Chain Stability in Trading Networks

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We consider general trading networks with bilateral contracts. We show that a suitably adapted *chain stability* concept [4] is equivalent to *stability* [1, 2] if all agents' preferences are jointly fully substitutable [3] and satisfy the Laws of Aggregate Supply and Demand (a condition we call *monotone-substitutability*). We also present three examples to show that the results are sharp, demonstrating that:

- If preferences of some agents do not satisfy the Laws of Aggregate Supply and Demand, then chain stable outcomes may not be stable.
- If preferences of some agents are not fully substitutable, then chain stable outcomes may likewise not be stable.
- If blocking sets are restricted to chains that do not “cross” themselves (i.e., chains that involve each agent in at most two contracts), then an outcome that is robust to such blocks may not be robust to richer blocks.

Our results imply that in trading networks with transferable utility, an outcome is consistent with competitive equilibrium if and only if it is not blocked by any chain of contracts.

We show moreover that, from a computational perspective, checking whether an outcome is chain stable is substantially easier than checking whether that outcome is stable directly. Indeed, we show that as the size of the economy grows, the number of chains of trades (corresponding to possible blocking chains) becomes infinitely smaller than the number of general sets of trades (corresponding to possible blocking sets).

Full Paper: <https://ssrn.com/abstract=3180740>

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