Priority Based Wireless Multi-Network Selection Games

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ABSTRACT

With increase in the number of connected wireless devices, simultaneous use of multiple wireless networks has been proposed as a solution to improve throughput and reduce congestion. The autonomous choice of networks by clients leads to a network selection game, where selfish clients strategize to find the best set of providers. In this network selection game, we define client utility as a function of the throughput achieved when the number of such network connections is constrained. The throughput model is based on capacity sharing and incorporates link dependent PHY rates and priority levels of the clients. This paper studies the stability of the network selection game and provides bounds on the inefficiency of such games. We also present results based on simulation studies.

CCS CONCEPTS

• **Networks** → **Mobile networks**; *Network resources allocation*; *Network economics*;

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1 INTRODUCTION

The explosive growth of wireless connected devices motivates the need to improve connectivity and throughput in wireless systems. This necessitates moving beyond the classical systems that utilize one base-station/provider for every user. Advances in wireless systems introduce the notion that wireless devices or clients can select multiple base-stations/providers as well as the type of access (Wi-Fi, LTE-Advanced/IEEE 802.16 schemes, WiMAX, femtocells etc.) to provide enhanced throughput rates (Google Fi services is an example). Increase of traffic demands [8] is expected to increase the density of base stations. The proposed coverage includes the use of Macrocells, Microcells, Picocells and Femtocells that provide coverage to kilometer ranges, hundreds of meter ranges and residential ranges, respectively. While traditional standard models have considered the association of single base stations with a user, in substantially heavy load scenarios, mechanisms that allow

clients devices to efficiently utilize multiple base stations have been considered. In this spirit, co-ordinated multi-point [1] mechanisms between base stations have been proposed. These mechanisms provide for co-ordination between base stations to service users concurrently [8]. Multi-homing clients or devices which can connect to multiple providers at the same time and distribute their demands over multiple connections (concurrent multi-access) have also been proposed so as to obtain *always best connected* characteristics [22].

In this paper, we consider multiple base station system servicing multiple clients. Each client-base station pair has a PHY rate based on device characteristics and SNR. The eventual throughput achieved will be jointly determined as a function of the congestion. To achieve a quality of service (QoS) via concurrent multi-access, individual client priorities can be ensured by providing priority agreements. Throughput analysis of accessing multiple bases stations with heterogeneous radio technologies has been studied in [2, 14, 15, 22]. In these models, clients utilize information from the access networks to determine the choice of base stations or network access points.

Mechanisms that utilize co-ordinated control via administrative oversight from multiple base stations, and algorithms to optimize the benefit to clients have been designed in [7, 8, 20]. However, in practice, clients are autonomous agents, with or without additional rules that may be imposed on the clients to regulate traffic. This system of autonomous agents, competing for a limited set of resources, leads to a congestion game. Such a complex system model where a user (client) would select a base station, based on priority weights and an instantaneous PHY rate provided by the base station, has been utilized in [15] to study this problem as the RAN (Radio Access Network) selection problem, first proposed in [2]. The throughput achieved depends on current conditions like base station load, location, or even radio bandwidth congestion. In this congestion game, every client seeks to maximize her own total throughput given a fixed strategy of the other clients. We enhance this model to apply to multi-point mechanisms by allowing strategy sets where clients can choose multiple base stations. In general, the study of congestion games was initiated in [17] where congestion was measured by a cost function that was a function of the number of players occupying a resource. Additional throughput or utility models can be found in the survey paper [22]. As distinct from the game where multiple base station access is allowed, the above papers consider a model termed as an *atomic throughput game*, also referred to as a RAN selection game. The previous results do not account for multiple base station choices for players and leave a number of unresolved issues regarding the existence of Nash equilibrium in the defined games and the inefficiency of the game.

Current research on congestion games places emphasis on the inefficiency of the Nash equilibrium. The inefficiency of Nash equilibrium compares the worst equilibrium solution to an optimal

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solution using a social objective function. The ratio is referred to as the *Price of Anarchy* (PoA) [13]. Alternatively an optimistic viewpoint considers the *Price of Stability* (PoS) that compares the best equilibrium to the optimal solution. These measures have become important measures in the performance analysis of networks. In the RAN selection game, we utilize two different metrics for social welfare, one metric being the minimum achievable throughput and the other the total throughput provided by the system. Recent research has also concentrated on the complexity of games. For congestion games the paper [6], proved that atomic congestion games with arbitrary cost functions are $\mathcal{PLS}$ -Complete.

The price of anarchy of weighted congestion games has been studied in [3] where the players have weights which contribute to the load on the resource. Wireless congestion games and cost network specific cost functions have been studied in [11, 14, 21]. Unlike prior studies, the model in [2, 15] directly addresses the throughput as a metric of performance. However, the price of anarchy has not been examined in this context. The price of anarchy in a related association game on IEEE 802.11 LAN networks with cost functions that is a linear function of the number of players and their rates has been studied in [5]. Additional game theoretic models using evolutionary games [16, 19] have been considered but are not relevant as these models correspond to non-atomic versions of the game with a large number of clients, each with infinitesimal impact.

In this paper we consider the selection game and formalize the notion of concurrent access to base stations as Atomic Splittable Throughput RAN Games and prove results regarding the computation and inefficiency of Nash Equilibrium. In order to characterize the complexity of computing Nash equilibrium in this model, we formulate a generalization of the game, termed the Utility Throughput Game (UTG), where the utility of each client is given by a function of her throughput. In this model we prove that finding a Nash Equilibrium is hard and, in fact, $\mathcal{PLS}$ -complete for non-decreasing utility functions. We also present a polynomial time algorithm for the case with uniform priorities and rates.

We also find bounds on the the Price of Anarchy in UTG games, utilizing two measures of social welfare, one where the optimum solution is obtained via maximizing the minimum throughput achieved by any player (termed the max-min measure) and the other where we utilize the sum of throughputs. Our results indicate that the Price of Anarchy worsens with increased disparity in client priorities.

Furthermore, we conducted numerical simulations on UTG games. We present results on simulating the effect of priority disparity on throughput using a network modeled on a real campus environment; we also show convergence of a greedy approach to determine Nash Equilibrium.

2 NETWORK SELECTION GAMES

The wireless selection problem has a set of clients P accessing a set wireless access points K , which we refer to as base stations (BS). The base stations represent the range of wireless access points. A client can connect to one or more base stations at a given PHY rate; and throughput on a BS is determined by the set of clients accessing

it. The objective is to schedule clients on base stations to optimize throughput.

We represent the problem using a network model where the underlying graph is bipartite, denoted by $G = (P, K, E)$. Vertex set P represents the set of clients, and the vertex set K represents the base stations. The edge set $E \subseteq P \times K$ represents the available client-base station pairs. An edge, $e = (i, k)$, $i \in P$, $k \in K$, is associated with two parameters:

- (i) Priority of service $\phi_{i,k}$ that client i has on base station k and
- (ii) PHY rate $R_{i,k}$ that client i obtains on base station k .

The throughput $\omega_{i,k}$ that client i obtains on base station k is defined using the model in [2, 15] as follows:

$$\omega_{i,k}(s(k)) = \frac{\phi_{i,k}}{\sum_{j \in s(k)} \frac{\phi_{j,k}}{R_{j,k}}}$$

where $s(k)$ is the set of clients that are currently accessing base station k . Note that when i is the only client on a base station k , her throughput is the same as her PHY rate, $R_{i,k}$.

This has been used to model fair access MAC protocols provided by the distributed coordination function (DCF) in IEEE 802.11 [2, 4, 9]. Throughput using round-robin mechanisms can be obtained by assigning $\phi_{i,k} = L_i$, where each transmission slot is of length L_i . In general, QoS priorities can be implemented by using proportional sharing. The above equation can also be used to model throughput achieved by assigning QoS based contention windows ($CW_{i,k}$) to clients in 802.11e (by setting $\phi_{i,k} = \frac{2}{CW_{i,k}}$), as illustrated in [15].

To provide for capabilities for concurrent multiple access, we assume that each client i can access a subset of base stations. Let S_i be the set of BS occupied by client i . The total throughput gained by the client is

$$\omega_i = \sum_{k \in S_i} \frac{\phi_{i,k}}{\sum_{j \in s(k)} \frac{\phi_{j,k}}{R_{j,k}}} \quad (1)$$

Since each client has an independent choice of scheduling her traffic on the available base stations, the rational autonomous decisions of the client can be modeled by a game:

A **Throughput Game** is defined by $TG(P, K, \phi, R)$ where P is the set of clients (players), K is set of base stations; the function $\phi : P \times K \rightarrow \mathbb{R}^+$ represents the priorities (weights) of clients on the base stations K ; the function $R : P \times K \rightarrow \mathbb{R}^+$ provides the rates the clients can obtain on the base stations. The utility function for client i is defined by the total throughput achieved ω_i , as specified in eq 1. In this game, given the strategies of other clients, client i selects a set of base stations to maximize her utility.

We consider generalized utilities of the players in the above game model. The utility for client i is defined as $U_i = \sum_{k \in S_i} f(\omega_{i,k})$, where $f : \mathbb{R} \rightarrow \mathbb{R}$ is a real-valued non-decreasing quasi-concave function. We term this generalized game as an **Utility Throughput Game**, defined by $UTG(P, S, \mathcal{U}, \phi, R, K)$ where P, ϕ, R, K denote the same sets and values as defined for a Throughput Game and $\mathcal{U} = \{U_1, U_2, \dots, U_P\}$ is the set of utilities of the players. Every player has a strategy set $\mathcal{S}$, comprising of subsets of $\mathcal{P}(K)$ (power set of the set of links), representing the set of BS that the client may access. In a realistic setting, clients have limits on the number of base stations they can access. Such games are termed **T-split Utility Throughput Games**, denoted by $UTG(P, S, \mathcal{U}, \phi, R, K, T)$, or

UTG for simplicity, where clients can select a subset of base stations of size at most T from the set K . This limit may depend on a number of factors, including hardware constraints, such as when a device cannot access more than T number of distinct radio channels simultaneously. The above games are distinguished by their respective parameter lists.

We also consider imposing rules on client priorities and rates across base stations, and define additional types of T-split games:

(i) **Uniform Priority UTG** games, $UTG_\phi(P, S, \mathcal{U}, \phi, R, K, T)$, or UTG_ϕ , are games where the priority levels are independent of the base stations, i.e. $\phi_{i,j} = \phi_{i,k} = \phi_i, \forall j, k$.

(ii) **Uniform Priority-Rate UTG**, $UTG_{\phi,R}(P, S, \mathcal{U}, \phi, R, K, T)$, or $UTG_{\phi,R}$, are games where, in addition to the priorities, the rates achieved by a client i are also independent of the base stations, i.e. $R_{i,k} = R_{i,j} = R_i$.

A game is at a **Pure Nash Equilibrium** when for every client i , the client cannot improve upon her total utility by changing her current strategy to occupy a different set of base stations, while keeping the other clients fixed. Let S_i represent the strategy at a Nash equilibrium for client i , and S_{-i} be the strategy profile for the other clients. Then, at Nash equilibrium

$$U_i(S_i, S_{-i}) = f\left(\sum_{k \in S_i} \omega_{i,k}(s(k))\right) \geq f\left(\sum_{k \in S'_i} \omega_{i,k}(s'(k))\right)$$

where $s(k)$ and $s'(k)$ are the sets of players on base station k under strategy profiles (S_i, S_{-i}) and (S'_i, S_{-i}) respectively. In this paper, we only consider pure Nash equilibrium solutions.

The *Social Optimum* strategy profile, S^* , is an assignment of clients to base stations such that a defined measure for Social Utility is maximized. We present two measures for the social utility $\mathcal{U}_{soc}(S)$, where S is a strategy profile:

(i) *Max-min measure*: The social utility of a solution is defined to be the minimum utility of any client.

$$\mathcal{U}_{soc}^M(S) = \min_i U_i(S_i, S_{-i}) \quad (2)$$

(ii) *Sum measure*: The social utility of a solution is defined to be the sum of utilities of all clients.

$$\mathcal{U}_{soc}^S(S) = \sum_{i \in P} U_i(S_i, S_{-i}) = \sum_{i \in P} f\left(\sum_{k \in S_i} \omega_{i,k}(s(k))\right) \quad (3)$$

Thus, the social optimum solution for both the measures is the strategy profile S^* such that $S^* = \arg \max_{S \in \mathcal{S}} \mathcal{U}_{soc}(S)$ where $\mathcal{S}$ is the set of all strategies. To understand the performance of these games we consider the *Price of Anarchy* (PoA) given by the lowest (worst) achievable ratio of the social utility at a Nash Equilibrium, which we denote by $\mathcal{U}_{soc}(S^{NE})$, to that of the Social optimum $\mathcal{U}_{soc}(S^*)$. It signifies the inefficiency of the equilibrium when players behave without co-operation as compared to a system which may be governed by an administrator to optimize the social utility measure.

$$PoA = \inf_{G \in \mathcal{G}} \inf_{S^{NE} \in NE_G} \frac{\mathcal{U}_{soc}(S^{NE})}{\mathcal{U}_{soc}(S^*)} \quad (4)$$

where $\mathcal{G}$ is the set of all UTG games and NE_G the set of all Nash equilibrium over the game G .

Alternatively, an optimistic viewpoint considers the *Price of Stability* (PoS), given by the best ratio of the the social utility at Nash

Equilibrium to that of the Social optimum,

$$PoS = \inf_{G \in \mathcal{G}} \sup_{S^{NE} \in NE_G} \frac{\mathcal{U}_{soc}(S^{NE})}{\mathcal{U}_{soc}(S^*)} \quad (5)$$

2.1 Terminology and Preliminaries

The effective load on a base station k when a set $s(k)$ of players are scheduled on base station k is defined as

$$G_{s(k)} = \sum_{i \in s(k)} \frac{\phi_{i,k}}{R_{i,k}}$$

where $\frac{\phi_{i,k}}{R_{i,k}}$ is the effective load contribution of client i . Additionally, we define

- $(\frac{\phi}{R})_{\max} = \max_{i \in P, k \in K} \frac{\phi_{i,k}}{R_{i,k}}$,
- $(\frac{\phi}{R})_{\min} = \min_{i \in P, k \in K} \frac{\phi_{i,k}}{R_{i,k}}$,
- $\phi^m = \frac{\max_{i,k} \phi_{i,k}}{\min_{i,k} \phi_{i,k}}$, and
- $\sigma = \sum_i \max_k \frac{\phi_{i,k}}{R_{i,k}} \leq |P|(\frac{\phi}{R})_{\max}$

An initial observation is that upon being given a choice to acquire more base stations, a user will always do so. We explain it in the following lemma.

LEMMA 1. *In any instance of T-split UTG, every Nash equilibrium solution will have clients occupying exactly T base stations.*

PROOF. Since client utility U_i is a non-decreasing function of the throughput, any change that does not decrease in total throughput results in improvement in utility.

Suppose i is currently occupying the set of base stations S_i , where $|S_i| < T$. If she were to occupy another link, say k' , from the set of available links, while the other clients remain unchanged in their actions, her total throughput would not decrease because,

$$\sum_{k \in S_i} \omega_{i,k} \leq \sum_{k \in S_i} \omega_{i,k} + \omega_{i,k'} \quad (6)$$

since $\omega_{i,k'} \geq 0$. This process can continue until i is scheduled on T links. $\square$

2.2 Results and Paper outline

In this paper we present the following results:

- We show that Nash equilibrium does not always exist in UTG Games (section 3). However, it does exist for uniform priority UTG_ϕ games. Moreover, we prove that determining Nash Equilibrium in general UTG games is $\mathcal{PLS}$ -Complete (section 3.2). A polynomial algorithm is also presented for the case of uniform-priority-rate games, $UTG_{\phi,R}$.
- We establish a rule, the Priority-Fair rule, under which stable points of the game are achieved (section 3.1). This rule is easily implementable.
- Social optimum is discussed in section 4, where we show that determining the social optimum, under the Max-min measure is NP-Complete.
- We present tight bounds on the *Price of Anarchy* (PoA) in section 5, considering both measures, Max-min and Sum. Our results show that, unfortunately, the *Price of Anarchy* can be arbitrarily low. We also show that the *Price of Stability* can

be arbitrarily low, thus quashing hopes of always ensuring a good Nash equilibrium.

- We also conducted simulations (section 6) which illustrated that POA increased with increasing disparity of priorities between clients (players) in UTG games. Our simulations involved a real-world campus network. We also design a *best-response method* to show convergence of an algorithm that determines Nash equilibrium. Synthetic networks are used to illustrate its rate of convergence.

3 NASH EQUILIBRIUM IN UTILITY THROUGHPUT GAMES

In this section, we consider the existence and computation of Nash equilibrium in *UTG*. We show that a system allowing players to make selfish moves will be unstable, i.e. may not converge to a solution. Fortunately, we can establish a simple rule that will ensure stability.

3.1 Existence of Nash Equilibrium and Stability in Utility Throughput Games

Our first result shows that Nash equilibrium may not exist in *UTG* games, for an arbitrary value of T ,

THEOREM 2. *For any value of T , there exists an instance of a T -split Utility throughput game, UTG , where a pure Nash equilibrium does not exist.*

PROOF. We show this by using an instance of a Utility throughput game, where $T = 1$, for which Nash equilibrium does not exist. This game is described in the appendix and adopted from [10].

Let $G = TG(P, K, \phi, R)$ be the instance with linear utilities for which no Nash equilibrium exists. Then, from this instance, we create an instance of an augmented game $G' = UTG(P, \mathcal{S}, \mathcal{U}, \phi, R, K \cup K', T)$ as follows: Add a set of base stations K' , where $|K'| = T - 1$, to the instance. Let $\phi_{i,k} = 1$ where $k \in K'$ and let $R_{i,k}$, where $k \in K'$, equal a large number M such that $\frac{M}{|P|} > \max_{i \in P, k \in K} R_{i,k}$. This ensures that all clients would obtain higher throughput, $\frac{M}{|P|}$, on each BS $k \in K'$ as compared to $k \in K$. Let f be a linear function. This leads to every client occupying all base stations in K' and an additional link in K . Therefore, the conditions for non-existence of a Nash equilibrium in G will still exist in the augmented game G' . $\square$

COROLLARY 1. *For any instance of UTG where $T \geq |K|$, a Nash equilibrium always exists.*

PROOF. In an arbitrary game where a client i occupies less than $|K|$ base stations, her total throughput can be increased by occupying additional base stations, similar to the condition shown in eq. 6. Once all the clients are using all available links, client i will not discard a link k' as it would decrease her total throughput since $\sum_{k \in S_i} \omega_{i,k} < \sum_{k \in S_i \setminus k'} \omega_{i,k}$. $\square$

On a positive note, we observe that for a restricted Utility throughput game, Nash equilibrium exists:

THEOREM 3. *A Nash equilibrium always exists in uniform priority games UTG_ϕ .*

PROOF. We show the convergence to a Nash equilibrium using a lexicographic ordering of effective loads of base stations. For every strategic move from one link to another that improves the throughput of a player, the lexicographic ordering decreases.

The proof is similar to the proof given in [10] and more details shall be provided in a complete version of the paper. $\square$

We next define a rule that will ensure convergence to a stable strategy profile in T-split Utility throughput games.

DEFINITION 1. *Priority-fair rule: A client may switch from BS k to another BS, k' , to improve her total throughput only if her priority on the new base station is no higher than the older one, i.e. $\phi_{i,k} \geq \phi_{i,k'}$.*

We can now show that an arbitrary utility throughput game will converge to a stable strategy profile.

THEOREM 4. *Given a T -split Utility throughput game, UTG , a stable strategy profile always exists when strategy changes are regulated by the priority-fair rule.*

PROOF. Suppose we start from an arbitrary strategy that is unstable, i.e. a client i chooses to leave base station k in favor of k' . Then, we have $\frac{\phi_{i,k}}{G_{s(k)}} < \frac{\phi_{i,k'}}{G_{s(k')} + \frac{\phi_{i,k'}}{R_{i,k'}}}$. Based on the priority-fair rule,

we know $\phi_{i,k} \geq \phi_{i,k'}$ which implies, $\frac{G_{s(k')} + \frac{\phi_{i,k'}}{R_{i,k'}}}{G_{s(k)}} < \frac{\phi_{i,k'}}{\phi_{i,k}} \leq 1$.

Therefore, we ensure that the load of base station k' after i moves from k to k' is less than the load of k before i moves. This property implies that the lexicographic ordering of effective loads (as defined in [10]) of base stations decreases and thus, the strategies converge to a stable strategy profile. $\square$

3.2 Complexity of Nash Equilibrium

In this subsection, we resolve the complexity of determining Nash equilibrium in Utility Throughput Games. Since pure Nash equilibrium may not exist in general Utility Throughput Games, we focus our attention on Uniform Priority Utility Throughput Games, where $\phi_{i,k} = \phi_{i,k'} = \phi_i, \forall k \in K$. We show that finding Nash equilibrium in this class of games is $\mathcal{PLS}$ -Complete. The complexity class $\mathcal{PLS}$ -Complete was defined in [18] and is indicative of the hardness of finding a pure Nash Equilibrium. Note that the following theorem applies to the general class of *UTG* games rather than the T-split *UTG* games.

THEOREM 5. *The problem of finding Nash Equilibrium in a general Utility Throughput Game, $UTG(P, \mathcal{S}, \mathcal{U}, \phi, R, K)$, is $\mathcal{PLS}$ -Complete.*

PROOF. We prove that determining Nash equilibrium in *UTG* games is $\mathcal{PLS}$ -Complete. In fact, we show that general *UTG* games with uniform priority are $\mathcal{PLS}$ -complete. We first show that the problem is in $\mathcal{PLS}$, and then reduce a known $\mathcal{PLS}$ -complete problem to our problem:

UTG is in $\mathcal{PLS}$ since we can satisfy the following three conditions in polynomial time: 1) For any instance of the uniform priority general *UTG*, I_{UTG} , a feasible solution is obtained by ensuring that every player has employed a strategy from her strategy set. 2) For each solution, the objective value is computed for each player as the total throughput she obtains by employing a strategy. 3) For every instance and solution, if any player cannot improve her total

throughput by changing her strategy, we obtain a local optimum. Otherwise, a better solution can be obtained by allowing only 1 player to change her strategy to improve her total throughput.

Next, we reduce the POSNAE3FLIP problem, which is known to be $\mathcal{PLS}$ -Complete in [6], to the uniform priority UTG problem.

We denote the known problem as $POSNAE3FLIP(X, C, Z)$, where $X = \{x_1, x_2, \dots, x_p\}$ is the set of $\{0, 1\}$ (boolean) variables, $C = \{c_1, c_2, \dots, c_m\}$ is the set of clauses and $Z = \{z_1, z_2, \dots, z_m\}$ is the set of weights of clauses. The goal of POSNAE3FLIP is to maximize the total weight of NAE (not-all-equal) clauses s.t. the total weight cannot be increased by changing the value of any one variable.

Given an instance I_p of $POSNAE3FLIP(X, C, Z)$, we construct an instance I_{UTG} of UTG . For every clause c_k of weight z_k , we have 2 base stations L_k and L'_k . Every variable x_i is a player i in I_{UTG} . The rate of i on base station L_k and L'_k is $R_{i,L_k} = R_{i,L'_k} = 1$, if $x_i \in c_k$ and $R_{i,L_k} = R_{i,L'_k} = \epsilon$ (an infinitesimal value) otherwise. The priority of each player is 1.

The strategy sets for every player i are: occupy L_k for all clauses x_i is present in, or, occupy base stations L'_k for the same set of clauses. Therefore, $S_i = \{s_i^1 = \{L_k \mid x_i \in c_k\}, s_i^2 = \{L'_k \mid x_i \in c_k\}\}$ and $|S_i| = 2$ for each player i . If the strategy of a player is to choose base stations corresponding to the first strategy, s_i^1 , then the variable x_i is assigned a true value, whereas if she chooses base stations corresponding to the second strategy, s_i^2 , the variable x_i is assigned a false value.

The utility function of a player i on base station k is defined as $U(\omega_{i,k}) = z_k$, if k is occupied by 2 or fewer players, or $\omega_{i,k} \geq \frac{1}{2}$, and $U(\omega_{i,k}) = 0$, if k is occupied by more than 2 players, or if $\omega_{i,k} < \frac{1}{2}$. Note that the step-function U can be replaced by a differentiable quasi-concave function (the smoothstep function is an example).

By definition, for a Nash Equilibrium solution in I_{UTG} , if player i changes her strategy, her utility U_i can either remain unchanged or decrease. If the utility decreases, from amongst the S_i base stations occupied by i , the number of base stations occupied by 3 players increases due to the change in strategy.

Therefore, in I_p , less clauses c corresponding to the pairs of base stations L_k and L'_k will now have an not-all-equal assignment, implying that the total weight of clauses having a not-all-equal truth assignment $\sum_{c \in NAE} z_c$ will only have decreased, thus proving our implication, and consequently the theorem. $\square$

On further investigation of the time complexity, we note that a polynomial time algorithm to determine the Nash equilibrium exists in the case of uniform throughput and uniform rates. This is shown in Algorithm 1.

LEMMA 6. *Given an instance of an uniform priority-rate Utility throughput game, $UTG_{\phi,R}$, Algorithm 1 determines a Nash Equilibrium in time $O(|P| \cdot T \cdot |K|^2 \log_2 |K|)$.*

PROOF. The proof relies on induction, and omitted here. $\square$

4 COMPLEXITY OF SOCIAL OPTIMUM

We consider the problem finding the social optimum solution. We focus on the Max-min measure, since it is a more reasonable measure for balancing throughput.

Algorithm 1 Uniform Priority-Rate $UTG_{\phi,R}$

- 1: Create T copies for each client i , with effective loads $\frac{\phi_i^t}{R_i^t} = \frac{\phi_i}{R_i}$ where $t = \{1, 2, \dots, T\}$
  - 2: Sort all effective loads, $\frac{\phi_i^t}{R_i^t}$, in non-increasing order O
  - 3: **for** each effective load $\frac{\phi_i^t}{R_i^t}$ in O
  - 4:   Sort base stations in decreasing preference order for adding client i
  - 5:   **for** each base station k in the sorted list
  - 6:   **if** player i is not occupying base station k
  - 7:   assign i to k and exit for loop
-

THEOREM 7. *Determining the Social Optimum solution for T -split UTG is NP-Complete when the social utility of strategy profile S is measured by $\mathcal{U}_{soc}^M(S)$, the Max-min measure (equation 2).*

PROOF. Showing that the problem is in NP is easy and we omit it from this version.

We reduce *Load balancing*, a known NP-complete problem to the problem of determining a social optimum solution.

The Load balancing problem is as follows- given a set of n jobs and m identical machines, with each job i having a processing time p_i , the objective is to minimize the makespan, i.e., minimize the load of the highest loaded machine.

For an arbitrary instance of the Load balancing problem, we create a client i in UTG for every job i . Let $\phi_i = 1, \forall i \in P, T = 1$, and $R_i = \frac{1}{p_i}$ where R_i is the rate of client i in the throughput game, and p_i is the processing time of job i in the Load balancing game. Lastly, we create a base station k in UTG for every machine k .

The load of a machine k in the load balancing game is $\sum_{i \in k} p_i$, and the effective load of a base station k in UTG is $\sum_{i \in k} \frac{\phi_i}{R_i} = \sum_{i \in k} p_i$.

It can be shown that minimizing the makespan of the load balancing problem is equivalent to maximizing the rate of the client facing the highest effective load on a base station in UTG . $\square$

We leave the problem of determining the complexity of the social optimum solution using the *sum* measure, as future work.

5 PRICE OF ANARCHY AND STABILITY

In this section we show a bound on the *Price of Anarchy* (PoA) in UTG . We also present examples where the PoA matches the bound, showing that the bound is tight. We also show that the *Price of Stability* (PoS) can become arbitrarily low. In order to show a lower bound on PoA for both measures, we need the following result -

LEMMA 8. *The lower bound for PoA, using both the Max-min and the Sum measure, in the class of Utility throughput games UTG can be obtained by using a linear utility function.*

PROOF. In a UTG game where the social utility uses the Max-min measure, we can still show that the worst PoA can be obtained using linear utility functions.

Let ω^* be the Max-min throughput at social optimum, and ω^{NE} be the max-min throughput at the worst Nash equilibrium.

Since f has been defined to be non-decreasing quasi-concave, we have

$$\begin{aligned} f(\omega^*) &\leq f(\omega^{NE}) + \int_{\omega=\omega^{NE}}^{\omega^*} \tilde{f}'(\omega) d\omega \\ &= f(\omega^{NE}) + \tilde{f}(\omega^*) - \tilde{f}(\omega^{NE}) \end{aligned}$$

where $\tilde{f}(\omega) = a_f \omega$ is a linear function, with $a_f = f'(\hat{\omega})$ where $\hat{\omega} = \arg \sup_{\omega \in [\omega^{NE}, \omega^*]} f'(\omega)$. Using eq 5, we now have

$$\frac{f(\omega^{NE})}{f(\omega^*)} \geq \frac{f(\omega^{NE}) - \tilde{f}(\omega^{NE}) + \tilde{f}(\omega^{NE})}{f(\omega^{NE}) - \tilde{f}(\omega^{NE}) + \tilde{f}(\omega^*)} \geq \frac{\tilde{f}(\omega^{NE})}{\tilde{f}(\omega^*)}$$

which proves the theorem for the Max-min measure.

For evaluating the PoA in games using the Sum measure, Johari [12] (theorem 3.8) has proven that the worst PoA, in games with non-decreasing concave utility functions for each client, occurs when the utility function is in fact linear. This holds for the case when the social utility function is the aggregate of all client utilities. Our proof for the case using the Sum measure, where the utilities are quasi-concave, is analogous to their proof and we omit it. $\square$

5.1 Price of Anarchy using the Max-min measure

We now focus on the Max-min measure and derive bounds on PoA to illustrate the inefficiency of equilibria.

Lower bounds on PoA (Max-min measure). We determine the PoA for the Max-min measure of social optimum. Since we restrict our attention to linear utility functions, we evaluate the PoA using linear utility functions.

We determine the PoA for any utility throughput game where a Nash equilibrium exists.

THEOREM 9. *The Price of Anarchy using the Max-min measure for the class of T-split Utility throughput games UTG is lower bounded as*

$$PoA(UTG) \geq \frac{|K|(\frac{\phi}{R})_{\min}}{\phi^m(T\sigma + |K|(\frac{\phi}{R})_{\max})}$$

where $(\frac{\phi}{R})_{\min}, (\frac{\phi}{R})_{\max}, \phi^m$ and σ are defined in section 2.1.

PROOF. We first provide a bound for the throughput at Nash equilibrium. Let k_{high} denote the base station with the maximum effective load at a Nash equilibrium solution and $i_{\min}$ denote the client with the least priority on k_{high} . Let $\phi^m = \frac{\phi_{i,k_{high}}}{\phi_{i,k}}$. Then, for any base station k ,

$$\begin{aligned} G_{s(k_{high})}^{NE} &\leq \phi^m(G_{s_k}^{NE} + (\frac{\phi}{R})_{\max}) \\ \Rightarrow |K|G_{s(k_{high})}^{NE} &\leq \phi^m(\sum_{k \in K} G_{s_k}^{NE} + |K|(\frac{\phi}{R})_{\max}) \\ \Rightarrow G_{s(k_{high})}^{NE} &\leq \phi^m(\frac{\sum_{k \in K} G_{s_k}^{NE}}{|K|} + (\frac{\phi}{R})_{\max}) \\ &= \phi^m(\frac{\sum_{i \in P} (\frac{\phi}{R})_i}{|K|} + (\frac{\phi}{R})_{\max}) = \frac{\phi^m(T\sigma + |K|(\frac{\phi}{R})_{\max})}{|K|} \end{aligned} \quad (7)$$

Thus, we get a bound on the throughput,

$$\omega_{i_{\min}}^{NE} \geq \frac{\phi_{i_{\min}}|K|}{\phi^m(T\sigma + |K|(\frac{\phi}{R})_{\max})}$$

For the social optimum bound, we get $\omega_{\min}^* \leq \frac{\phi_{i_{\min}}}{(\frac{\phi}{R})_{\min}}$. Therefore,

$$PoA \geq \frac{\frac{\phi_{i_{\min}}|K|}{\phi^m(T\sigma + |K|(\frac{\phi}{R})_{\max})}}{\frac{\phi_{i_{\min}}}{(\frac{\phi}{R})_{\min}}} = \frac{|K|(\frac{\phi}{R})_{\min}}{\phi^m(T\sigma + |K|(\frac{\phi}{R})_{\max})}$$

$\square$

COROLLARY 2. *For uniform priority games and games which must abide by the priority-fair rule,*

$$PoA \geq \frac{(\frac{\phi}{R})_{\min}}{(\frac{\phi}{R})_{\max}(|P| \frac{T}{|K|} + 1)}$$

PROOF. This follows from the fact that $\phi^m \geq 1$. $\square$

Interestingly, for the case when all clients have the same priority, we obtain a constant bound.

LEMMA 10. *When all clients have the same priority, $\phi_i = 1$ (w.l.o.g), for the class of uniform priority-rate games $UTG_{\phi,R}$ the Price of Anarchy using the Max-min measure is bounded by a constant.*

$$PoA \geq \frac{3}{4}$$

PROOF. Consider a bound on the throughput of the client, $i_{\min}$, with the minimum value at Nash equilibrium $\omega_{i_{\min}}^{NE}$, i.e.,

$$\omega_{i_{\min}}^{NE} \geq \frac{1}{G_{s(k_{high})}^{NE}}$$

For social optimum, we have a bound on the maximum effective load on links, given by

$$G_{s(k)}^* \geq \frac{\sum_i \frac{1}{R_i}}{|P|} = \frac{\sum_{k=K \setminus k_{high}} G_{s(k)}^{NE} + G_{s(k_{high})}^{NE}}{|K|}$$

At Nash equilibrium, if the BS with the highest effective load contains only 1 client $i_{\min}$, then the Nash equilibrium is also the social optimum and $PoA = 1$ (trivial case). This is because $i_{\min}$ gets a throughput of $\frac{1}{G_{s(k_{high})}^{NE}}$, and since k_{high} is the BS with the highest effective load, the minimum throughput can only be lower if $i_{\min}$ shares a BS with another client. Therefore, we must consider that k_{high} is occupied by 2 or more clients, which implies $G_{s(k)} \geq \frac{1}{2} G_{s(k_{high})}$. Thus,

$$\begin{aligned} G_{s(k)}^* &\geq \frac{\frac{|K|-1}{2} G_{s(k_{high})}^{NE} + G_{s(k_{high})}^{NE}}{|K|} \geq \frac{|K|+1}{2|K|} G_{s(k_{high})}^{NE} \\ \Rightarrow \omega_{i_{\min}}^* &\leq \frac{2|K|}{(|K|+1)G_{s(k_{high})}^{NE}} \\ \Rightarrow PoA &= \frac{\omega_{i_{\min}}^{NE}}{\omega_{\min}^*} \geq \frac{|K|+1}{2|K|} \geq \frac{3}{4} \text{ (when } k=2) \end{aligned}$$

$\square$

Upper bounds on PoA (Max-min measure). We next illustrate that the lower bounds on PoA, under the Max-min measure, are essentially tight by constructing instances that upper bound the PoA. We first address uniform priority-rate games.

CLAIM 1. *There exists a uniform priority-rate game $UTG_{\phi, R}$ where, under the Max-min measure, $PoA \leq \frac{3}{4}$*

PROOF. We create a game with 4 clients and 2 base stations, where $\phi_i = 1, \forall i$ and $R_1 = \epsilon, R_2 = \epsilon, R_3 = 2\epsilon$ and $R_4 = 2\epsilon$, where ϵ is any constant.

At the worst Nash equilibrium, clients 1 and 2 will be assigned to base station 1 and clients 3 and 4 will be assigned to base station 2. The client, $i_{\min}$, with the minimum throughput, has $\omega_{i_{\min}} = \frac{\epsilon}{2}$. At social optimum, clients 1 and 3 will be assigned to base station 1 and clients 2 and 4 will be assigned to base station 2. Now, the client, $i_{\min}^*$, with the minimum throughput has $\omega_{i_{\min}} = \frac{2}{3\epsilon}$. The result follows. $\square$

Now, we show that PoA for UTG games can be arbitrarily low.

CLAIM 2. *There exists an instance of uniform priority UTG_{ϕ} where, under the Max-min measure, for all $\epsilon < 1$, $PoA \leq \epsilon$.*

PROOF. We create a game with 4 clients and 2 base stations, where $\phi_i = 1, \forall i$ and $R_{1,1} = 1, R_{1,2} = \frac{1}{\epsilon}, R_{2,1} = 1, R_{2,2} = \frac{1}{\epsilon}, R_{3,1} = \frac{1}{\epsilon}, R_{3,2} = 1, R_{4,1} = \frac{1}{\epsilon}$ and $R_{4,2} = 1$,

At the worst Nash equilibrium, clients 1 and 2 will be assigned to base stations 1 and clients 3 and 4 will be assigned to base stations 2, with the minimum throughput client $i_{\min}$ obtaining $\omega_{i_{\min}} = \frac{1}{2}$. At social optimum, clients 1 and 2 will be assigned to base stations 2 and clients 3 and 4 will be assigned to base stations 1, with the minimum throughput client $i_{\min}$ obtaining $\omega_{i_{\min}} = \frac{1}{2\epsilon}$, which yields the result. $\square$

5.2 Price of Anarchy using the Sum measure

Lower bound on PoA (Sum measure). We analyze the bounds on the *Price of Anarchy* for the Sum measure. Note that $T \leq |K|$, since a player cannot be scheduled on more base stations than those available.

THEOREM 11. *The Price of Anarchy using the Sum measure for the class of T-split Utility throughput games UTG is lower bounded as*

$$PoA \geq \frac{|K||P|\phi_{\min}}{\phi^m(T\sigma + |K|(\frac{\phi}{R})_{\max}) \cdot \mathcal{R}_\tau}$$

where $\mathcal{R}_\tau$ is the sum of the highest $\frac{|K|}{T}$ rates from among all $R_{i,k}$, and $(\frac{\phi}{R})_{\min}, (\frac{\phi}{R})_{\max}, \phi^m$ and σ are defined in section 2.1.

PROOF. We have shown an upper bound for the load in eq 7. Next, the total throughput of all clients at Nash equilibrium is given by

$$\sum_{i \in P} \omega_i^{NE} \geq \frac{|K| \sum_i \min\{\phi_{i,k}\}}{\phi^m(T\sigma + |K|(\frac{\phi}{R})_{\max})} \quad (8)$$

Under conditions for social optimum, the total throughput is given by $SO^* = \sum \frac{\phi_i}{G_{s(k_i)}^*}$. We must minimize the load $G_{s(k)}^*$ observed by every player. For an upper bound, we can consider that we

have assigned the client with the highest rate on each base station (allowing for T repetitions), and ignore the remaining clients. Then,

$$\sum_i \omega_i^* \leq T\mathcal{R}_\tau$$

Using the definition for the Sum measure,

$$PoA = \frac{\sum_i \omega_i^{NE}}{\sum_i \omega_i^*} \geq \frac{|K| \sum_i \min\{\phi_{i,k}\}}{\phi^m(T\sigma + |K|(\frac{\phi}{R})_{\max}) \cdot \mathcal{R}_\tau}$$

We can bound the value of $\sum_i \min\{\phi_{i,k}\}$ as $\sum_i \min\{\phi_{i,k}\} \geq |P|\phi_{\min}$, and thus, obtain the result. $\square$

Upper bound on PoA (Sum measure). In this subsection, we illustrate the tightness of our bounds on the PoA under the Sum measure. We describe an example that upper bounds the PoA for the class of UTG games.

CLAIM 3. *There exists an instance of a Uniform Priority-Rate $UTG_{\phi, R}$ such that under the Sum measure, for any constant $\epsilon < 1$, $PoA \leq \epsilon$*

PROOF. We create a game to have $|P| = 2|K|$ clients (and $|K|$ base stations). Choose $\phi_i = 1, \forall i \in P$ and the rates of the P clients be partitioned into two classes, A and B . Let $R_a = \frac{1}{\epsilon}, a \in A$ and for the other class let $R_b = \epsilon, b \in B$. The worst NE is obtained when each base station has 2 players, 1 of rate R_a and the other of rate R_b . Total throughput at NE is $K \frac{2}{\frac{1}{\epsilon} + \epsilon}$.

In a socially optimum solution, we put all the clients in class A alone on as many base stations as possible, and put the remaining players on the last base station. So, the total throughput at Social Optimum is $(K-1)\frac{1}{\epsilon} + \frac{P+1}{P\frac{1}{\epsilon} + \epsilon}$. Therefore, we obtain a bound,

$$PoA \leq \frac{K \frac{2}{\frac{1}{\epsilon} + \epsilon}}{(K-1)\frac{1}{\epsilon} + \frac{P+1}{P\frac{1}{\epsilon} + \epsilon}} \leq \epsilon$$

$\square$

5.3 Upper bounds on Price of Stability (PoS)

An interesting question that arises in the study of Nash equilibrium is to determine the inefficiency of an equilibrium with the best objective function w.r.t to the optimum. However, the following negative result shows that the *Price of Stability* for the class UTG games can be arbitrarily low.

PoS (Max-min measure). We show that PoS using the Max-min measure can be arbitrarily low.

CLAIM 4. *There exists an instance of uniform priority UTG_{ϕ} where, under the Max-min measure, for any constant $\epsilon \leq \frac{1}{4}$, $PoS \leq \frac{2\epsilon}{1+\epsilon}$.*

PROOF. We create a game with 3 clients and 3 base stations. Let $\phi_1 = 1, \phi_2 = 1$ and $\phi_3 = \epsilon$. Let $R_{1,1} = 1, R_{1,2} = \epsilon, R_{1,3} = \epsilon, R_{2,1} = \epsilon, R_{2,2} = 1 + \epsilon, R_{2,3} = 2, R_{3,1} = \epsilon, R_{3,2} = \epsilon$, and $R_{3,3} = 2$. The Nash equilibrium with the best social utility occurs when client 1 is on base station 1, and clients 2 and 3 are on base station 3, yielding the *max-min* throughput of $\frac{2\epsilon}{1+\epsilon}$. The social optimum yields a max-min throughput of 1, when client 1 is on base station 1, client 2 is on base station 2, and client 3 is on base station 3, implying the result. $\square$

PoS (Sum measure). A similar bound on the *Price of Stability* under the Sum measure for the class of *UTG* games also holds.

CLAIM 5. *There exists an instance of a Uniform Priority-Rate $UTG_{\phi, R}$ such that, under the Sum measure, for any constant $\epsilon < \frac{1}{4}$, $PoS \leq 4\epsilon$.*

PROOF. We create a game with 3 clients and 2 base stations. Let $\phi_1 = 1, \phi_2 = 1$ and $\phi_3 = 1$. Let $R_{1,1} = \frac{1}{\epsilon}, R_{1,2} = \epsilon, R_{2,1} = 1, R_{2,2} = 1, R_{3,1} = \epsilon$ and $R_{3,2} = 1$. The Nash equilibrium with the best social utility occurs when clients 1 and 2 are on base station 1 and client 3 is on base station 2, yielding the *sum* throughput of $\frac{2}{\epsilon+1} + 1$. The social optimum yields a total throughput of $\frac{1}{\epsilon} + 1$, when clients 1 is on base station 1, and clients 2 and 3 are on base station 2, yielding the result. $\square$

6 NUMERICAL SIMULATIONS

We conducted a set of simulations to observe the behavior of the price of anarchy with respect to changes in priorities and rates. We also provide a heuristic for finding Nash equilibrium and simulation results to illustrate the speed of convergence. All simulations were implemented in Java, on a modern day desktop machine.

In the first set of simulations, we show the impact of priority disparity between clients on the price of anarchy. For the second set of simulations, we provide a *best response method* to determine Nash equilibrium and implement it on larger, randomly generated instances, demonstrating the time in which stability can be achieved in a system.

Our simulations indicate that for the max-min measure, the PoA is adversely affected by the ratio $\frac{\phi_{max}}{\phi_{min}}$; interestingly, PoA actually does not decrease with the disparity in the ratio $\frac{\phi_{max}}{\phi_{min}}$ for the sum measure. This is evidence of the domination of the higher priority players in the Sum measure. The disparity between the throughput achieved by the players is thus illustrated. These simulations used connectivity data obtained from a real-world campus environment.

Further using synthetic data we also illustrate that the PoA suffers w.r.t the disparity in rates, measured by $\frac{R_{max}}{R_{min}}$, between competing clients. These results are in consonance with the theoretical lower bounds in theorems 9 and 11 respectively.

Data gathering: We obtained PHY data transfer rates available on 4 base stations for 6 clients, located across a university campus. All base stations were accessible via 802.11ac wifi, and based on the distance from the router, the PHY rates locked on to different levels, ranging from 7Mbps to 300 Mbps. We assume that each client would be able to connect to the 4 base stations, varying from 1 BS at a time to 4 (depending on the specified T-split factor). We introduced synthetic priorities for the clients, and incorporated it into the RAN selection model. We denote the network thus obtained as $\mathcal{N}_R$.

6.1 Impact of Priorities on PoA

In this subsection, we study the behavior of PoA in the campus network $\mathcal{N}_R$.

We initially assigned all clients the same priority, and incremented the priority of a subset of clients over each iteration. We chose subsets of size varying from 1 to 5 and increased the priority of these clients together.

Results for Max-min Measure. In figure 1, we illustrate the effect of the disparity in client priorities on PoA. The results reflect that an increase in priorities of a subset of clients adversely affects the PoA for the Max-min measure.

Figure 1a illustrates the impact of increasing priority of exactly 1 client, keeping others fixed. The PoA worsens with increasing priority of the client. The differently colored plots further illustrate the impact of the T-split factor. As the T-split factor increases, the PoA decreases (worsens) but increases again past the middle value of $T = 2$. In figure 1b, we increased the priority of 2 clients, keeping the others fixed. The impact of priority increase on PoA is more dramatic. Finally, in figures 1c, 1d, and 1e, we increment the priorities of a selected subset of clients of size 3, 4, 5, respectively and note that the PoA worsens at a faster rate with increase in priority levels of the selected subsets. All simulations are run for T-split factors 1, 2, 3, and 4 and show a similar impact of T.

Results for Sum Measure. The figures 2a, 2b, 2c, 2d, 2e show the impact on varying priorities on PoA using the sum-measure, when subsets of size 1, 2, 3, 4 and 5 were used to increase the priorities together over the remainder clients.

The impact is actually reversed with the PoA somewhat improving, not worsening with increase of priorities.

Discussion of results. From the simulations (figures 1 and 2), we observe that the PoA decreased for the Max-min measure but actually increased somewhat for the Sum measure as the disparity in the priorities increased. We also consider the throughput disparity, as measured by $\frac{\phi_{max}}{\phi_{min}}$, between the maximum throughput to the minimum throughput in each of the scenarios. We computed this disparity as we increased the disparity in priorities, measured by $\frac{\phi_{max}}{\phi_{min}}$, where ω_{max} (ω_{min}) is the maximum (minimum) throughput achieved. The computations are illustrated in the figures 3a, 3b, 3c, 3d and 3e, where the priorities of subsets of size 1, 2, 3, 4 and 5, respectively, were increased.

The increase in throughput disparity and the unresponsive behavior of the PoA using the Sum measure serves to illustrate that the Max-min measure is better suited to understand the behavior of the disparity introduced by varying priorities.

6.2 Impact of Rate disparity on PoA

We next show how the PoA using the Sum measure was impacted by increasing disparity in rates measured by $\frac{R_{max}}{R_{min}}$. This was studied by considering the same network $\mathcal{N}_R$, simulating 4 base stations which offered equal priority to 6 clients. Each client could connect to each of the 4 base stations.

Initially, all clients had the same rate. The rate of a set of clients was incremented over each iteration. The simulations were run for T-split factors 1, 2, 3, and 4. Figures 4a, 4b, 4c, 4d and 4e, illustrate the behavior of PoA by increasing the rates of 1, 2, 3, 4, and 5 clients together, respectively. The results indicate that PoA becomes worse as the rate disparity between clients increases. The T-split factor has a similar impact as in the previous section.

We omit the details of the simulation for the Max-min measure as that behavior was uneventful.

Figure 1: Effect of $\frac{\phi_{max}}{\phi_{min}}$ on PoA (Max-min measure) using real-world data. T = 1 (orange), 2 (red), 3 (blue) and 4 (green)

Figure 2: Effect of $\frac{\phi_{max}}{\phi_{min}}$ on PoA (Sum measure) using real-world data. T = 1 (orange), 2 (red), 3 (blue) and 4 (green)

Figure 3: Effect of variation in $\frac{\phi_{max}}{\phi_{min}}$ on $\frac{\omega_{max}}{\omega_{min}}$ using real-world data. T = 1 (orange), 2 (red), 3 (blue) and 4 (green)

Figure 4: Effect of variation in $\frac{R_{max}}{R_{min}}$ on PoA (Sum measure). T = 1 (orange), 2 (red), 3 (blue) and 4 (green)

6.3 Complexity of finding Nash Equilibrium: Best Response Method

In this set of simulation we consider the complexity of determining Nash equilibrium. We have shown that, in general, finding a Nash equilibrium solution is hard. So, we developed a *best response method* which allows us to discover a Nash equilibrium solution for instances of *UTG* games.

The method aims to accelerate the decrease of the load vector L in lexicographic ordering, facilitating convergence to a Nash equilibrium (Theorem 3). The method repeatedly selects the base

station with the highest effective load and selects players whose throughput can be improved by switching base stations. This potentially decreases the number of steps required to converge to an NE. We describe the method in algorithm 2. Note that while switching base stations, the method ensures that a client cannot have multiple copies on a base station. We created multiple instances of the throughput games with each instance having the number of base stations, clients and T varying from 1 to 30, 1 to 100 and 1, 4, 8, 16 respectively. Priorities and rates were randomly assigned to clients. The method was able to find Nash equilibrium in less

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Algorithm 2 Best Response Method: Finding Nash equilibrium in uniform priority UTG_ϕ

- 1: Start with any random assignment of T copies of clients on base stations
  - 2: **while** $\exists$ Client i s.t. i can change her set of occupied base stations to improve total throughput ω_i
  - 3:   Pick the base station with the highest effective load k_{high} in the current system state; $G_{s(k_{high})} = \max_{k \in K} G_{s(k)}$
  - 4:   From base station k , move all those clients $i \in k$ who can improve their throughput by discarding base station k in favor of another.
-

Figure 5: Average convergence time using the method for different number of players and base stations with values of $T = 1$ (orange), 4 (blue), 8 (maroon) and 16 (green) to find Nash equilibrium

than 2 seconds for even the largest instance with 100 players, 30 base stations and a T -split factor of 16. Convergence of the method is illustrated in figure 5.

7 CONCLUSIONS AND ACKNOWLEDGEMENTS

In this paper we have extended the RAN model to allow for concurrent multiple access and investigated the inefficiency of equilibrium. Further, we have shown tight bounds on the *Price of Anarchy*. An issue of further interest is resolving the complexity of Nash equilibrium for additional cases.

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8 APPENDIX

CLAIM 6. *There exists an instance of $UTG(P, S, \mathcal{U}, \phi, R, K, T)$, where $T = 1$, for which there is no pure Nash equilibrium.*

PROOF.

$$\Phi = \begin{bmatrix} L_1 & L_2 & L_3 \\ P_1 & 9.8 & 1.6 & 5.1 \\ P_2 & 8.1 & 0.2 & 8.6 \\ P_3 & 4.6 & 3.9 & 8.8 \end{bmatrix} R = \begin{bmatrix} L_1 & L_2 & L_3 \\ P_1 & 98.3 & 80.8 & 12.6 \\ P_2 & 27.6 & 32.6 & 21.2 \\ P_3 & 65.8 & 14.9 & 9.8 \end{bmatrix}$$

In a game involving 3 clients and 3 base stations, the above values of ϕ and R present a scenario such that no configuration of assignments result in any client being satisfied on the base station she occupies. The matrix Φ represents the priorities $\phi_{i,k}$ and the matrix R represents the rates $R_{i,k}$. $\square$