Parametric RBAC Maintenance via Max-SAT

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ABSTRACT

In the past decade, many organizations have adopted a Role-Based Access Control model (RBAC) to reduce their administration costs and increase security. The migration to RBAC requires a role engineering phase aimed at generating “good” initial roles starting from direct assignments of permissions to users. For an RBAC approach to be effective, however, it is also necessary to update roles and keep them compliant with the dynamic nature of the business processes; not only this, but errors and misalignments between the current RBAC state and reality need to be promptly detected and fixed.

In this paper, we propose a new maintenance process to fix and refine an RBAC state when “exceptions” are detected. Exceptions are permissions some users realize they miss that are instrumental to their job and should be granted as soon as possible. They are caught by a monitoring system as unexpected “access denied” conditions and then validated by the RBAC administrator. The fix we produce aims at balancing two conflicting objectives, i.e., (i) simplifying the current RBAC state, and (ii) reducing the transition cost. Our approach is based on a Max-SAT formalization of this trade-off and it exploits *incomplete* solvers that quickly provide approximations of optimal solutions.

Experiments show good performance on real-world benchmarks.

Disclaimer: The opinions expressed and conclusions drawn are those of the authors and do not necessarily reflect the views of the authors’ company.

CCS CONCEPTS

• **Security and privacy** → *Logic and verification; Access control; Authorization;*

KEYWORDS

Role Maintenance; Role Based Access Control (RBAC); Max-SAT

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1 INTRODUCTION

Complex organizations have to guarantee the correct access to software, systems, and data to a large number of employees involved in heterogeneous and evolving tasks. Classical access control policies based on a direct assignment of permissions to users have shown their limits: Granting and revoking individual permissions as tasks evolve and employees move is inefficient and error-prone.

Role Based Access Control - RBAC [3] has emerged as a solution to this issue and is currently widely adopted; it simplifies the management of permissions by defining *roles*, which include the permissions required to execute certain tasks. Users gain the permissions which are included in at least one of the roles assigned to them. In case users are moved to different tasks or if tasks change, it is sufficient to operate at the role level without modifying several Permission-to-User assignments. In addition, basic security principles such as *Least Privilege* and *Separation of Duties* may be easily enforced by posing conditions on roles and their assignments.

In order for an organization to adopt the RBAC model, it has to implement a *role management lifecycle* [14, 15] process, aimed at designing, implementing, and maintaining roles. To design (initial) roles, it is necessary to analyze Permission-to-User assignments and group together permissions into functional roles compactly representing the current access control policy. This is typically done via an automatic heuristic process called *role mining* [8]. While role mining supports the discovery of roles from scratch, in many cases organizations need to just *tune* or *maintain* the current set of roles: Roles may have to be revised to support new User-to-Permission assignments; or, a (possibly hand-made) RBAC state may contain errors, which need to be first discovered and then incorporated into the current state (after validation by a security administrator).

While tuning an RBAC model, the company has to consider conflicting objectives. It certainly benefits from a *simplification* of the current Permission-to-Role and Role-to-User assignments, i.e., of the so called *RBAC state*. Different metrics have been proposed in the literature to characterize the complexity of an RBAC state; they are often based on a combination of the number of roles and the number of assignments in it [1, 9]. Nevertheless, any *uninformed* simplification of a given RBAC state may heavily modify the current assignments, thus disrupting organizational processes and/or separation of duty constraints [13], not to mention the muscle memory of any human actor involved in managing and assigning roles. This may be acceptable if the organization is very small or at a preliminary stage of RBAC adoption; not so much in case of a large company with its own well-defined, perhaps “cherished” working set of roles, which have been costly defined and negotiated.

In this paper we define a new *parametric RBAC maintenance process* that supports both from-scratch design and maintenance of an existing role model. When performing maintenance, the in-place RBAC state is provided as input to the algorithm (together with

new requirements to satisfy) and an adjusted model is produced as output, which accommodates the new requirements by “patching” the original state. When starting with a clean slate, a fully compliant RBAC state is grown out of the empty state by a sequence of patches.

As a key feature, the new algorithm aims at balancing two possibly conflicting metrics: *similarity* and *simplicity*. Similarity is a measure of how different the patched state is from the pre-patch state: the closer the two states the less disrupting the update. Simplicity is a measure of how close the patched state is to the unconstrained, *optimal* RBAC state: the more a patch strives for optimality, the larger the potential impact on the organization. As we will see, in our approach the RBAC administrator is provided with means to strike the best balance between these two objectives.

The algorithm generates Max-SAT [6] instances and solves them via publicly-available, state-of-the-art solvers. Several well known datasets are adapted to this new incremental setting and are used to establish the practicality of our solution in real-world cases.

The rest of this paper is organized as follows. Section 2 frames the problem and presents a working example. Section 3 defines the Max-SAT formalization and introduces two metrics we use to evaluate the quality of an RBAC state. Section 4 describes the datasets we use in the experiments and presents the results of our experimental evaluation. Section 5 discusses related works. Section 6 concludes the paper and proposes directions for future work.

2 PARAMETRIC RBAC MAINTENANCE

2.1 Preliminaries

Role mining is a preliminary step to enable the adoption of an RBAC model. It can be executed bottom-up or top-down.

The bottom-up approach takes as input an existing set of direct Permission-to-User assignments. Then, mining roles consists in defining a collection of roles (sets of permissions) and then assigning one or more roles to each user. The permissions each user is granted by any of his role(s) have to be exactly the same Permissions-to-User assignments he had before roles were introduced.

Formally, starting from a binary matrix $UPA \in \{0, 1\}^{m,n}$ representing the assignments of n permissions to m users, role mining factorizes it into a Role-to-User matrix $UA \in \{0, 1\}^{m,k}$ and a Permission-to-User matrix $PA \in \{0, 1\}^{k,n}$, for some positive integer k (number of roles), such that $UA \otimes PA = UPA$, where the boolean matrix multiplication operator $\otimes$ is defined as $UPA_{ij} = \bigvee_{l=1}^k (UA_{il} \wedge PA_{lj})$.

Top-down approaches—employed as an alternative or in combination with bottom-up mining—define roles starting from an analysis of the business processes to support. Top-down roles are typically hand-crafted by business stakeholders after an analysis of the organization processes and of the permissions users require to execute them. Hand-made roles, though validated and closely inspected, may of course contain (several) errors.

Whether the synthesis is top-down or bottom-up, once an RBAC state is in place, a *role maintenance* process is required to adapt the state to intervening organizational changes (employees move, new systems are deployed, etc.) or to fix errors that become apparent during actual usage.

Figure 1: The Parametric Role Maintenance process.

2.2 Maintenance of an RBAC state

Errors show up as either Permission-to-User assignments that are not implied by the current state but should be, or as unnecessary permissions granted to users. In this paper, we address missing permissions only, though the formalization in Section 3 is capable of dealing with extra permissions (the issue with extra permissions is how to discover them in the first place, a subject out of scope here). Figure 1 describes the process of discovering missing permissions. The logs captured by the monitoring systems record (among the other things) events corresponding to missing permissions. The security administrator comes into play at this stage by *validating* prohibited accesses; of particular interest are false positives, i.e., permissions a user misses but should be allowed to have, which we call *exceptions*. The administrator may also directly specify further exceptions by generalizing the issue at hand.

Starting from a (validated) exception $e : p \rightarrow u$ to be incorporated into an existing RBAC state S_0 , the *maintenance* phase generates a target RBAC state with the following properties:

- (a) it is semantically equivalent to S_0 , aside from accommodating exception e , i.e., u now has permission p ;
- (b) among all states that satisfy (a), it is either (i) the *most similar* to S_0 , or (ii) the *simplest possible* state, or (iii) any state *in between* these two extremes.

Section 3 formalizes the meaning of “most similar”, “simplest possible”, and “in between”. Maintenance is applied as new exceptions arrive. If necessary, several exceptions may be dealt with at once, in a single step (however, we only show the exception-by-exception workflow in the following sections). The degree of freedom at (b) is exploited by the security administrator to strike the best balance between evolving the state towards simplicity and preserving the status quo, according to criteria external to the algorithm.

2.3 Scope of application

Role maintenance may also act as a *fixing* or *mining* tool, depending on how it is applied; indeed, the state fed into it can be:

- Fix A hand-made, top-down RBAC state, which despite possibly containing (several) errors cannot be radically overhauled because it has been agreed on by several stakeholders; here, role maintenance is basically used to gradually shepherd the hand-made model to the reality of business processes;

Tune An in-place, fully satisfactory RBAC state which needs to undergo maintenance to accommodate exogenous events, such as updates in business processes, deployment/replacement of information systems, relocation of employees, etc.;

Mine An empty state granting no permission to anyone; in this case, role maintenance is used as an incremental role mining procedure whereby the administrator generates a well-behaved and well-structured RBAC state by monitoring missing permissions on-line and judiciously granting them.

We focus on the “tune” use case in the rest of the paper.

2.4 A working example

Let us consider a consulting firm with dozens of employees. They use two applications that store data into a company database. The first application supports the *publishing* activity of the company. The second application is used for *marketing* purposes, specifically in the definition of campaigns for mainstream media. An independent in-house *quality assurance* process ensures the compliance of the actions taken through the publishing and the marketing platform with all relevant guidelines and regulations. Employees also use a *general-purpose* Internet service and an in-house e-mail server, through which they *communicate* with institutional stakeholders. An Intranet is available for internal communication and to access *business* applications. The *HR* department manages a database with employees’ data. A *web site* describes the services offered by the company on the web. The application server and the *databases* are managed by different administrators.

An RBAC solution is in place at this company: Roles have been crafted so that each employee is enabled to perform only the task(s) he is involved in. The set of (business and technical) roles are: *publishing*, *marketing*, *quality assurance*, *general communication*, *business communication*, *HR management*, *server administration* and *database administration*. As usual, each role is associated with the set of permissions required to carry out the corresponding task(s). For example, the marketing (publishing) role entails the permission to access the marketing (publishing) application, in addition to the permission to access the shared business database. The business communication role grants the permission to use the mail server and the Intranet, while the general communication role gives access to the Internet. Finally, the database administration role is required to operate on the business and the HR databases.

Let us suppose the RBAC manager at this company is now faced with these urgent requests: An employee is about to join the marketing division and needs the corresponding permissions; at the same time, another employee who is already in that division just realized he lacks some much needed permissions to operate; another employee from the marketing division is required to (also) work in the publishing process for a while. Finally, yet another employee is to be temporarily granted permissions to help the database administrator, but only on a non-critical dataset.

The RBAC administrator would like to be supported in the process of re-configuring the RBAC state. In particular, he would like to automatically obtain good answers to questions such as:

- “How to modify the current RBAC state to implement the necessary variations and nothing more?”

- “Is it possible to operate on the Permission-to-Role and/or Role-to-User assignments without adding new roles?”
- “Would it be better to just create new custom roles?”
- “Is this an occasion to merge some existing roles into less and more clear-cut business figures?”
- “Once I’m at it, can I simplify the current role set while keeping the solution stable w.r.t. to the desired RBAC state?”

3 FORMALIZATION

We use the language of propositional logic to represent RBAC maintenance instances. The question we ask is not simply about the satisfiability of a logic statement by some propositional model (SAT), because we need to also express preferences between conflicting objectives (similarity and simplicity of the output RBAC state). It turns out a slightly different, “optimizing” framework perfectly suited to our needs is Max-SAT, which considers all possible truth assignments to the input formula and picks the best one according to a fitness metric defined in the Max-SAT language itself.

3.1 SAT and Max-SAT

A SAT problem [2] is solved by assigning a truth value in $\{True, False\}$ to each Boolean variable that appears in the input propositional formula in such a way that the formula as a whole evaluates to *True*. If at least one such assignment exists, the formula is *satisfiable* and the satisfying assignment is called a *model*; otherwise, the formula is *unsatisfiable* (inconsistent) and it has no model. This problem is intractable (NP-complete) in general, yet several highly efficient solvers exist that in practice solve real-world problems with millions of variables in reasonable time.

Given any *UPA* matrix, its factorization into $UA \otimes PA = UPA$ for some number of roles k can be directly expressed in SAT as:

$$\bigwedge_{ij} [UPA_{ij} \leftrightarrow \bigvee_{l=1}^k (UA_{il} \wedge PA_{lj})] \quad (1)$$

(1) is a SAT instance with $k(m+n)$ variables. Without loss of generality, most SAT solvers require input formulas in Conjunctive Normal Form (CNF), i.e., a conjunction of clauses, each clause being a disjunction of literals. By either reworking the formula (possibly enlarging its size) or adding auxiliary variables, it is possible to rewrite any non-CNF problem like (1) in CNF.

In SAT, all clauses have to be satisfied for the formula to be declared satisfiable. The *Maximum Satisfiability problem* (Max-SAT) is a variant of SAT that relaxes this premise. The goal of Max-SAT is to find an assignment that makes true the *largest possible subset* of clauses: some may remain unsatisfied. *Partial* Max-SAT is another variant in which some clauses are declared *hard* and must be satisfied no matter what, as in SAT, while others are declared *soft*: As many of them as possible must be satisfied, à la Max-SAT. Yet another variant is *Weighted* Max-SAT, that generalizes Max-SAT by associating a positive real or integer weight to each input clause: An assignment that *maximizes the sum of the weights* of satisfied clauses, rather than just their number, is sought (clauses left unsatisfied contribute nothing to the sum).

Finally, the *Weighted Partial* Max-SAT (WPMS) language combines both features: It finds an assignment that satisfies all hard clauses while maximizing the cumulated weight of satisfied soft clauses. We employ WPMS here (and call it simply Max-SAT).

3.2 Hard constraints

Hard clauses are well suited to represent invariants that must hold on any RBAC state we may possibly output, and in particular to capture condition (a) from Section 2.2.

The input RBAC state is represented by two boolean matrices UA^0 and PA^0 such that $UA^0 \otimes PA^0 = UPA^0$. UA^0 and PA^0 are respectively a $m \times k^0$ and a $k^0 \times n$ boolean matrices having non-empty roles, i.e., $\forall t=1, \dots, k^0 (\bigvee_{i=1, \dots, m} UA_{i,t} = \text{True}) \wedge (\bigvee_{j=1, \dots, n} PA_{t,j} = \text{True})$. Given an exception $e : p_i \rightarrow u_j$ meant to grant a new permission p_i to user u_j (it was $UPA_{i,j}^0 = \text{False}$), we define $EXC(e)$ as the $m \times n$ boolean matrix that is *True* at (i, j) and *False* everywhere else. We then need $k(m + n)$ variables to represent the unknown elements of the updated Role-to-User (UA) and of the Permissions-to-Role (PA) matrices, where m , k , and n are the number of users, roles, and permissions.

We have to find two boolean matrices UA and PA such that:

$$UA \otimes PA = UPA^0 \oplus EXC(e) = UPA \quad (2)$$

where $\oplus$ is the boolean matrix sum operator. In terms of the variables defining UA and PA , this can be written:

$$\bigwedge_{i,j} F_{i,j}, \text{ where } F_{i,j} := (\bigvee_{t \in [1,k]} UA_{i,t} \wedge PA_{t,j}) = UPA_{i,j}^0 \vee EXC_{i,j}(e)$$

Formulas $F_{i,j}$ are not in CNF. To turn them into CNF we observe that only two cases have to be considered:

$$\begin{pmatrix} \dots & \dots & \dots \\ UA_{i_0,1} & \dots & UA_{i_0,k} \\ \vdots & \vdots & \vdots \\ UA_{i_1,1} & \dots & UA_{i_1,k} \\ \dots & \dots & \dots \end{pmatrix} \begin{pmatrix} \dots & PA_{1,j_0} & \dots & PA_{1,j_1} & \dots \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ \dots & PA_{k,j_0} & \dots & PA_{k,j_1} & \dots \end{pmatrix} =$$

$$\begin{matrix} & j_0 & j_1 \\ i_0 & \begin{pmatrix} \dots & \dots & \dots & \dots & \dots \\ \dots & \text{False} & \dots & \dots & \dots \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ \dots & \dots & \dots & \text{True} & \dots \end{pmatrix} \\ i_1 & \begin{pmatrix} \dots & \dots & \dots & \dots & \dots \\ \dots & \dots & \dots & \dots & \dots \end{pmatrix} \end{matrix}$$

If $UPA_{i,j}^0 \vee EXC_{i,j}$ is *False*, such as at index (i_0, j_0) above, we have:

$$\bigwedge_{t \in [1,k]} (UA_{i,t} \wedge PA_{t,j}) = \text{False}$$

which can be rewritten as a set of clauses by negating both sides:

$$has_{i,j}(UA, PA) := \bigwedge_{t \in [1,k]} (\neg UA_{i,t} \vee \neg PA_{t,j}) \quad (3)$$

If $UPA_{i,j}^0 \vee EXC_{i,j}$ is *True*, such as at index (i_1, j_1) , we have:

$$\bigwedge_{t \in [1,k]} (UA_{i,t} \wedge PA_{t,j}) = \text{True}$$

Here we apply the Tseytin transformation [12] by inserting k auxiliary variables $\{aux_1, \dots, aux_k\}$, one for each disjunct:

$$\begin{cases} \bigvee_{t \in [1,k]} aux_t \\ \bigwedge_{t \in [1,k]} aux_t \leftrightarrow (UA_{i,t} \wedge PA_{t,j}) \end{cases}$$

which is equivalent to the CNF form:

$$hasnt_{i,j}(UA, PA) := \begin{cases} \bigvee_{t \in [1,k]} aux_t \\ \bigwedge_{t \in [1,k]} (\neg aux_t \vee UA_{i,t}) \wedge (\neg aux_t \vee PA_{t,j}) \\ \bigwedge_{t \in [1,k]} (aux_t \vee \neg UA_{i,t} \vee \neg PA_{t,j}) \end{cases} \quad (4)$$

Overall, the hard clauses that guarantee we are incorporating the exception(s) in EXC are:

$$exc(UA, PA) := \bigwedge_{i,j | UPA_{i,j}^0} has_{i,j}(UA, PA) \wedge \bigwedge_{i,j | \neg UPA_{i,j}^0} hasnt_{i,j}(UA, PA) \quad (5)$$

3.3 Soft constraints

We use soft clauses to express constraints that we know are in trade-off and cannot always be satisfied simultaneously. In particular, we want to capture the conditions (i) and (ii) at point (b) in Section 2.2.

Identity. To express the condition that the output RBAC state (UA, PA) must be identical to the input state (UA^0, PA^0) we write:

$$\begin{cases} \bigwedge_{i \in [1,m], j \in [1,k]} UA_{i,j} \leftrightarrow UA_{i,j}^0 \\ \bigwedge_{i \in [1,k], j \in [1,n]} PA_{i,j} \leftrightarrow PA_{i,j}^0 \end{cases}$$

which in CNF is:

$$eq(UA, PA) := \begin{cases} \bigwedge_{i,j} (UA_{i,j} \vee \neg UA_{i,j}^0) \wedge (\neg UA_{i,j} \vee UA_{i,j}^0) \\ \bigwedge_{i,j} (PA_{i,j} \vee \neg PA_{i,j}^0) \wedge (\neg PA_{i,j} \vee PA_{i,j}^0) \end{cases} \quad (6)$$

Note that $exc(UA, PA) \wedge eq(UA, PA)$ is unsatisfiable by construction in the traditional SAT sense.

Sparsification. To express the condition that the output RBAC state (UA, PA) must be as “simple” as possible we need to adopt some notion of simplicity. The two simplifying conditions we consider are: (i) a reduction in the number of Roles-to-User and/or Permission-to-Role assignments (“sparsification”), and (ii) a reduction in the number of roles (see next section, “Contraction”).

According to (i), the sparsest possible RBAC state would be:

$$sparse(UA, PA) := \begin{cases} \bigwedge_{i \in [1,m], j \in [1,k], UA_{i,j}^0 = 1} \neg UA_{i,j} \\ \bigwedge_{i \in [1,k], j \in [1,n], PA_{i,j}^0 = 1} \neg PA_{i,j} \end{cases} \quad (7)$$

Contraction. Sometimes, incorporating an exception makes it possible to satisfy equation (2) with one less role than those present in (UA^0, PA^0) . The soft constraint $sparse(UA, PA)$ is not sufficient to prefer a reduction in the number of roles over similarly-sized but sparse unassignments from UA . The fact that role j is no longer in use in the target RBAC state is represented by a column of 0's in the Role-to-User assignment matrix (UA); so the condition is:

$$contr(UA) := \bigwedge_{j \in [1,k]} unused_j(UA) \quad (8)$$

where $\{unused_j\}$ are auxiliary variables telling whether role j is assigned to someone ($unused_j = \text{False}$) or not ($unused_j = \text{True}$):

$$unused_{contr}(UA) := \bigwedge_{j \in [1,k]} [unused_j(UA) \leftrightarrow \bigwedge_{t \in [1,m]} \neg UA_{t,j}] \quad (9)$$

Expansion. Dually, it is possible that the only way to incorporate an exception is by adding one role to the state, i.e., one column

to UA and one row to PA . In a propositional encoding, the only way to accommodate for a possible expansion of the matrices we handle is to “reserve space” in advance for the additional row and column. This means we actually work with an RBAC state made by a matrix UA^+ with $k + 1$ columns and a matrix PA^+ with $k + 1$ rows. The condition that this additional role stays unused is:

$$noexp(UA) := unused_{k+1} \quad (10)$$

where $unused_{k+1}$ is an auxiliary variable telling whether the additional role remains unused:

$$unused_{exp}(UA) := unused_{k+1} \leftrightarrow \bigwedge_{i \in [1, m]} \neg UA_{i, k+1}^+ \quad (11)$$

$$unused(UA) := unused_{contr}(UA) \wedge unused_{exp} \quad (12)$$

In an encoding where both contraction and expansion constraints are added, we enforce a coherent interplay between the two opposite effects by saying they must not happen simultaneously:

$$mutex(UA) := noexp(UA) \vee \neg leastcontr(UA) \quad (13)$$

where $leastcontr$ tells whether at least a role is no longer in use:

$$leastcontr(UA) := \bigvee_{j \in [1, k]} unused_j(UA)$$

3.4 The complete encoding

We have all the pieces to present the complete encoding of the RBAC maintenance problem as a Weighted Partial Max-SAT instance. First, let us list the hard constraints (HC) that any feasible solution must comply with. In addition to those from Section 3.2, all the auxiliary constraints from Section 3.3 are hard, because auxiliary propositional variables cannot cause inconsistencies on their own, yet they must always be assigned a consistent value:

$$HC(UA, PA) := exc(UA^+, PA^+) \wedge unused(UA) \wedge mutex(UA) \quad (14)$$

Soft constraints (SC) are used to express and balance two potentially conflicting objectives of the RBAC maintenance procedure:

- (1) maximizing the *similarity* between the target and the origin RBAC state; in this sense, the higher the weight we assign to clauses in (6), the better;
- (2) maximizing the *simplicity* of the target RBAC state (independently of the origin RBAC state); in this sense, the higher the weight we assign to clauses in (7), (8), and (10), the better.

Let us assume the weight of a given clause C is a real value¹ $w \in [0, 1]$, noted $w : C$. The notation $w : (C_1 \wedge \dots \wedge C_n)$ is a shorthand for $w/n : C_1 \wedge \dots \wedge w/n : C_n$, while $w_1 : (w_2 : C)$ is interpreted as $(w_1 \cdot w_2) : C$. The three components (7), (8), and (10) of the “simplicity” objective are merged into one weighted set of clauses, noted $SIMP_{k^+, k^-}(UP, PA)$ and defined as:

$$v : [k^- : contr(UA, PA) \wedge sparse(UA, PA) \wedge k^+ : noexp(UA, PA)] \quad (15)$$

Here, $k^- \geq 0$ is a parameter meant to quantify how much we reward the elimination of a role versus to the sparsification of the matrix; $k^+ \geq 0$ measures our aversion to introduce a new role

if not strictly necessary. $v := (1 + k^- + k^+)^{-1}$ is a normalization factor meant to ensure the weights of the three components sum up to 1. We assume $k^- > k^+ > 1$, i.e., we value the possibility to expunge a role more than the removal of an equivalent number of assignments; adding a role is a last resort.

Finally, let $\beta \in [0, 1]$ be a balancing parameter that measures the extent to which the RBAC administrator is interested in a simplified (β closer to 1) VS a stable (β closer to 0) RBAC state; we pose:

$$SC_{\beta, k^-, k^+}(UA, PA) := (1 - \beta) : eq(UA, PA) \wedge \beta : SIMP_{k^+, k^-}(UA, PA) \quad (16)$$

3.5 Solutions and their quality

Given (i) the input RBAC state (UA^0, PA^0) , (ii) the exception matrix EXC to incorporate, and (iii) some specific values for the parameters β , k^+ , and k^- , we submit to a Max-SAT solver the hard clauses (14) plus the soft clauses (16). The Max-SAT solver returns a *fixed* RBAC state (UA, PA) , i.e., a truth assignment to all the $k(m + n)$ variables that define UA and PA . Note that the encoding is satisfiable by construction, so we always obtain a fixed state.

To assess the quality of UA, PA (both per se and w.r.t. UA^0, PA^0) we need some synthetic indicators for the two dimensions we are after: simplicity and similarity.

Similarity is computed according to the metric defined in [13], which is based on the Jaccard coefficient.

Role-to-Role similarity. The similarity between two roles r_1 and r_2 granting permissions P_1 and P_2 respectively is defined as:

$$sim_{1-1}(r_1, r_2) := \frac{|P_1 \cap P_2|}{|P_1 \cup P_2|}$$

Role-to-RoleSet similarity. The similarity between a role r and a set of roles R is defined as:

$$sim_{1-N}(r, R) := \max_{r_x \in R} sim_{1-1}(r, r_x)$$

RoleSet-to-RoleSet similarity. The similarity between a set of roles R_1 and a set of roles R_2 is defined as:

$$sim_{N-N}(R_1, R_2) := \text{avg}_{r \in R_1} sim_{1-N}(r, R_2)$$

Building on these definitions, we pose:

Similarity. The similarity of the target RBAC state (with roles R) to the source RBAC state (with roles R_0) as:

$$sim(R_0, R) := \frac{sim_{N-N}(R_0, R) + sim_{N-N}(R, R_0)}{2} \quad (17)$$

This similarity function is always between 0 and 1; in particular, the value 1 is obtained iff $R \equiv R_0$.

Simplicity is defined in different ways in the relevant literature; metrics taken into account to define it include, but are not limited to, the number of roles, assignments, erroneous permission granted, relations among roles, permission assigned directly, and any combination of these metrics. We start from a definition of “absolute complexity” that grows with the number of roles and the number of assignments in UA and PA :

$$comp(UA, PA) := (|UA| + |PA|) + |R|^2 \quad (18)$$

¹ Some Max-SAT solvers only accept positive integer values as weights; we can map real values onto integers that are equivalent to the effect of the optimization we perform as $weight_{int} = \lceil \delta * weight_{real} \rceil$ where δ is the minimum absolute difference between the sum of the weights of any two disjoint subsets of the real weights mentioned in the formula.

where R is the set of roles, and UA and PA are the assignments of permissions and roles². A *relative* complexity is then computed as the ratio between the complexity of the target state and the complexity of the “trivial” admissible state; in such state, each user is assigned one and only one custom role which entails exactly the set of permissions that user has in UPA . In this configuration, the UA matrix becomes the identity matrix, there are as many roles as users, and all the knowledge about permissions is in PA . The absolute complexity of such trivial state is thus $(|UPA| + |U|) + |U|^2$. The (relative) simplicity³ of a state is then:

$$opt = 1 - \frac{|UA| + |PA| + |R|^2}{|UPA| + |U| + |U|^2} \quad (19)$$

4 VALIDATION

In this section we validate our maintenance algorithm (i) at a *small-scale* in Section 4.1, where we apply it to our motivating example from Section 2.4; (ii) at a *larger scale* in Section 4.2, where we present experimental results showing its viability in real-word cases.

4.1 Maintenance of our working example

Table 1 and Table 2 formalize (a small version of) the example from Section 2.4. Table 3 represents the corresponding RBAC state as Permission-to-Role and Role-to-User assignments.

We now imagine that four exceptions to such initial RBAC state—listed in Table 4—are captured by the monitoring system or directly specified by the RBAC administrator. We have to incorporate them.

e_1 : According to the first exception, it is necessary to augment the permissions of user u_4 by granting him access to the marketing application (permission p_7). The user already has access to the publishing application and to the business database. Different RBAC states result from different values of the balance parameter β . For example, if (i) the administrator prefers not to alter the current state too much and he submits the value $\beta = 0.1$ to the algorithm, the exception is incorporated by simply assigning role *marketingFunct* to u_4 . The variation over the current RBAC state is minimal ($sim = 1$) and the complexity of the state is almost unchanged at $opt = 0.388$ (same number of roles and the addition of a single assignment); in case (ii) the administrator values simplicity more and sets the value $\beta = 0.5$, the algorithm again answers by assigning role *marketingFunct* to u_4 , but then carries out further simplifying role and adjustments to the input state. Namely, the *busComm* role is augmented with the Internet access permission thus making it possible to reduce the assignments of role *genComm* to just u_6 and u_7 . Simplicity raises to $opt = 0.432$ at the price of lowering the similarity to $sim = 0.958$. Let us suppose in what follows that the RBAC administrator picks option (ii).

e_2 : It is now required that the marketing application (p_7) is made accessible to user u_5 too. Assuming (i) a high interest in similarity

($\beta = 0.1$), the algorithm assigns role *marketingFunct* to u_5 , thus maintaining a stable state ($sim = 1$) and essentially the same simplicity ($opt = 0.429$). Alternatively, (ii) the assignment $\beta = 0.5$ causes the algorithm to extend the role *publishingFunct*—which is already assigned to u_5 —with the missing permission p_7 . This is possible since no user is granted p_8 but not p_7 anymore. Thus, the new *publishingFunct* role enables both business applications, while *marketingFunct* gives access to the marketing application only, and is assigned to u_3 . This new RBAC state has a lower similarity at $sim = 0.958$, but only a slightly improved simplicity at 0.446. Since the optimization is marginal and it comes with a damage to similarity, the administrator picks option (i) over option (ii).

e_3 : User u_3 needs to access the publishing application (permission p_7). It turns out that in the current state all users granted p_8 should also have p_7 . It follows that the distinction between *marketingFunct* and *publishingFunct* makes no longer sense. It is thus not surprising that with even moderate importance assigned to simplicity ($\beta = 0.3$), the algorithm answers by joining such two roles and reducing the state complexity to $opt = 0.541$, at the price of some state variation ($sim = 0.935$). In particular, role *publishingFunct* is erased and role *marketingFunct*, already assigned to u_3 , is augmented with p_8 , thus enabling the access to both business applications.

e_4 : User u_6 is assigned the responsibility of managing the business DB (permission p_{10}). A role enabling the management of all the databases (business and HR, p_{10} and p_9) already exists: *DBAdmin*. However, u_6 is to be granted access to the business database only; it may be necessary to augment the role set. By applying the role-maintenance algorithm with $\beta = 0.1$, we are indeed returned a target RBAC state with a new role (we call it *BussDBAdmin*) assigned to u_6 and including permission p_{10} only. The complexity of the RBAC state is augmented since a new role is introduced ($opt = 0.452$); there is a minimal variation from the initial state ($sim = 0.958$).

4.2 Experimental Evaluation

To experiment at a larger scale we exercise our algorithm on the semi-synthetic datasets described in Section 4.2.1. Different MaxSAT solvers are compared in Section 4.2.2 to select the one that best suits our needs. Experimental results, performed on a 20-core Intel CPU with 138GB of memory, are presented in Section 4.2.3. Unless noted otherwise, we set $k^+ = k^- = 1$. All the results and datasets can be downloaded from the web site [10].

4.2.1 Synthetic datasets. In order to experiment with our maintenance procedure, we require datasets that include: (i) some initial RBAC state defined in terms of a set of roles and their assignment to users, and (ii) a list of exceptions: couples of Permission-to-User assignments that are either missing or in excess in such RBAC state. To the best of our knowledge, no public dataset provides such information. Most datasets only consist of a binary matrix of Permission-to-User assignments, with no associated RBAC state. Moreover, exception histories are never included.

In order to synthesize a benchmark for the maintenance problem, we start with four existing datasets: our example from Section 4.1, named *SmallComp* here, plus three classical problems of increasing

² By squaring the role number, we penalize the growth of roles more than linearly compared to the role and permission assignments, without introducing arbitrary weighting factors. Alternative definitions from the literature have been tested, with minimal impact on the results from Section 4.

³ Another possibility here would be to measure the absolute complexity via (18) as a percentage of the complexity of the initial state. We prefer measure (19) because it is independent of the unknown quality of the initial state (which we'll synthesize via an approximate miner) and stays in the range [0, 1] with a clear meaning at the extremes.

U/P	p1	p2	p3	p4	p5	p6	p7	p8	p9	p10	p11
u1	1	1	0	1	0	1	1	1	0	0	0
u2	1	1	0	1	0	1	1	1	0	0	0
u3	1	1	0	1	0	1	1	0[1] ³	0	0	0
u4	1	1	0	1	0	1	0[1] ¹	1	0	0	0
u5	1	1	0	1	0	1	0[1] ²	1	0	0	0
u6	1	0	0	0	0	0	0	0	0	0[1] ⁴	0
u7	1	0	0	0	0	0	0	0	0	0	0
u8	1	1	0	1	0	1	0	0	1	1	0
u9	1	1	0	0	0	1	1	1	0	0	1
u10	1	1	0	1	1	1	0	0	0	0	0
u11	1	1	1	0	0	1	0	0	0	0	0

Table 1: Permission-to-User assignment matrix (UPA).

Permission	Meaning
p1	Internet Access
p2	Mail Access
p3	HR DB Access
p4	Business DB Access
p5	High Perf. Comp. Resource Access
p6	Intranet Access
p7	Marketing Application Access
p8	Publishing Application Access
p9	HR DB Admin
p10	Business DB Admin
p11	Application Server Admin

Table 2: Permissions and their meaning.

R/P	p1	p2	p3	p4	p5	p6	p7	p8	p9	p10	p11	u1	u2	u3	u4	u5	u6	u7	u8	u9	u10	u11
bussComm	0[1] ¹	1	0	0	0	1	0	0	0	0	0	1	1	1	1	1	0	0	1	1	1	1
genComm	1	0	0	0	0	0	0	0	0	0	0	1[0] ¹	1[0] ¹	1[0] ¹	1[0] ¹	1[0] ¹	1	1	1[0] ¹	1[0] ¹	1[0] ¹	1[0] ¹
marketingFunct	0	0	0	1	0	0	1	0[1] ³	0	0	0	1	1	1	0[1] ¹	0[1] ²	0	0	0	0	0	0
publishingFunct	0	0	0	1	0	0	0	1	0	0	0	1[0] ³	1[0] ³	0[0] ³	1[0] ³	1[0] ³	0[0] ³	0[0] ³	0[0] ³	0[0] ³	0[0] ³	0[0] ³
HRManagement	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1
QualityAssurance	0	0	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0
DBAdmin	0	0	0	1	0	0	0	0	0	1	1	0	0	0	0	0	0	1	0	0	0	0
ServerAdmin	0	0	0	0	0	0	1	1	0	0	1	0	0	0	0	0	0	0	1	0	0	0
BussDBAdmin	[0] ⁴	[0] ⁴	[0] ⁴	[0] ⁴	[0] ⁴	[0] ⁴	[0] ⁴	[0] ⁴	[0] ⁴	[0] ⁴	[1] ⁴	[0] ⁴	[0] ⁴	[0] ⁴	[0] ⁴	[1] ⁴	[0] ⁴	[0] ⁴	[0] ⁴	[0] ⁴	[0] ⁴	[0] ⁴

Table 3: Permission-to-Role matrix PA (left) and User-to-Role matrix UA^T (right).

dimension used in the role-mining literature, called *Domino*, *University*, and *Firewall1*. All these instances are defined in terms of their User-to-Permission matrix. Starting from one of these UPA matrices, we generate (purely additive) maintenance instances as follows:

- (1) Given a positive integer k , we randomly select k user permissions in UPA and remove them, thus obtaining UPA^0 . A random order is assigned to such permissions to obtain the list of exceptions which we ask our algorithm to incorporate;
- (2) We synthesize a complete RBAC state out of UPA^0 via one of the role mining algorithms available in the literature. Given that we aim at an arbitrary (not necessarily optimal) initial state, we adopt *Fast Miner*, a heuristic procedure which returns a sub-optimal set of roles⁴. The corresponding UA^0 matrix is then iteratively generated, and the resulting RBAC state is used as initial state for the maintenance algorithm.

The resulting benchmark is described in Table 5.

⁴ We choose *Fast Miner* over the alternatives—which we tried—because it quickly returns solutions of good quality to medium and large problems. In preliminary experiments done with other miners, we observed different initial absolute values but quite similar trends (as a function of beta and of the timeout) in all the experiments.

Exception	β	UPA	RBAC state			
			R	UA + PA	opt	sim
-	-	50	8	47	0.390	-
$e_1 : p_7 \rightarrow u_4$	0.5	51	8	40	0.432	0.958
$e_2 : p_7 \rightarrow u_5$	0.1	52	8	41	0.429	1
$e_3 : p_8 \rightarrow u_3$	0.3	53	7	36	0.541	0.935
$e_4 : p_{10} \rightarrow u_6$	0.1	54	8	38	0.452	0.958

Table 4: List of exceptions to incorporate in the working example. In Table 1 and Table 3, we note $old[new]^i$ an assignment that is changed from value *old* to value *new* by applying exception e^i .

4.2.2 Choosing a Max-SAT solver. Max-SAT solvers are either complete or incomplete. Complete solvers always identify the optimal solution (if one exists), given enough time. Incomplete solvers determine an approximate solution, with no guarantee on how distant it is from the optimum. In practice, on satisfiable non-random instances, they often produce good approximations, and quickly. Furthermore, some solvers work as anytime algorithms, i.e., given any timeout as input, they return the best solution they could possibly find (if any) within the assigned timeout.

Complete solvers. Solving large instances of the role-maintenance problem may be unfeasible in practice for complete solvers. To check if this is the case, we chose a few of the best performers at the Max-SAT 2016 international competition⁵. Then, we exercise these promising solvers on the datasets from Section 4.2.1, imposing a time limit of 1 hour. Results are in the following table.

Solver	SmallComp	Domino	University	Firewall1
Maximo	$\beta \leq 0.5$	$\beta = 0$	$\beta = 0$	$\beta = 0$
MaxHS	$\beta \leq 0.4$	$\beta = 0$	$\beta = 0$	-
LMHS	$\beta \leq 0.3$	$\beta = 0$	$\beta = 0$	-
Ahmaxsat	$\beta \leq 0.25$	-	-	-

The table shows the maximum value of β for which solvers are able to incorporate all the exceptions. The symbol “-” means the solver failed (timeout or memout) on one or more instances.

The *SmallComp* dataset is the only one for which we obtain some results across the entire panel of solvers. Even with such dataset of small instances, as β grows most solvers quickly stop responding within the allotted timeout. For example, even the best solver of *SmallComp* (i.e., Maximo) fails instances as $\beta > 0.5$.

As expected, the larger the instances in the benchmark, the sooner complete solvers stop responding. More surprising is how

⁵Max-Sat competition web-site - <http://maxsat.ia.udl.cat/introduction/>

Dataset	#U	#P	Density	RBAC state		#Excs	Max-SAT encoding			
				$ R^0 $	$ UA^0 + PA^0 $		#V	#C	#C _h	#C _s
SmallComp	11	11	0.207	13	65	12	$6.9 \cdot 10^2$	$3 \cdot 10^3$	$2.6 \cdot 10^3$	$3.7 \cdot 10^2$
Domino	79	231	0.039	71	1803	19	$7.4 \cdot 10^4$	$1.4 \cdot 10^6$	$1.4 \cdot 10^6$	$2.4 \cdot 10^4$
University	493	56	0.143	71	8769	10	$3.2 \cdot 10^5$	$2.6 \cdot 10^6$	$2.6 \cdot 10^6$	$4.8 \cdot 10^4$
Firewall1	365	709	0.123	580	99713	32	$1.9 \cdot 10^7$	$1.9 \cdot 10^8$	$1.9 \cdot 10^8$	$7.2 \cdot 10^5$

Table 5: Datasets in our benchmark. #U, #P, and *Density* are the number of users, permissions, and the percentage of assignments in the UPA^0 matrix, respectively, after the removal of #Excs exceptions. $|R^0|$ and $|UA^0| + |PA^0|$ are the number of roles and assignments in the initial RBAC state. #V, #C, #C_h, and #C_s are the number of variables, clauses, hard clauses, and soft clauses in the Max-SAT encoding, respectively.

quickly performance deteriorates: By the time we try to solve *Firewall1*, even the best solver doesn't return solutions unless $\beta = 0$.

We conclude that it is not feasible to employ complete solvers to tackle real-world instances of our RBAC maintenance encoding, except perhaps for very small values of β .

Incomplete solvers. Let us shift our attention to state-of-the-art incomplete solvers, as represented by the best performers at Max-SAT 2016. We tested: *Dsat*, *CCLS2015*, *CCEHC*, *OptiRiss*, *Dist*, *WPM3*.

CCEHC [7] stands out here, because it is the only solver that: (1) showed a strong performance at Max-SAT 2016, (2) accepts command-line options to tune its solving behavior for industrial benchmarks, and (3) works as an anytime algorithm.

Let us first measure how CCEHC behaves in a case clearly beyond reach for complete solvers. We generate 90 maintenance instances of increasing size from *Firewall1* by selecting more and more of its users (i.e., rows); each instance is associated with a single exception to incorporate and generates a Max-SAT encoding of growing size. We ask CCEHC to incorporate the exception leaning strongly towards on optimized, simpler RBAC state ($\beta = 0.8$). Figure 2 shows the minimum timeout needed to obtain a feasible solution for these inputs as a function of their size.

While the minimum timeout grows exponentially, performances over instances in a real-world size range are acceptable; for example, it is possible to obtain a solution in less than one hour for a 337.2MB

Figure 2: Minimum time to compute feasible solutions to Firewall1 (y axis, secs) as a function of the number of users (x axis). Along the x axis we also note the size of the corresponding CNF encoding in Megabytes.

formula that encodes the problem of incorporating one exception into an RBAC state with 165 users and 709 permissions.

But how good are these solutions? To provide an estimate we compare them to optimal solutions returned by complete solvers. As we have shown, *Firewall1* is completely out of reach, so we resort to *SmallComp*. Given that hard constraints are always satisfied by feasible solutions from both complete and incomplete solvers, we focus on the ability of the incomplete solver to satisfy soft constraints. In particular, we compute the average weight of satisfied soft constraints over the total sum of input weights for the 12 different exceptions in the dataset. In Figure 3 we report this metric measured after CCEHC has worked for $t = 2sec$ and then for $t = 180sec$. We include the same metric computed on optimal solutions by complete solvers, which we could obtain for $\beta \leq 0.5$ (1 hour timeout). As expected, the complete solver outputs (slightly) better answers across the line, independently of β , but CCEHC is not far. These results, though quite comforting, have to be taken with a grain of salt because the small instances in *SmallComp* may not be representative of the general behavior of CCEHC.

4.2.3 Experimental Results. Incremental Max-SAT solvers seem capable of providing good approximate solutions within reasonable time on real-world instances. In the rest of this section we use them (namely, CCEHC) to explore the behavior of our encoding. We set $k^- = 7$ and $k^+ = 2$ to prefer role reduction to matrix simplification.

Figure 3: Average percentage of satisfied soft clauses (y axis) as a function of the balance β (x axis) in the *SmallComp* dataset.

Figure 4: Average similarity and simplicity (y axis) as a function of the balance β (x axis) for different datasets. Twenty-one values of $\beta \in [0, 1]$ are sampled, at regular intervals (step 0.05).

Impact of β . The first thing we assess experimentally is the impact the value of β has on the structure of the fixed RBAC state. Figure 4 shows that—for all datasets—similarity decreases (and simplicity increases) almost monotonically as β grows. Figure 6 shows how the corresponding number of roles decreases on average by increasing the preference towards optimized solutions (β close to 1). Conversely, if the encoding leans towards preserving the original RBAC state (β close to 0) these numbers stay close to the input values. None of the instances in the largest of the four datasets could be solved within 1 hour: Firewall1 thus constitute a *hard* problem at present (see Conclusions). Overall, by tuning β , it seems actually possible to steer the quality of the solution towards similarity or simplicity during maintenance.

Impact of timeout. We show how the output RBAC state (simplicity, similarity) changes by granting more time to the solver, at different balance points. Figure 5 and 7 show the results for timeouts in 10s–600s on the dataset *Domino* (representative of the entire benchmark). Simplicity increases for $\beta \geq 0.25$ (we are optimizing the state as a side effect of the maintenance) while it drifts towards lower values when $\beta < 0.25$ (the price we pay to avoid reworking the RBAC state too much as exceptions arrive). Conversely, similarity improves for low values of β while it remains almost stable

Figure 5: Average simplicity in *Domino* (y axis) as a function of the timeout (x axis, secs) at difference balance points (β).

Figure 6: Average number of roles after incorporating exceptions (y axis) as a function of the balance β (x axis) for different datasets.

for $\beta \geq 0.25$. It follows that if the interest towards simplification is high, it always pays to allot more reasoning time to the solver. Conversely, if the interest is in preserving the input state, increasing the reasoning time pays for $\beta < 0.25$, where similarity increases at the expense of simplicity.

The order of exceptions. In these experiments, exceptions are incorporated sequentially, one-by-one, as they show up (though bulk incorporation is also possible, as already noted). We are interested in understanding how much the order in which they manifest affects the quality of the eventual state. In principle, the number of roles can either increase (small values of β) or decrease (large β). To confirm this, we select the *Domino* dataset and pick a string of 6 exceptions to be incorporated. We generate all their permutations (720) as incorporation sequences. For each sequence, we record the final number of roles, assignments, simplicity and similarity to the initial state (that has 73 roles). Figure 8 shows the distribution generated by 715 paths (each solvable in 60 seconds) with four different β configurations. While by construction the final states are equivalent in terms of Permission-to-User assignments, the average value and the variance of the distributions are profoundly impacted by β ; the variance in particular widens as β goes close to 0; for example, with $\beta = 0.1$ the number of roles ranges from 28 to

Figure 7: Average similarity in *Domino* (y axis) as a function of the timeout (x axis, secs) at difference balance points (β).

Figure 8: Distribution of the number of roles in *Domino* (x axis) after incorporating 6 exceptions in all possible orderings.

43, corresponding respectively to a 61.6% to 41.1% reduction with respect to initial RBAC state.

This behavior suggests that—independently of the order in which exceptions arrive—by leveraging β the RBAC administrator can direct the state over time to become simpler, to accumulate “clutter,” or to remain stable in terms of number of roles.

5 RELATED WORK

The literature on Role Mining and RBAC state tuning is quite extensive. The works most closely related to this paper are [5, 13, 16, 17]. These approaches are similar to ours in that they aim at reconfiguring an existing set of roles with the goal of simplifying them.

In [16] the simplification procedure works as follows: A candidate set of new roles are iteratively generated starting from the trivial role set (one role per permission) and combining them in several ways (pairwise, etc.). The candidate set is kept consistent with the input RBAC state, i.e., each old role must be representable as the union of some new roles. Individual candidates with the lowest “management cost” are preferred, where the management cost is the sum of a fixed component (the same for all roles) plus a variable component which depends on the role cardinality and granularity. A parameter is introduced to balance their relative importance.

The approach presented in [13] more directly balances the quality of the new RBAC state against its similarity to the current one. It first applies a canonical role mining algorithm (Fast Miner) to generate a starting set of candidates. Then it iteratively evaluates roles according to a fitness function and selects the best ones until a solution is obtained that covers all the Permission-to-User assignments of the input RBAC state. The fitness function is a weighted sum of a measure of similarity to the original roles and a measure of coverage of the input Permission-to-User matrix. This algorithm has been extended to work with role hierarchies [4, 11].

A different perspective is adopted in [17]. This procedure analyzes logs of actual permission utilizations over time and exploits this data to inform the next RBAC state. There are two phases: The first phase applies a variant of the subset enumeration technique of Fast Miner to iteratively generate a set of roles, which are then sorted according to a fitness function. The fitness function is

where the traces of real permission usage come into play: It aims at balancing similarity to the original roles with “homogeneity” to actual permission utilization. At each iteration, top ranked roles which cover the input Permission-to-User matrix are selected; this is repeated until fixpoint (two subsequent steps result in the same set of roles) or until a maximum number of iterations is reached. The second phase then assigns roles to users based on a heuristic process which aims to contain redundancy.

A first difference between our approach and [13, 16] is that those procedures ignore how the simplified Permission-to-Role matrix they output will impact (in terms of administration costs) the correspondingly adjusted Permission-to-User assignments. This is evident because at no point those procedures take into account the UA matrix or how complex it would be for the administrator to edit it. Furthermore, in [16] the management cost of a set of roles is defined by only looking at roles per se, and not at the distance from the original. Overall, the link between the input matrices and the output matrices is very indirect: Both couples represent the same Permission-to-User relations, but the difficulty for an administrator to transform UA^0 into UA and PA^0 into PA is not modeled. Conversely, our encoding explicitly captures the difference of UA from UA^0 and PA from PA^0 , so it relates more directly to the amount of work an administrator will have to do.

Another major difference from [13, 16, 17] is that our approach captures the RBAC optimization problem declaratively within a logic formalism, and neatly separates the declaration of the constraints and objective function from the search for an optimal solution. This provides several benefits and additional guarantees versus employing custom multi-phase heuristics. One advantage is that future breakthroughs from the community developing complete (and incomplete) Max-SAT solvers will result in an improved RBAC maintenance process without any change to our encoding. Another advantage is that the mechanism controlling the trade-off between quality and similarity provides strong guarantees about the output RBAC state; for example, for $\beta = 0$ we are sure that the output RBAC state is the least variation to the input that is capable of accommodating the intervening exception, whereas for $\beta = 1$ we have the guarantee that the output state is optimal w.r.t. a certain metric and that the input state has been completely ignored⁶.

A logic-based formalization of the RBAC state similar to ours is presented in [5]. In that case, the Satisfiability Modulo Theory (SMT) framework is adopted; the theory used to expand the expressive power of SAT is Integer Linear Programming (ILP), employed to capture certain boolean-unfriendly quality metrics. The paper shows how to exploit SMT to solve several role mining variants, including one that generates an RBAC state optimal in terms of a combination of WSC [9] and similarity to an input RBAC state. To this end, a heuristic technique is presented to iteratively generate role sets, which are evaluated according to a fitness function that balances the conflicting WSC-vs-similarity objectives. The complete RBAC state is thus taken into account, and configurations that disrupt the Role-to-User assignments are penalized. Other

⁶It is worth noticing that while the *encoding* we propose offers these guarantees, we may lose them by solving the resulting problems with *incomplete* solvers.

variants consider the permission usage or the RBAC hierarchy as optimization metrics, resulting in a hybrid role mining approach.

The major difference between this approach and ours (beyond the definitions of certain quality metrics) is that we embrace the inherent nature of the role maintenance process as an *optimization procedure* by using a logic framework (Max-SAT) meant to optimize an objective function, whereas Satisfiability Modulo Theory (SMT) is a decision procedure meant to prove the consistency of statements. As a result, while the entire problem is captured by a single Max-SAT instance and the reasoning/searching/optimization stage is decoupled and offloaded to an external solver, [5] generates a long sequence of SMT problems in the context of a complex, custom algorithm to achieve the trade-off between quality and similarity. Another difference is that instead of generating candidate role sets and testing them against the original RBAC state to assess their similarity, we directly embed the original RBAC state as-is in the encoding together with penalties for diverging from it.

Another key difference from previous approaches is in when and how the RBAC reconfiguration is supposed to happen. Previous algorithms are presented as off-line procedures; administrators are supposed to first let the complexity of the RBAC state increase, perhaps by creating several ad-hoc new roles to quickly fix exceptions. Then, from time to time, a procedure is applied to simplify everything. Large discontinuities are acceptable at these “reconfiguration points”. Conversely, the approach presented here can be leveraged to continuously introduce small optimizing changes: Applied on-line, in an exception-driven way, it steers the trajectory of the RBAC state towards simplicity without any costly or stakeholder-adverse update.

6 CONCLUSION AND FUTURE WORK

We presented a novel approach to perform maintenance of RBAC states, that is, to incorporate exceptions as they show up and improve the quality of the role model along the way. Based upon generating Max-SAT instances and solving them via state-of-the-art solvers, this method seems capable of dealing with real-life instances.

We are motivated to pursue optimized RBAC maintenance by actual issues at our company. Seldom there is the time and opportunity to perform large, impactful RBAC updates; conversely, errors show up on a daily basis, and if any push towards optimization can be exerted during maintenance, the RBAC state may actually start to converge to an (almost) optimal version of itself.

Directions for future work are as follows. Large datasets (e.g., *Firewall1*) still require too much time to solve; improvements to the encodings and/or the solving stage are of the essence here⁷. We haven't fully investigated yet how our technique behaves in the case of permissions in excess, to be withdrawn; these cases are indeed more difficult to capture automatically and we have less examples at our disposal. An optimal, fully-specified (and automatically synthesized) course of action for the RBAC administrator to implement a fix into an RBAC management system in use is another objective of interest, which seems within reach.

⁷We submitted our datasets to the organizers of the Max-SAT competition to provide them with challenging real-world instances on which to compare solvers.

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