unbiased state estimates can be asymptotically achieved when $p^* = l$. $\square$

The theorem above implies that for each mode of operation that results from switching attacks, the total number of vulnerable actuators and sensors must not exceed the number of measurements. Moreover, it is worth reiterating that the necessity of strong detectability can serve as a guide for preventative attack mitigation by determining which actuators or sensors need to be safeguarded to guarantee resilient estimation. Since strong detectability is a system property that is independent of the filter design, the necessity of this property can be viewed as a fundamental limitation for resilient estimation, i.e., the ability to asymptotically/exponentially obtain unbiased estimates.

4.2.2 Number of Required Models for Estimation Resilience. Then, in a similar spirit as the attack set identification approach of Weimer et al. (2012) and Pasqualetti et al. (2013), in which a bank of deterministic residuals are computed to determine the true attack set (but not the magnitude of the attacks), we consider a bank of filters to find the most probable model/mode. We now characterize the maximum number of models $\mathfrak{N}^*$ that need to be considered with the MM approach in Section 4.1 (which is independent of the size of the system, e.g., the number of buses in a power system):

THEOREM 4.4 (MAXIMUM NUMBER OF MODELS/MODES). *Suppose there are t_a actuators and t_s sensors, and at most $p \leq l$ of these signals are attacked. Suppose also that there are t_m possible attack modes (mode attack). Then, the combinatorial number of all possible models, and hence the maximum number of models that need to be considered with the MM approach, is*

$$\mathfrak{N}^* = t_m \binom{t_a + t_s}{p} = t_m \binom{t_a + t_s}{t_a + t_s - p}.$$

PROOF. It is sufficient to consider only models corresponding to the maximum number of attacks p . All models with strictly less than p attacks are contained in this set of models with the attack vectors having some identically zero elements for which our estimation algorithm is still applicable. Thus, we only need to consider combinations of p attacks among $t_a + t_s$ sensors and actuators for each of the t_m attack modes of operation/topologies. Note that this number is the maximum because resilience may be achievable with less models: For instance, when $t_m = 1$, $t_a = 0$ and $t_s = 2 = l$, $p = 1$, $A = \begin{bmatrix} 0.1 & 1 \\ 0 & 0.2 \end{bmatrix}$, and $C = I_2$, we have $\mathfrak{N}^* = 2$, but it can be verified that with $G = 0_{2 \times 2}$ and $H = I_2$ (only one model, i.e., $1 = \mathfrak{N} < \mathfrak{N}^*$), the system is strongly detectable. $\square$

Remark 4.5. If $\mathfrak{N} > 1$, the MM approach requires that the number of attacks is strictly less than the number of sensor measurements, i.e., $p < l$. Otherwise, the generalized innovation Equation (5) is empty and we have no means of selecting the “best” model, i.e., of computing mode probabilities.

We now discuss how the availability of additional knowledge about the data injection attack strategies may influence the number of models that needs to be considered in relation to the number of models $\mathfrak{N}^*$ in Theorem 4.4 when such knowledge is not available. Suppose we have additional knowledge that there are at most $n_a \leq t_a$ and $n_s \leq t_s$ attacks on the actuators and sensors, respectively, with a total of p attacks (where $p \leq l$ and $p \leq n_a + n_s$), then the maximum number of models that are required,

$$\mathfrak{N}^* = t_m \sum_{i=0}^{\min\{n_a, p\}} \binom{t_a}{i} \binom{t_s}{\min\{p-i, n_s\}},$$

is less than the number required in the combinatorial case in Theorem 4.4.

On the other hand, the knowledge that less actuators or sensors are vulnerable may actually increase the number of models, as shown in the following example with $t_m = 1$ (one mode of operation), $n_a = 0$ (no attacks on actuators), $A = \begin{bmatrix} 0.1 & 1 \\ 0 & 1.2 \end{bmatrix}$, and $C = I$. Suppose only one of the two sensors is vulnerable, $n_s = p = 1 < l = 2$, then we have to consider two models with $G = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$, $H_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$, and $H_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$. On the other hand, if both sensors are vulnerable $n_s = p = 2$, then only one model is required with $G = 0$ and $H = I$. Note, however, that the latter case is not strongly detectable with zeros at $\{0.1, 1.2\}$, thus this system violates the necessary condition in Yong et al. (2016c) for obtaining resilient estimates; but both systems in the former case can be verified to be strongly detectable.

5 ATTACK DETECTION AND IDENTIFICATION

In this section, we consider Problem 1.3 and study the consequence of the asymptotic properties of the resilient state estimation algorithm (static MM filter) in Section 4.1.2 on attack detection and identification.

First, note that the resilient state estimation algorithm we presented in the previous section is oblivious to whether the switching and false data injection attacks on the system are strategic. Nonetheless, we would like to understand how strategic attacks can be detected or identified by our algorithm. Specifically, we consider strategic attackers whose goal is to choose data injection signals d_k and the true mode $* \in Q$ to mislead the system operator/defender into believing that the mode of operation is $q \in Q$, $q \neq *$. If such an attack action cannot be reconstructed/identified by the system operator, then we refer to this attack as *unidentifiable*. If, in addition, the attack is not noticeable, then this attack is *undetectable*, formally defined as follows:

Definition 5.1 (Switching and Data Injection Attack Detection). A switching and data injection attack is detected if the true mode $* \in Q$ (chosen by attacker) has the maximum mean probability when using the resilient state estimation algorithm in Algorithm 2 or is not distinguishable from another mode $q \in Q$, $q \neq *$ (chosen by defender) on average.

Definition 5.2 (Switching and Data Injection Attack Identification). A switching and data injection attack strategy is identified if the attack is detected and in addition, the true mode $* \in Q$ is uniquely determined on average, which reveals the chosen *mode attack* and *signal attack location*, and asymptotically unbiased estimates of attack signals d_k can be obtained, i.e., the *signal magnitude attack* is reliably estimated.

From the above definitions, it is clear that if an attack is undetectable, then it will also be unidentifiable. On the flip side, if an attack is identifiable, then it is detectable. Note, however, that attack detection or identification is not needed for obtaining resilient state estimates. For instance, in the trivial case that there are no attacks $d_k = 0$ for all k , the state estimates of all models would

perform equally well. This means that the attacks need not be detected or identified for obtaining resilient estimates.

5.1 Attack Detection

Fortunately, our resilient state estimation algorithm in Algorithm 2 guarantees that an attack will always be detected by Definition 5.1.

THEOREM 5.3 (ATTACK DETECTION). *The resilient state estimation algorithm in Algorithm 2 (with ratios of prior being identically 1) guarantees that switching and data injection attacks are always detectable.*

PROOF. Since the Kullback-Leibler divergence $D(f_\ell^* \| f_\ell^q)$ is greater than or equal to 0 with equality if and only if $f_\ell^* = f_\ell^q$ (Kullback and Leibler (1951, Lemma 3.1)), with $j = * \in \mathcal{Q}$ as the true model and $i \in \mathcal{Q}, i \neq *$, the summand in the exponent of the ratio of geometric means whose expression is given in Yong et al. (2016b, Lemma 14) is always non-negative, i.e., $D(f_\ell^* \| f_\ell^i) - D(f_\ell^* \| f_\ell^*) = D(f_\ell^* \| f_\ell^i) \geq 0$. In other words, the ratio of the true model mean probability to the model mean probabilities of any other mode ($i \in \mathcal{Q}, i \neq *$) cannot decrease and can at best remain the same as the ratio of their priors which is 1 by assumption. Thus, either the true model is identified or both modes are indistinguishable and an alarm can be raised for attack detection. $\square$

5.2 Attack Identification

On the other hand, even when a combined switching and false data injection attack is detectable, it may not be identifiable. To identify an attack strategy and action, the mean consistency property of our estimation algorithm is a sufficient condition, which follows directly by Definition 5.2.

THEOREM 5.4 (ATTACK IDENTIFICATION). *Suppose mean consistency, i.e., Yong et al. (2016a, Theorem 8) holds (and hence Yong et al. (2016b, Corollary 13) also holds). Then, the switching and data injection attack strategy can be identified using the resilient state estimation algorithm in Algorithm 2.*

On the other hand, if the estimator is not mean consistent but the true mode is in the set of models, then there must exist some models with generalized innovations that have identical probability distributions as the generalized innovation of the true model (since their KL-divergences are identically zero), and that are hence Gaussian white sequences (Yong et al. (2016b)). In other words, to remain unidentifiable for some mode $q \in \mathcal{Q}$, an attacker seeks to choose another “true” mode $* \in \mathcal{Q}, * \neq q$ and the attack signal d_k such that the distributions of their generalized innovations v_k^* and $v_k^{q|*}$ are identical, i.e., Gaussian white sequences with $\mathbb{E}[v_k^{q|*}] = \mathbb{E}[v_k^*] = 0$ and $\mathbb{E}[v_k^{q|*} v_k^{q|*T}] = \mathbb{E}[v_k^* v_k^{*T}] = S_k^* \triangleq \tilde{\Gamma}_k^* \tilde{R}_{2,k}^* \tilde{\Gamma}_k^{*T}$ for all k . Using this observation, we now investigate some conditions under which attackers can be unidentifiable, as well as some other conditions under which the defenders/system operators can guarantee that the attacks are identifiable.

5.2.1 A Sufficient Condition for Unidentifiable Attacks. Given that mean consistency guarantees that an attack is identifiable, the goal of an attacker would be to ensure that mean consistency does not hold by a strategic choice of data injection signals d_k and the true mode $* \in \mathcal{Q}$ to mislead the system operator/defender into believing that the mode of operation is $q \in \mathcal{Q}, q \neq *$. The following is a sufficient condition for an attacker to synthesize an unidentifiable switching and data injection attack.

THEOREM 5.5 (UNIDENTIFIABLE ATTACK). *Suppose $\tilde{\Gamma}_k^q T_{2,k}^q H_k^*$ has linearly independent rows and there exists $* \neq q \in \mathcal{Q}$ such that*

$$\mathcal{D}_k^s \triangleq (\tilde{\Gamma}_k^q T_{2,k}^q H_k^*)^\dagger (S_k^* - \tilde{\Gamma}_k^q T_{2,k}^q (\mathbb{E}[\mu_k^{q|*} \mu_k^{q|*T}] + R_k) (\tilde{\Gamma}_k^q T_{2,k}^q)^\top) (\tilde{\Gamma}_k^q T_{2,k}^q H_k^*)^{\dagger T} \quad (10)$$

is positive definite (≥ 0) for all k . Moreover, we assume that $\mu_0^* = \mu_0^q$. Then, the attack is unidentifiable if the attacker chooses this mode $* \neq q$ as well as the attack signal d_k as a Gaussian sequence

$$d_k \sim \mathcal{N}(d_k^d, \mathcal{D}_k^s), \quad \forall k \quad (11)$$

with $\mathcal{D}_k^s$ defined in Equation (10) and d_k^d is given by

$$\begin{aligned} d_k^d &\triangleq \mathbb{E}[d_k] = -(\tilde{\Gamma}_k^q T_{2,k}^q H_k^*)^\dagger \tilde{\Gamma}_k^q T_{2,k}^q (C_k^* \mathbb{E}[x_k] - C_k^q \hat{x}_{k|k}^{*,q} + (D_k^* - D_k^q) \mathbb{E}[u_k]) \\ &= -(\tilde{\Gamma}_k^q T_{2,k}^q H_k^*)^\dagger \tilde{\Gamma}_k^q T_{2,k}^q (C_k^* \hat{x}_{k|k}^* - C_k^q \hat{x}_{k|k}^{*,q} + (D_k^* - D_k^q) \mathbb{E}[u_k]), \quad \forall k. \end{aligned} \quad (12)$$

PROOF. First, we compute what the generalized innovation for $q \in \mathcal{Q}$ would be when the attacker chooses $*$ as the true mode:

$$v_k^{q|*} = \tilde{\Gamma}_k^q T_{2,k}^q (y_k - C_k^q \hat{x}_{k|k}^{*,q} - D_k^q u_k) = \tilde{\Gamma}_k^q T_{2,k}^q (\mu_k^{q|*} + H_k^* (d_k - \mathbb{E}[d_k]) + v_k), \quad (13)$$

where $\mu_k^{q|*} \triangleq C_k^* x_k - C_k^q \hat{x}_{k|k}^{*,q} + (D_k^* - D_k^q) u_k + H_k^* d_k^d$, whose first and second moments, $\mathbb{E}[\mu_k^{q|*}]$ and $\mathbb{E}[\mu_k^{q|*} \mu_k^{q|*\top}]$, are assumed to be known to the attacker, while $T_{2,k}^q$ is the transformation matrix for mode q as described in detail in Yong et al. (2016c).

Substituting Equation (11) into Equation (13) and computing its first and second moments using Equations (10) and (12), we obtain

$$\begin{aligned} \mathbb{E}[v_k^{q|*}] &= \tilde{\Gamma}_k^q T_{2,k}^q \mathbb{E}[\mu_k^{q|*}] = 0, \\ \mathbb{E}[v_k^{q|*} v_k^{q|*\top}] &= \tilde{\Gamma}_k^q T_{2,k}^q (\mathbb{E}[\mu_k^{q|*} \mu_k^{q|*\top}] + H_k^* \mathcal{D}_k^s H_k^{*\top} + R_k) (\tilde{\Gamma}_k^q T_{2,k}^q)^\top = S_k^*. \end{aligned}$$

Since we assumed that $\mu_0^* = \mu_0^q$, we observe that the ratio of the geometric means of model probabilities given in Yong et al. (2016b, Lemma 14) equals 1. In other words, the attacked system cannot be distinguished from one under normal operation, i.e., the attack is unidentifiable by Definition 5.2. $\square$

From the above theorem, we observe that the unidentifiable attack strategy relies on two factors. First, the system has vulnerabilities that can be exploited, if the sufficient conditions of Theorem 5.5 are allowed to hold. Thus, as a system designer, these conditions serve as a preventative mitigation guide for securing the system. Second, the attacker needs computational capability and system knowledge that are comparable to that of the system operator/defender.

5.2.2 A Sufficient Condition for Resilient State Estimation. From the perspective of the system designer/operator/defender, the main objective of resilient state estimation is to obtain unbiased state estimates to preserve the integrity and functionality of the system despite attacks. Attack identification is a secondary goal and is only optional.

First and foremost, the system vulnerabilities need to be eliminated. Thus, the system needs to be strongly detectable for all modes $q \in \mathcal{Q}$, as discussed in Section 3.2. Next, since Theorem 5.5 presents yet another system vulnerability, a sufficient condition is needed such that this theorem does not hold.

LEMMA 5.6. *Theorem 5.5 does not hold if $H_k^q = H_k$ for all $q \in \mathcal{Q}$.*

PROOF. Since $H_k^q = H_k$ for all $q \in \mathcal{Q}$, $T_{2,k}^q = T_{2,k}^*$ and thus, $T_{2,k}^q H_k^* = T_{2,k}^* H_k^* = 0$ and $\tilde{\Gamma}_k^q T_{2,k}^q H_k^* = 0$. Hence, $\tilde{\Gamma}_k^q T_{2,k}^q H_k^* = 0$ cannot have linearly independent rows, as assumed in Theorem 5.5. $\square$

In addition to requiring $H_k^q = H_k$ for all $q \in \mathcal{Q}$, without loss of generality and for simplicity, we also assume that $D_k^q = D_k^*$ for all $q \in \mathcal{Q}$. Since Theorem 5.5 presents only sufficient conditions, a strategic attacker may somehow still be able to make the distributions of the generalized innovations v_k^* and $v_k^{q|*}$ identical. Thus, even in this case where $H_k^q = H_k$ and $D_k^q = D_k$ for all $q \in \mathcal{Q}$, it is interesting to investigate further sufficient conditions for the system defender to ensure that the main objective of resilient state estimation is still achieved.

THEOREM 5.7 (RESILIENCE GUARANTEE). *Suppose $H_k^q = H_k$ and $D_k^q = D_k$ for all $q \in \mathcal{Q}$. Moreover, for all $q, q' \in \mathcal{Q}$, if there exists T such that for all $k \geq T$ and the following hold:*

- (i) $\text{rank}[\tilde{\Gamma}_k^q T_{2,k}^q C_k^{q'} \tilde{\Gamma}_k^{q'} T_{2,k}^{q'} C_k^q] = 2n$, if $C_k^q \neq C_k^{q'}$,
- (ii) $\text{rank}(\tilde{\Gamma}_k^q T_{2,k}^q C_k^{q'}) = \text{rank}(\tilde{\Gamma}_k^{q'} T_{2,k}^{q'} C_k^q) = n$, if $C_k^q = C_k^{q'}$,

then the state estimates obtained using Algorithm 2 are guaranteed to be resilient (i.e., asymptotically unbiased).

PROOF. These sufficient conditions are derived by making sure that $\mathbb{E}[v_k^{q|*}] \neq \mathbb{E}[v_k^*] = 0$ such that Yong et al. (2016b, Theorem 8) does not hold. First, by the assumptions of this theorem, Equation (13) simplifies to

$$v_k^{q|*} = \tilde{\Gamma}_k^q T_{2,k}^q (C_k^* x_k - C_k^q \hat{x}_{k|k}^{*,q} + v_k). \quad (14)$$

In Case (i), i.e., when $C_k^q \neq C_k^*$, we have $\mathbb{E}[v_k^{q|*}] = [\tilde{\Gamma}_k^q T_{2,k}^q C_k^* \tilde{\Gamma}_k^{q'} T_{2,k}^{q'} C_k^q] \begin{bmatrix} \mathbb{E}[x_k] \\ -\mathbb{E}[\hat{x}_{k|k}^{*,q}] \end{bmatrix}$. Hence, since the rank condition holds, $\mathbb{E}[v_k^{q|*}] \neq 0$ unless $\begin{bmatrix} \mathbb{E}[x_k] \\ -\mathbb{E}[\hat{x}_{k|k}^{*,q}] \end{bmatrix} = 0$, in which case we have an unbiased (and thus resilient) estimate $\mathbb{E}[\hat{x}_{k|k}^{*,q}] = \mathbb{E}[x_k] = 0$.

In Case (ii), i.e., when $C_k^q = C_k^*$, we have $\mathbb{E}[v_k^{q|*}] = \tilde{\Gamma}_k^q T_{2,k}^q C_k^q \mathbb{E}[x_k - \hat{x}_{k|k}^{*,q}]$. By the rank assumption, $\mathbb{E}[v_k^{q|*}] \neq 0$ unless $\mathbb{E}[x_k - \hat{x}_{k|k}^{*,q}] = 0$, in which case we again have an unbiased (and thus resilient) estimate $\mathbb{E}[\hat{x}_{k|k}^{*,q}] = \mathbb{E}[x_k]$. $\square$

Note that Theorem 5.7 only guarantees that the state estimates are unbiased, but the true mode cannot be uniquely distinguished and the attack signal cannot be estimated. Hence, the conditions in Theorem 5.7 are not sufficient for attack identification.

6 ATTACK MITIGATION

We now turn to the next step beyond attack detection and identification, and investigate how we can mitigate the effects of attacks (Problem 1.4). Specifically, we study the problem of rejecting/canceling data injection attacks assuming that the attack mode can be identified (thus, the superscript q is omitted throughout this section), while using the resilient state estimates for feedback stabilization, in the sense of guaranteeing the boundedness of the expected states for bounded attack signals. To this end, we consider a linear state feedback controller with attack/disturbance rejection terms, where the true state and unknown input are replaced by their estimated values:

$$u_k = -K_k^c \hat{x}_{k|k} - J_{1,k} \hat{d}_{1,k} - J_{2,k} \hat{d}_{2,k-1}, \quad (15)$$

where K_k^c is the state feedback gain, while $J_{1,k}$ and $J_{2,k}$ are the attack/disturbance rejection gains. Note that we have used a delayed estimate of $d_{2,k-1}$ given in Equation (2), which is the only estimate we can obtain in light of Yong et al. (2016c, Equation (6)).

THEOREM 6.1 (ATTACK-MITIGATING AND STABILIZING CONTROLLER). *Suppose the system is controllable in the true mode $q \in \mathcal{Q}$ (known or detected), and the expected values of attack signals $\mathbb{E}[d_{2,k}]$ and their rates of variation $\|\mathbb{E}[d_{2,k} - d_{2,k-1}]\|$ are bounded for all k , i.e., $\|\mathbb{E}[d_{2,k}]\| \leq d_2^B$ and $\|\mathbb{E}[d_{2,k} - d_{2,k-1}]\| \leq \delta d_2^B$ with $\min\{d_2^B, \delta d_2^B\} < \infty$. Then, a feedback controller that mitigates the effects of data injection attacks and guarantees the boundedness of the expected system states is given by*

$$u_k = -K_k^c \hat{x}_{k|k} - \begin{bmatrix} J_{1,k} & J_{2,k} \end{bmatrix} \tilde{J}_k^{-1} \begin{bmatrix} M_{1,k}(z_{1,k} - C_{1,k} \hat{x}_{k|k}) \\ M_{2,k}(z_{2,k} - C_{2,k} \hat{x}_{k|k-1}) \end{bmatrix}, \quad (16)$$

where K_k^c is any state feedback gain such that $A_k - B_k K_k^c$ is stable and the attack/disturbance rejection gain $J_{1,k}$ is chosen to minimize $\gamma_1 \triangleq \|G_{1,k} - B_k J_{1,k}\|_2$, which can be solved with a semidefinite program² (with $i = 1$) as follows:

$$\begin{aligned} & \text{minimize } \gamma_i \\ & \text{subject to } \begin{bmatrix} \gamma_i I & G_{i,k} - B_k J_{i,k} \\ (G_{i,k} - B_k J_{i,k})^\top & \gamma_i I \end{bmatrix} \geq 0, \end{aligned} \quad (17)$$

while $J_{2,k}$ is chosen as 0 if $\delta d_2^B > d_2^B$, and otherwise, to minimize $\gamma_2 \triangleq \|G_{2,k} - B_k J_{2,k}\|_2$ by solving the semidefinite program Equation (17) with $i = 2$. It is assumed that $\tilde{J}_k \triangleq \begin{bmatrix} I - M_{1,k} D_{1,k} J_{1,k} & -M_{1,k} D_{1,k} J_{2,k} \\ -M_{2,k} D_{2,k} J_{1,k} & I - M_{2,k} D_{2,k} J_{2,k} \end{bmatrix}$ is invertible.

To prove Theorem 6.1, we first show that there exists a separation principle for linear discrete-time stochastic systems with unknown inputs, i.e., when the true mode is known, which allows us to choose the state feedback gain K_k^c and attack/disturbance rejection gains $J_{1,k}$ and $J_{2,k}$ independently.

LEMMA 6.2 (SEPARATION PRINCIPLE). *The state feedback controller gain K_k^c in Equation (15) can be designed independently of the state and input estimator gains L_k , $M_{1,k}$ and $M_{2,k}$ in Algorithm 1, as well as the disturbance rejection gains $J_{1,k}$ and $J_{2,k}$.*

PROOF. Using the control law Equation (15) and the filter equations in Equations (2), (3) and (4), it can be verified that the system states and estimator error dynamics are given by

$$\begin{aligned} \begin{bmatrix} x_{k+1} \\ \tilde{x}_{k+1|k+1} \end{bmatrix} &= \begin{bmatrix} A_k - B_k K_k^c & B_k(K_k^c - J_{1,k} M_{1,k} C_{1,k}) \\ 0 & (I - \tilde{L}_{k+1} C_{2,k}) \bar{A}_k \end{bmatrix} \begin{bmatrix} x_k \\ \tilde{x}_{k|k} \end{bmatrix} \\ &+ \begin{bmatrix} G_{1,k} - B_k J_{1,k} & G_{2,k} - B_k J_{2,k} \\ 0 & 0 \end{bmatrix} \begin{bmatrix} d_{1,k} \\ d_{2,k} \end{bmatrix} + \begin{bmatrix} B_k J_{2,k} \\ 0 \end{bmatrix} (d_{2,k} - \hat{d}_{2,k-1}) \\ &+ \begin{bmatrix} I - B_k J_{2,k} & -B_k J_{1,k} M_{1,k} + B_k J_{2,k} & -B_k J_{2,k} M_{2,k+1} \\ M_{2,k+1} C_{2,k+1} & M_{2,k+1} C_{2,k+1} G_{1,k} M_{1,k} & \\ (I - \tilde{L}_{k+1} C_{2,k+1}) & -(I - \tilde{L}_{k+1} C_{2,k+1}) & -(I - \tilde{L}_{k+1} C_{2,k+1}) \\ (I - G_{2,k} M_{2,k+1} C_{2,k+1}) & (I - G_{2,k} M_{2,k+1} C_{2,k+1}) & G_{2,k} M_{2,k+1} - \tilde{L}_{k+1} \\ C_{2,k+1} & G_{1,k} M_{1,k} & \end{bmatrix} \mathbf{w}_k, \end{aligned} \quad (18)$$

²Semidefinite programs are convex optimization problems for which software packages, e.g., CVX (CVX Research, Inc. (2012) and Grant and Boyd (2008)), are available.

where $\mathbf{w}_k \triangleq [\mathbf{w}_k^\top \mathbf{v}_{1,k}^\top \mathbf{v}_{2,k+1}^\top]^\top$ has zero mean and $\bar{A}_k \triangleq (I - G_{2,k-1}M_{2,k}C_{2,k})(A_k - G_{1,k}M_{1,k}C_{1,k})$. Since the state matrix has a block diagonal structure, the eigenvalues of the controller and estimator are independent of each other. $\square$

Armed with the above lemma, we now show how the state feedback and attack rejection gains can be independently chosen.

PROOF OF THEOREM 6.1. By Lemma 6.2, the state feedback gain, K_k^c , can be independently designed with no effect on the stability of the resilient state estimator and independent of the choice of the disturbance rejection gains $J_{1,k}$ and $J_{2,k}$. In other words, K_k^c can be chosen as any state feedback gain (e.g., with linear-quadratic regulator or pole placement) such that $A_k - B_k K_k^c$ is stable, thus the expected system states is bounded since the expected values of the attack signals and their variation rates are bounded by assumption.

On the other hand, J_1 and J_2 are chosen such that the effect of injected attack signal d_k on the closed loop system is minimized/reduced. Since $d_{1,k}$ affects the closed loop dynamics through the matrix $G_{1,k} - B_k J_{1,k}$, we choose $J_{1,k}$ such that the induced 2-norm of $G_{1,k} - B_k J_{1,k}$ is minimized, which can be obtained by the semidefinite program Equation (17) with $i = 1$. Similarly, $J_{2,k}$ can be chosen to minimize the induced 2-norm of $G_{2,k} - B_k J_{2,k}$ using the semidefinite program Equation (17) with $i = 2$. However, we have an additional $d_{2,k} - \hat{d}_{2,k-1}$ in the closed loop dynamics Equation (18), which disappears if $J_{2,k} = 0$. Thus, if we assume that $\mathbb{E}[d_{2,k}]$ is bounded for all k , i.e., $\|\mathbb{E}[d_{2,k}]\| \leq d_2^B$, and/or its rate of variation is bounded, i.e., $\|\mathbb{E}[d_{2,k} - d_{2,k-1}]\| \leq \delta d_2^B$ is bounded for all k , one would choose $J_{2,k}$ as 0 or use Equation (17) with $i = 2$, depending on the lower of the two bounds, d_2^B or δd_2^B .

In addition, $J_{1,k}$ and $J_{2,k}$ must also be chosen so that u , $\hat{d}_{1,k}$ and $\hat{d}_{2,k-1}$ can be uniquely determined since $\hat{d}_{1,k}$ and $\hat{d}_{2,k-1}$ become implicit equations. Thus, the choices of $J_{1,k}$ and $J_{2,k}$ must also be such that $\tilde{J}_k$ is invertible. The explicit expressions for $\hat{d}_{1,k}$ and $\hat{d}_{2,k-1}$ (to be substituted into Algorithm 1) is then

$$\begin{bmatrix} \hat{d}_{1,k} \\ \hat{d}_{2,k-1} \end{bmatrix} = \tilde{J}_k^{-1} \begin{bmatrix} M_{1,k}(z_{1,k} - C_{1,k}\hat{x}_{k|k}) \\ M_{2,k}(z_{2,k} - C_{2,k}\hat{x}_{k|k-1}) \end{bmatrix}. \quad (19)$$

Substituting Equation (19) back into Equation (15), we obtain the feedback controller in Equation (16). $\square$

Note that if the system in Equation (1) for each $q_k \in \mathcal{Q}$ fulfills a *matching condition* for $d_{1,k}$ ³, i.e., $\exists J_{1,k}$ such that $B_k J_{1,k} d_{1,k} = G_{1,k} d_{1,k}$, the above minimization procedure will exactly cancel out the attack signal $d_{1,k}$.

7 SIMULATION EXAMPLES

7.1 Benchmark System (Signal Magnitude and Location Attacks)

In this example, we consider the resilient state estimation problem for a system (modified from Yong et al. (2016c)) that has been used as a benchmark for many state and input filters, with only one mode of operation ($t_m = 1$) and with possible attacks on the actuator and four of the five

³The matching condition assumption is common for disturbance rejection in the sliding mode and adaptive control literature.

Fig. 5. Mode probabilities for Example 7.1 with switching from $q = 3$ to $q = 2$ after 500s (left) and with alternating switchings between $q = 3$ and $q = 2$ every 150s (right).

sensors ($t_a = 1, t_s = 4$):

$$\begin{aligned}
 A &= \begin{bmatrix} 0.5 & 2 & 0 & 0 & 0 \\ 0 & 0.2 & 1 & 0 & 1 \\ 0 & 0 & 0.3 & 0 & 1 \\ 0 & 0 & 0 & 0.7 & 1 \\ 0 & 0 & 0 & 0 & 0.1 \end{bmatrix}; \quad B=G=\begin{bmatrix} 1 \\ 0.1 \\ 0.1 \\ 1 \\ 0 \end{bmatrix}; \quad C=\begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & -0.1 & 0 & 0 \\ 0 & 0 & 1 & -0.5 & 0.2 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0.25 & 0 & 0 & 1 \end{bmatrix}; \\
 H &= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}; \quad Q = 10^{-4} \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0.5 & 0 & 0 \\ 0 & 0.5 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}; \quad R = 10^{-4} \begin{bmatrix} 1 & 0 & 0 & 0.5 & 0 \\ 0 & 1 & 0 & 0 & 0.3 \\ 0 & 0 & 1 & 0 & 0 \\ 0.5 & 0 & 0 & 1 & 0 \\ 0 & 0.3 & 0 & 0 & 1 \end{bmatrix}.
 \end{aligned}$$

The known input u_k is 2 for $100 \leq k \leq 300$, -2 for $500 \leq k \leq 700$ and 0 otherwise, whereas the unknown inputs are as depicted in Figure 6. We also assume that there are at most $p = 4$ attacks with no constraints on n_a and n_s ; as a result, we have to consider $\mathfrak{N} = 1 \cdot \binom{5}{4} = 5$ models.

Due to space limitation, we only provide simulation results for the case when the signal attack locations are switched from $q = 3$ (attack on actuator and sensors 1,3,4) to $q = 2$ (attack on actuator and sensors 1,2,4) at time $t = 500$ s. From Figure 5 (left), we observe that the mode probabilities converge to their true values, while Figure 6 shows the estimates of states as well as the unknown attack signal magnitudes. The state estimates, which are our main concern, are seen to be good even before the mode probabilities converge, while the unknown attack signals are also reasonably well estimated, with the exception of little jumps in its estimates during the switch in attack locations at $t = 500$ s. Similar results (not shown) are obtained for all other attack modes, $q = 1$ (attack on actuator and sensors 1,2,3), $q = 4$ (attack on actuator and sensors 2,3,4) and $q = 5$ (attack on sensors 1,2,3,4).

On the other hand, if the frequency of mode switchings between $q = 3$ and $q = 2$ is increased such that the dwell time decreases from 500s to 150s, Figure 5 (right) shows that the true mode $q = 2$ is wrongly estimated as $\hat{q} = 4$ between 150s and 300s. This results in poor estimation of d_2 and d_3 in that time interval, although all other states and inputs are well estimated, with an accuracy that is comparable to Figure 6. Thus, this example illustrates a limitation of our proposed method in that it may not work well when switching attacks and signal location attacks change too quickly/frequently.

7.2 IEEE 68-Bus Test System (Mode & Signal Magnitude Attacks)

Next, we apply our approach to the IEEE 68-bus test system shown in Figure 7 to empirically illustrate that the proposed algorithm can scale to large systems.

Fig. 6. State and attack magnitude estimates in Example 7.1 with switching from $q = 3$ to $q = 2$ after 500s.

Fig. 7. IEEE 68-bus test system with locations of potential actuator signal and mode/transmission line attacks (adapted from Pal and Chaudhuri (2006)).

A power network is generally represented by undirected graph $(\mathcal{V}, \mathcal{E})$ with the set of buses $\mathcal{V} \triangleq \{1, \dots, N\}$ and the set of transmission/tie lines $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$. Each bus is either a generator bus $i \in \mathcal{G}$, or a load bus $i \in \mathcal{L}$. The set of neighboring buses of $i \in \mathcal{V}$ is denoted as $\mathcal{S}_i \triangleq \{j \in \mathcal{V} \setminus \{i\} | (i, j) \in \mathcal{E}\}$. For the IEEE 68-bus test system, there are 16 generator buses and 52 load buses (i.e., $|\mathcal{G}| = 16$,

$|\mathcal{L}| = 52$ and $|\mathcal{V}| = 68$). Each bus, $i \in \mathcal{V}$, is described by (as in Wood et al. (2013, Chap. 10)):

$$\begin{aligned}\dot{\theta}_i(t) &= \omega_i(t), \\ \dot{\omega}_i(t) &= -\frac{1}{m_i} [D_i \omega_i(t) + \sum_{j \in \mathcal{S}_i} P_{tie}^{ij}(t) - (P_{M_i}(t) + d_{a,i}(t)) + P_{L_i}(t) + w_i(t)],\end{aligned}\quad (20)$$

with the phase angle $\theta_i(t)$ and angular frequency $\omega_i(t)$ as system states (thus, the state dimension is $n = 136$) and an actuator attack signal $d_{a,i}(t)$. The power flow between neighboring buses $(i, j) \in \mathcal{E}$ is given by $P_{tie}^{ij}(t) = -P_{tie}^{ji}(t) = t_{ij}(\theta_i(t) - \theta_j(t))$, while the mechanical power and power demand are denoted as $P_{M_i}(t)$ and $P_{L_i}(t)$, respectively. The mechanical power $P_{M_i}(t)$ is the control input for the generator bus $i \in \mathcal{G}$ and is zero at load bus $i \in \mathcal{L}$. On the other hand, since power demand $P_{L_i}(t)$ can be obtained using load forecasting methods (e.g., Alfares and Nazeeruddin (2002)), it is assumed to be a known input to the system. It is assumed that the noise $w_i(t)$ is a zero-mean Gaussian signal with covariance matrix $Q_i(t) = 0.01$ and the system parameters are adopted from Kundur et al. (1994, page 598): $D_i = 1$, $t_{ij} = 1.5$ for all $i \in \mathcal{V}$, $j \in \mathcal{S}_i$ and $t_{ij} = 0$ otherwise. Angular momentums are $m_i = 10$ for $i \in \mathcal{G}$ and a larger value $m_i = 100$ for load buses $i \in \mathcal{L}$.

The measurements are sampled at discrete times (with sampling time $\Delta t = 0.01$ s):

$$y_{i,k} = \begin{bmatrix} P_{elec,i,k} & \theta_{i,k} & \omega_{i,k} \end{bmatrix}^\top + v_{i,k}, \quad (21)$$

where $P_{elec,i,k} = D_i \omega_{i,k} + P_{L_i,k}$ is the electrical power output and $v_{i,k}$ is a zero-mean Gaussian noise signal with covariance matrix $R_i(t) = 0.01^4 I_3$. The continuous system dynamics Equation (20) is also discretized with a sampling time of $\Delta t = 0.01$ s so that it is compatible with the measurement model. Moreover, in this example, we choose stabilizing control inputs $P_{M_i,k}$ and $P_{L_i,k}$ to regulate the phase angles to $\theta_i = 10$ rad with system eigenvalues at -0.05 using standard linear control design tools, which is combined with the attack-mitigating controller described in Theorem 6.1.

The attacker could launch actuator attacks and mode/transmission line attacks as shown in Figure 7. For this case study, we consider eight potential attacks modes ($|\mathcal{Q}| = 8$):

- Mode $q = 1$. Lines $\{27,53\}, \{53,54\}, \{60,61\}$, and actuator G1.
- Mode $q = 2$. Lines $\{18,49\}, \{18,50\}$, and actuator G2.
- Mode $q = 3$. Line $\{40,41\}$, and actuator G3.
- Mode $q = 4$. Lines $\{18,49\}, \{18,50\}, \{27,53\}, \{53,54\}, \{60,61\}$, and actuator G4.
- Mode $q = 5$. Lines $\{27,53\}, \{40,41\}, \{53,54\}, \{60,61\}$, and actuator G5.
- Mode $q = 6$. Lines $\{18,49\}, \{18,50\}, \{40,41\}$, and actuator G6.
- Mode $q = 7$. Lines $\{18,49\}, \{18,50\}, \{27,53\}, \{40,41\}, \{53,54\}, \{60,61\}$, and actuator G7.
- Mode $q = 8$. Actuator G8.

We study a time-varying attack scenario where the attack mode is $q = 2$ for $t = [0, 2.5)$ s followed by $q = 5$ for $t = [2.5, 5)$ s, while the actuator attack signal is given by $d_{a,i} = 10^3 t$ for $t = [0, 1.25)$ s, $d_{a,i} = 10^3(2.5 - t)$ for $t = [1.25, 2.5)$ s, and $d_{a,i} = -500(t - 2.5)$ for $t = [2.5, 5)$ s. The goal of this case study is to demonstrate that our proposed approach can detect, identify and mitigate attacks. First, Figure 8(a) shows that the attacks are almost instantaneously detected and the attack modes are quickly identified. In addition, Figure 8(b) shows that the actuator attack signal is successfully identified and similarly, all system states can be well estimated (not depicted for brevity). Finally, the attack mitigation scheme is successful at keeping the phase angles regulated to 10rad/s despite attacks, whereas in the absence of attack mitigation, the phase angles can be significantly influenced by the attackers, as shown in Figure 9.

Fig. 8. Estimates of attack signal and mode probabilities when the attack mode switches from $q = 2$ to $q = 5$ at 2.5s in Example 7.2.

Fig. 9. A comparison of system states with and without the proposed attack mitigation.

8 CONCLUSION

We addressed the problem of resilient state estimation for switching (mode/topology) attacks and attacks on actuator and sensor signals of stochastic CPS, which is especially important given the proliferation of the IoT. We first modeled the problem as a hidden mode switched linear stochastic system with unknown inputs and showed that the MM inference algorithm in Yong et al. (2016b) is a suitable solution to these issues. Moreover, we provided an achievable upper bound on the maximum number of asymptotically correctable signal attacks and also the maximum number of required models for the MM approach. We also found sufficient conditions for attack (un-)detectability and attack identification, as well as designed an attack-mitigating feedback controller. Simulation examples, including one with the IEEE 68-bus test system, demonstrated the effectiveness of our approach for resilient estimation and attack identification and mitigation.

APPENDIX

A.1 System Transformation

To obtain the mode-matched input and state estimator Equations (2), (3), and (4), we will consider a system transformation for the continuous system dynamics and output equation in Equation (1) for each mode q_k (Yong et al. (2016c)). First, we rewrite the direct feedthrough matrix H_k using singular value decomposition as $H_k = [U_{1,k} \ U_{2,k}] \begin{bmatrix} \Sigma_k & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} V_{1,k}^\top \\ V_{2,k}^\top \end{bmatrix}$, where $\Sigma_k \in \mathbb{R}^{p_{H_k} \times p_{H_k}}$ is a diagonal matrix of full rank, $U_{1,k} \in \mathbb{R}^{l \times p_{H_k}}$, $U_{2,k} \in \mathbb{R}^{l \times (l - p_{H_k})}$, $V_{1,k} \in \mathbb{R}^{p \times p_{H_k}}$, and $V_{2,k} \in \mathbb{R}^{p \times (p - p_{H_k})}$ with $p_{H_k} := \text{rk}(H_k)$, while $U_k := [U_{1,k} \ U_{2,k}]$ and $V_k := [V_{1,k} \ V_{2,k}]$ are unitary matrices. When there is

no direct feedthrough, Σ_k , $U_{1,k}$, and $V_{1,k}$ are empty matrices,⁴ and $U_{2,k}$ and $V_{2,k}$ are arbitrary unitary matrices.

Moreover, we define two orthogonal components of the unknown input d_k given by

$$d_{1,k} = V_{1,k}^\top d_k, \quad d_{2,k} = V_{2,k}^\top d_k. \quad (22)$$

Since V_k is unitary, $d_k = V_{1,k}d_{1,k} + V_{2,k}d_{2,k}$. Thus, the continuous system dynamics and output equation in Equation (1) for each mode q_k can be rewritten as

$$x_{k+1} = A_k x_k + B_k u_k + G_{1,k} d_{1,k} + G_{2,k} d_{2,k} + w_k, \quad (23)$$

$$y_k = C_k x_k + D_k u_k + H_{1,k} d_{1,k} + v_k, \quad (24)$$

where $G_{1,k} := G_k V_{1,k}$, $G_{2,k} := G_k V_{2,k}$, and $H_{1,k} := H_k V_{1,k} = U_{1,k} \Sigma_k$. Next, we decouple the output y_k using a nonsingular transformation $T_k = \begin{bmatrix} T_{1,k}^\top & T_{2,k}^\top \end{bmatrix}^\top = \begin{bmatrix} I_{p_{H_k}} & -U_{1,k}^\top R_k U_{2,k} (U_{2,k}^\top R_k U_{2,k})^{-1} \\ 0 & I_{(l-p_{H_k})} \end{bmatrix} \begin{bmatrix} U_{1,k}^\top \\ U_{2,k}^\top \end{bmatrix}$ to obtain $z_{1,k} \in \mathbb{R}^{p_{H_k}}$ and $z_{2,k} \in \mathbb{R}^{l-p_{H_k}}$ as

$$\begin{aligned} z_{1,k} &= T_{1,k} y_k = C_{1,k} x_k + D_{1,k} u_k + \Sigma_k d_{1,k} + v_{1,k}, \\ z_{2,k} &= T_{2,k} y_k = C_{2,k} x_k + D_{2,k} u_k + v_{2,k}, \end{aligned} \quad (25)$$

where $C_{1,k} := T_{1,k} C_k$, $C_{2,k} := T_{2,k} C_k = U_{2,k}^\top C_k$, $D_{1,k} := T_{1,k} D_k$, $D_{2,k} := T_{2,k} D_k = U_{2,k}^\top D_k$, $v_{1,k} := T_{1,k} v_k$, and $v_{2,k} := T_{2,k} v_k = U_{2,k}^\top v_k$. This system transformation essentially decouples the output equation involving y_k into two components, one with a full rank direct feedthrough matrix and the other without direct feedthrough. The transformation is also chosen such that the measurement noise terms for the decoupled outputs are uncorrelated. The covariances of $v_{1,k}$ and $v_{2,k}$ are

$$\begin{aligned} R_{1,k} &:= \mathbb{E}[v_{1,k} v_{1,k}^\top] = T_{1,k} R_k T_{1,k}^\top > 0, \\ R_{2,k} &:= \mathbb{E}[v_{2,k} v_{2,k}^\top] = T_{2,k} R_k T_{2,k}^\top = U_{2,k}^\top R_k U_{2,k} > 0, \\ R_{12,k} &:= \mathbb{E}[v_{1,k} v_{2,k}^\top] = T_{1,k} R_k T_{2,k}^\top = 0, \\ R_{12,(k,i)} &:= \mathbb{E}[v_{1,k} v_{2,i}^\top] = T_{1,k} \mathbb{E}[v_k v_i^\top] T_{2,i}^\top = 0, \quad \forall k \neq i. \end{aligned} \quad (26)$$

Moreover, $v_{1,k}$ and $v_{2,k}$ are uncorrelated with the initial state x_0 and process noise w_k .

REFERENCES

- H. K. Alfares and M. Nazeeruddin. 2002. Electric load forecasting: Literature survey and classification of methods. *International Journal of Systems Science* 33, 1 (2002), 23–34.
- Y. Bar-Shalom, X. R. Li, and T. Kirubarajan. 2004. *Estimation with Applications to Tracking and Navigation: Theory, Algorithms and Software*. John Wiley & Sons.
- A. A. Cárdenas, S. Amin, and S. Sastry. 2008a. Research challenges for the security of control systems. In *3rd Conference on Hot Topics in Security (HOTSEC'08)*. 6:1–6:6.
- A. A. Cárdenas, S. Amin, and S. Sastry. 2008b. Secure control: Towards survivable cyber-physical systems. In *International Conference on Distributed Computing Systems Workshops*. 495–500.
- L. F. Cómbita, J. Giraldo, A. A. Cárdenas, and N. Quijano. 2015. Response and reconfiguration of cyber-physical control systems: A survey. In *IEEE Colombian Conference on Automatic Control (CCAC'15)*. 1–6.
- CVX Research, Inc. 2012. CVX: Matlab software for disciplined convex programming, version 2.0. <http://cvxr.com/cvx>. (2012).
- G. Dan and H. Sandberg. 2010. Stealth attacks and protection schemes for state estimators in power systems. In *IEEE International Conference on Smart Grid Communications (SmartGridComm'10)*. 214–219.
- J. P. Farwell and R. Rohozinski. 2011. Stuxnet and the future of cyber war. *Survival* 53, 1 (2011), 23–40.
- H. Fawzi, P. Tabuada, and S. Diggavi. 2014. Secure estimation and control for cyber-physical systems under adversarial attacks. *IEEE Transactions on Automatic Control* 59, 6 (2014), 1454–1467.

⁴We adopt the convention that the inverse of an empty matrix is also an empty matrix and assume that operations with empty matrices are possible.

- B. Ghena, W. Beyer, A. Hillaker, J. Pevarnek, and J. A. Halderman. 2014. Green lights forever: Analyzing the security of traffic infrastructure. In *8th USENIX Workshop on Offensive Technologies*, Vol. 14. 7–7.
- S. Gillijns and B. De Moor. 2007a. Unbiased minimum-variance input and state estimation for linear discrete-time systems. *Automatica* 43, 1 (2007), 111–116.
- S. Gillijns and B. De Moor. 2007b. Unbiased minimum-variance input and state estimation for linear discrete-time systems with direct feedthrough. *Automatica* 43, 5 (2007), 934–937.
- R. Goebel, R. G. Sanfelice, and A. Teel. 2009. Hybrid dynamical systems. *IEEE Control Systems Magazine* 29, 2 (2009), 28–93.
- M. Grant and S. Boyd. 2008. Graph implementations for nonsmooth convex programs. In *Recent Advances in Learning and Control*, V. Blondel, S. Boyd, and H. Kimura (Eds.). Springer-Verlag Limited, 95–110.
- A. Humayed, J. Lin, F. Li, and B. Luo. 2017. Cyber-physical systems security—A survey. *IEEE Internet of Things Journal* 4, 6 (2017), 1802–1831.
- J. Kim and L. Tong. 2013. On topology attack of a smart grid: Undetectable attacks and countermeasures. *IEEE Journal on Selected Areas in Communications* 31, 7 (2013), 1294–1305.
- O. Kosut, L. Jia, R. J. Thomas, and L. Tong. 2011. Malicious data attacks on the smart grid. *IEEE Transactions on Smart Grid* 2, 4 (2011), 645–658.
- S. Kullback and R. A. Leibler. 1951. On information and sufficiency. *Annals of Mathematical Statistics* 22 (1951), 49–86.
- P. Kundur, N. J. Balu, and M. G. Lauby. 1994. *Power System Stability and Control*. McGraw-Hill New York.
- C. Kwon, W. Liu, and I. Hwang. 2013. Security analysis for cyber-physical systems against stealthy deception attacks. In *American Control Conference (ACC'13)*. 3344–3349.
- G. Liang, J. Zhao, F. Luo, S. R. Weller, and Z. Y. Dong. 2017. A review of false data injection attacks against modern power systems. *IEEE Transactions on Smart Grid* 8, 4 (2017), 1630–1638.
- S. Liu, S. Mashayekh, D. Kundur, T. Zourntos, and K. Butler-Purpy. 2013. A framework for modeling cyber-physical switching attacks in smart grid. *IEEE Transactions on Emerging Topics in Computing* 1, 2 (2013), 273–285.
- Y. Liu, P. Ning, and M. K. Reiter. 2011. False data injection attacks against state estimation in electric power grids. *ACM Transactions on Information and System Security* 14, 1 (2011), 13.
- C. Y. T. Ma, D. K. Y. Yau, X. Lou, and N. S. V. Rao. 2013. Markov game analysis for attack-defense of power networks under possible misinformation. *IEEE Transactions on Power Systems* 28, 2 (2013), 1676–1686.
- E. Mazor, A. Averbuch, Y. Bar-Shalom, and J. Dayan. 1998. Interacting multiple model methods in target tracking: A survey. *IEEE Transactions on Aerospace and Electronic Systems* 34, 1 (1998), 103–123.
- S. Mishra, Y. Shoukry, N. Karamchandani, S. Diggavi, and P. Tabuada. 2015. Secure state estimation: Optimal guarantees against sensor attacks in the presence of noise. In *IEEE International Symposium on Information Theory (ISIT'15)*. 2929–2933.
- Y. Mo and B. Sinopoli. 2010. False data injection attacks in control systems. In *First Workshop on Secure Control Systems, CPS Week*.
- M. Pajic, P. Tabuada, I. Lee, and G. Pappas. 2015. Attack-resilient state estimation in the presence of noise. In *IEEE Conference on Decision and Control*. 5827–5832.
- M. Pajic, J. Weimer, M. Bezzo, P. Tabuada, O. Sokolsky, I. Lee, and G. Pappas. 2014. Robustness of attack-resilient state estimators. In *ACM/IEEE International Conference on Cyber-Physical Systems*. 163–174.
- B. Pal and B. Chaudhuri. 2006. *Robust Control in Power Systems*. Springer Science & Business Media.
- F. Pasqualetti, F. Dörfler, and F. Bullo. 2013. Attack detection and identification in cyber-physical systems. *IEEE Transactions on Automatic Control* 58, 11 (2013), 2715–2729.
- S. Sundaram and C. N. Hadjicostis. 2007. Delayed observers for linear systems with unknown inputs. *IEEE Transactions on Automatic Control* 52, 2 (2007), 334–339.
- J. Weimer, S. Kar, and K. H. Johansson. 2012. Distributed detection and isolation of topology attacks in power networks. In *1st International Conference on High Confidence Networked Systems (HiCoNS'12)*. ACM, 65–72.
- A. J. Wood, B. F. Wollenberg, and G. B. Sheble. 2013. *Power Generation, Operation, and Control*. John Wiley & Sons.
- S. Z. Yong, M. Q. Foo, and E. Frazzoli. 2016a. Robust and resilient estimation for cyber-physical systems under adversarial attacks. In *American Control Conference (ACC'16)*. 308–315.
- S. Z. Yong, M. Zhu, and E. Frazzoli. 2015. Resilient state estimation against switching attacks on stochastic cyber-physical systems. In *IEEE Conference on Decision and Control (CDC'15)*. 5162–5169.
- S. Z. Yong, M. Zhu, and E. Frazzoli. 2016b. Simultaneous mode, input and state estimation for switched linear stochastic systems. *SIAM Journal on Control and Optimization* (2016). In Review. Retrieved from <http://arxiv.org/abs/1606.08323>.
- S. Z. Yong, M. Zhu, and E. Frazzoli. 2016c. A unified filter for simultaneous input and state estimation of linear discrete-time stochastic systems. *Automatica* 63 (2016), 321–329. Extended version first appeared in September 2013 and is available from: <http://arxiv.org/abs/1309.6627>.
- S. Z. Yong, M. Zhu, and E. Frazzoli. 2017. Simultaneous input and state estimation for linear time-varying continuous-time stochastic systems. *IEEE Transactions on Automatic Control* 62, 5 (2017), 2531–2538.

- M. Zhu and S. Martínez. 2011. Stackelberg-game analysis of correlated attacks in cyber-physical systems. In *American Control Conference (ACC'11)*. 4063–4068.
- M. Zhu and S. Martínez. 2013. On distributed constrained formation control in operator-vehicle adversarial networks. *Automatica* 49, 12 (2013), 3571–3582.
- Q. Zhu and T. Basar. 2015. Game-theoretic methods for robustness, security, and resilience of cyberphysical control systems: Games-in-games principle for optimal cross-layer resilient control systems. *IEEE Control Systems Magazine* 35, 1 (2015), 46–65.

Received July 2016; revised March 2018; accepted March 2018