Adaptive Sensitivity Analysis with Nonlinear Power Load Modeling

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ABSTRACT

Voltage fluctuation in power networks is a critical issue for VLSI designs. The analysis and optimization of the voltage drops rely on accurate sensitivity calculation. Due to the high complexity of large-scale circuits, in practice active devices are simplified with power load models. In order to grasp the details of voltage fluctuation, we propose a nonlinear power load model as a function of the supply voltage. We adopt an adaptive analysis algorithm to accelerate the calculation. The power load model provides efficient and accurate noise prediction at system-level.

KEYWORDS

power load model, sensitivity analysis, noise prediction, system-level

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1 INTRODUCTION

Due to the ever shrinking size of VLSI design, the effect of voltage fluctuation in power distribution network (PDN) has become increasingly significant. As shown in the ITRS roadmap [4], the current density of a single chip keeps increasing while the operating voltage of high performance processors is gradually dropping, which makes the target impedance of a PDN lower than ever. Meanwhile, Kim *et al.* [6] observed that the full-chip leakage power has increased by three times in recent years. Consequently, we can no longer neglect the IR drop and simultaneous switching noise (SSN) caused by parasitic capacitances and the short circuit currents that occur when NMOS and PMOS devices are switching.

With the increasing need of accurate PDN analysis, several works have been proposed for impedance modeling and noise minimization. Zhang *et al.* [12] minimized power supply noise in 3D IC using power transmission line based design. Charles *et al.* [1] studied

PDN effect of four chip-stacking topologies. Tanaka *et al.* [11] measured the PDN impedance profile and SSN of 3D system in a single package. Smith [10] proposed to characterize on-die PDN noise and generate the worst-case current stimuli. Kim [7] estimated the supply noise from the frequency-domain PDN impedance profile. Kim *et al.* [5] gave the closed-form expressions of the power supply noise caused by IC switching current for a complete PDN structure.

In previous works, the model of short circuit current is either ignored [7] [5] or considered as a constant [10] [8] in system-level PDN design. Zhang *et al.* [13] modeled the resistance as a linear function of supply voltage. Although it is demonstrated to predict PDN noise accurately and efficiently, the high performance is limited to the sub-threshold region of an NMOS.

In this work we focus on voltage-dependent power load modeling to accelerate large-scale simulation while maintaining high accuracy. Specifically, power loads are represented by a combination of Resistance and Current Source (RI) activity model and Resistance and Capacitance (RC) noise model. The equivalent short circuit resistance, capacitance, and transient current are modeled as nonlinear functions of supply voltage. During simulation, the proposed model can be applied adaptively, that is, in a step-wise manner for each small voltage range. To further validate the performance of our model, we perform sensitivity analysis on the proposed model. With a simple yet accurate power load model, the effect of short circuit current can be investigated, and potentially, system power consumption with billions of transistors can be optimized efficiently.

The remainder of the paper is organized as follows. The problem of power load modeling is formulated in Section 2.1. In Section 2.2 and 2.3 we introduce the proposed RI activity model and RC noise model, followed by sensitivity analysis of our model in Section 2.5. SPICE simulation for empirical value of the equivalent short circuit resistance, capacitance, and transient current is discussed in Section 3. Performance comparison between the proposed method and the reference is shown in Section 4. Finally, we conclude the paper in Section 5.

2 NONLINEAR POWER LOAD MODELING METHOD

A nonlinear power load modeling method is proposed. Such modeling method is capable of speeding up the simulation by adaptive sensitivity analysis.

2.1 Problem Formulation

The target is to capture the current model of the CMOS circuits with noise consideration. In this model, the current is a function of

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the supply voltage:

$$i = f(v_{dd}) \quad (1)$$

and we use the summation of the activity power dissipation and the noise fluctuation to represent the total current drop:

$$i = a(v_{dd}) + n(v_{dd}) \quad (2)$$

where $a(v_{dd})$ denotes the activity current load and $n(v_{dd})$ denotes the current fluctuation caused by noise.

2.2 RI Activity Model

For the activity current load, our circuit model is shown in Fig. 1a, and our method to build the activity model is displayed in Algorithm 1. In our model as Fig. 1a shown, we assume the cycle time is given. With the given activity current load, Algorithm 1 is able to approximate the voltage-dependent current source $i(v_{dd})$ and the resistance $R(v_{dd})$. The inputs of Algorithm 1 are the activity current load along with the corresponding supply voltage and a user-defined error tolerance, and the outputs are the parameters of the RI model: current source, resistance, and their corresponding piecewise linear supply voltage. The mechanism of the algorithm is to create a user-defined lookup table. Both the nonlinear function and the error tolerance are determined by the user, and thus, the size of the lookup table is flexible.

The spotlights of the proposed algorithm are the nonlinearity of the power load model and the acceleration of the calculation with the support of adaptive piecewise linear slices. The acceleration is achieved with Taylor expansion at a given point, and the region is determined by a given error tolerance ϵ along with the specified order n as depicted in Equation 3.

$$f(x_0 + \Delta x) = f(x_0) + \sum_{k=1}^{\infty} \frac{f^{(k)}(x_0)}{k!} (\Delta x)^k \quad (3)$$

$$= f(x_0) + \sum_{k=1}^n \frac{f^{(k)}(x_0)}{k!} (\Delta x)^k + \epsilon \quad (4)$$

$$\Delta x \approx \left(\left| \frac{n!}{f^{(n)}(x_0)} \epsilon \right| \right)^{(1/n)} \quad (5)$$

Figure 1: The proposed RI and RC models

2.3 RC Noise Model

For the noise analysis, our circuit model is shown in Fig. 1b, and our method to build the noise model is displayed in Algorithm 2. We assume the averaged amplitude and frequency of the noise are provided. To obtain the amplitude and frequency information, phasor analysis is utilized at a pre-processing stage to capture the resistance R and the capacitance C from the noise information.

Algorithm 1: RI Activity Model Computation

Input: v_{dd} , IDrop, ϵ

// IDrop is the activity current drop at each specified v_{dd} , ϵ is the user-defined tolerance

Output: Vdd , $CurrentSource$, $Resistance$

Curve fit IDrop as a function of v_{dd} , // differentiable nonlinear function is acceptable

$$IDrop := a(v_{dd})$$

$Vdd = []$ // assign Vdd as an empty vector

$Vdd[1] = v_{dd}[1]$

$CurrentSource = []$

$Resistance = []$

Start from $v_{dd}[1]$, $i = 1$ **repeat**

Solve δ in $|\nabla_{v_{dd}} a(Vdd[i])| = \epsilon$; // determine voltage step, high order approximation is allowed

$Vdd[i + 1] = Vdd[i] + \delta$;

Solve G, I_o in $a(Vdd[i + 1]) = G \times Vdd[i] + I_o$;

$CurrentSource[i] = I_o$;

$Resistance[i] = \frac{1}{G}$;

$i = i + 1$;

until $v_{dd}[end]$ is reached;

Subsequently, one is able to calculate the adaptive resistance and capacitance at arbitrary supply voltage with Algorithm 2. Algorithm 2 is similar to Algorithm 1 with the difference in the orientations. The inputs of Algorithm 2 are R, C in the noise model, and the outputs are two piecewise linear supply voltages (one for R , the other for C). In other words, two lookup tables are created: one for the resistance, and one for the capacitance. The advantages of RC noise model are allowing nonlinearity and providing adaptive piecewise linear supply voltages.

2.4 Nonlinear Power Load Model Simulator

The circuit diagram of our nonlinear power load model is depicted in Figure 2. We compute the total current under supply voltage fluctuation based on RI and RC models. The simulation of power load circuits is described in Algorithm 3, which adaptively employs the estimated resistance and current source at each simulation step. The strategy of the simulation is to utilize the best corresponding supply voltage (from lookup tables) to complete the calculation at the given time point.

Figure 2: The proposed power load model

Algorithm 2: RC Noise Model Computation

Input: $v_{dd}, R, C, \epsilon_R, \epsilon_C$
 /* R, C are gained from the operating frequency in phasor analysis at each specified v_{dd} */
 /* ϵ_R, ϵ_C are the user-defined tolerances */
Output: $V_{dd_C}, Adapt_C, V_{dd_R}, Adapt_R$
 Curve fit R, C as a function of v_{dd} , // differentiable nonlinear function is acceptable
 $R := nR(v_{dd})$, $C := nC(v_{dd})$
 $V_{dd_C} = []$, $V_{dd_R} = []$ // assign empty vectors
 $V_{dd_C}[1] = v_{dd}[1]$, $V_{dd_R}[1] = v_{dd}[1]$
 $Adapt_C = []$, $Adapt_C[1] = C[1]$
 $Adapt_R = []$, $Adapt_R[1] = R[1]$
Start from $v_{dd}[1], i = 1$ // Calculation of R
repeat
 Solve δ_R in $\delta_R | \nabla_{v_{dd}} nR(V_{dd}[i])| = \epsilon_R$;
 Solve δ_C in $\delta_C | \nabla_{v_{dd}} nC(V_{dd}[i])| = \epsilon_C$; // set voltage step, high order approximation is allowed
 $V_{dd_R}[i+1] = V_{dd_R}[i] + \delta_R$;
 $V_{dd_C}[i+1] = V_{dd_C}[i] + \delta_C$;
 $Adapt_R[i+1] = nR(V_{dd_R}[i+1])$;
 $Adapt_C[i+1] = nC(V_{dd_C}[i+1])$;
 $i = i + 1$;
until $v_{dd}[end]$ is reached for both R and C ;

Algorithm 3: Nonlinear Power Load Model Computation

Input: $T, V, RI_v_{dd}, RC_v_{dd}, R, I_o, R_{noise}, C_{noise}$
 /* T is time; V is the fluctuated supply voltage ;
 RI_v_{dd}, R, I_o are obtained from the RI model ;
 $RC_v_{dd}, R_{noise}, C_{noise}$ are gained from the RC model */
Output: Voltage of every node; Current of each branch
Initialization of the circuit simulation: set the parameters value to the corresponding supply voltage at $T[1]$
 $N = T.element_number$
 $ptr_I = \text{index of the piecewise linear region such that}$
 $RI_v_{dd}[ptr_I] \leq V[1] < RI_v_{dd}[ptr_I + 1]$
for $i = 1$ **to** N **do**
 if $V[i] \notin [RI_v_{dd}[ptr_I], RI_v_{dd}[ptr_I + 1]]$ **then**
 $ptr_I = \text{index of the piecewise linear region such that}$
 $RI_v_{dd}[ptr_I] \leq V[i] < RI_v_{dd}[ptr_I + 1]$
 Update R, I_o values to the corresponding supply voltage $V[i]$
 Run Circuit Simulation at time $T[i]$
 end
 else
 Run Circuit Simulation at time $T[i]$
 end
end

2.5 Sensitivity Analysis

Sensitivity equations for the targeted problem can be expressed as Equation 6 [9]. In the sensitivity analysis equations, i , the current load, is a variable dependent of time, v_{dd} , the supply voltage, represents the parameter of interest, and i_o stands for the initial current value. In the second row of Equation 6, $\partial_{v_{dd}} i$ and $\partial_{i_o} i$ are treated as variables.

$$D \begin{bmatrix} i \\ \partial_{v_{dd}} i \\ \partial_{i_o} i \end{bmatrix} = \begin{bmatrix} f(t; i(t; \mathbf{v}_{dd}, i_o)) \\ \partial_{v_{dd}} f(t; i(t; \mathbf{v}_{dd}, i_o), \mathbf{v}_{dd}) + \\ \partial_i f(t; i(t; \mathbf{v}_{dd}, i_o), \mathbf{v}_{dd}) \partial_{v_{dd}} i(t; \mathbf{v}_{dd}, i_o) \\ \partial_i f(t; i(t; \mathbf{v}_{dd}, i_o), \mathbf{v}_{dd}) \partial_{i_o} i(t; \mathbf{v}_{dd}, i_o) \end{bmatrix} \quad (6)$$

with the corresponding initial conditions

$$D \begin{bmatrix} i(t_0) \\ \partial_{v_{dd}} i(t_0) \\ \partial_{i_o} i(t_0) \end{bmatrix} = \begin{bmatrix} i_0 \\ \mathbf{0} \\ \mathbf{I} \end{bmatrix} \quad (7)$$

In our model, the initial conditions are given, so the calculation of the third row in the matrix 6 is not required. From the generalized adjoint network sensitivity calculation [2], the resistive networks' sensitivity is

$$\frac{-\partial f_R}{\partial p_R} \phi_R(\tau) + \frac{\partial g_R}{\partial p_R} \psi_R(\tau).$$

In this research, the interest is on the sensitivity of the load current. Therefore, given the fact that the load current is dependent of the supply voltage, the sensitivity equation in interest is the term $\frac{\partial g_R}{\partial p_R} \phi_R(\tau)$. By definition of the adjoint network, all the adjoint branches contribute to the current and voltage change, and the expression for the current load sensitivity becomes

$$\frac{d}{dt} \frac{di}{dv_{dd}} = \frac{d}{dt} \left(\frac{da(v_{dd})}{dv_{dd}} + \frac{dn(v_{dd})}{dv_{dd}} \right) \quad (8)$$

$$= \frac{d}{dt} \left(\frac{dI(v_{dd})}{dv_{dd}} + \left(\frac{dG(v_{dd})}{dv_{dd}} v_{dd} + G(v_{dd}) \right) + \frac{dI_{noise}(v_{dd})}{dv_{dd}} \right) \quad (9)$$

3 EXPERIMENTAL RESULTS

The test cases in this paper were an inverter chain formed of 12 inverters as shown in Fig. 3. In our calculation of the total current, we emphasized on the internal state of the circuit behavior. Therefore, the currents in the first stage (inv 1) and the last stage (inv 12), which were contaminated by the proximity to the input pulse and the output loading capacitor respectively, were removed. The inverter chain simulations were carried out with HSPICE [3].

Figure 3: Inverter Chain

To construct the RI Activity model, RC Noise model, and the metrics of the supply voltage fluctuation, we generated the simulation data with HSPICE. In our simulations, two different models from

ASU predictive model were used: 16nm High Performance model and 16nm Low Power model. For RI model, the activity current in our experiments was calculated by the averaged current in each pulse cycle (8ns per cycle, 1ns for each transition stage, 3ns for each stable state) specified in transient analysis of the inverter chain. For RC model, a sinusoidal wave (100GHz, $0.05 * v_{dd} * \sin(2\pi ft)$) was utilized as the noise input to simulate the fluctuating supply voltage.

The results of the low power and high performance RI, RC models are shown in Fig. 4 and Fig. 5 respectively. Considering RI model, Fig. 4a and Fig. 5a display the global view of the results of the inverter chain's activity current load associated with different supply voltages. Both Fig. 4a and Fig. 5a illustrate the advantages of the proposed RI Activity Model Computation. The X axis is the supply voltage; the Y axis is the corresponding activity current load. From the figure, we can see that big leaps can be made at the inclined slope, whereas dense piecewise linear regions are observed at the sharp slope. Such adaptive piecewise linear steps not only embrace the accuracy but also reduce the computation time. We observe that larger supply voltage induces larger activity current load, and a nonlinear curve should be employed to fit the curve. For both models, the nonlinear fitting curves in the activity model are polynomials of second order fitting piece-wisely and the tolerances are empirically determined.

For RC model, the performances are demonstrated in Fig. 4b and 5b. For both figures, the top subplot shows the relation between the supply voltage and the resistance, while the bottom subplot displays the association between the supply voltage and the capacitance. For the low power model, as the supply voltage increases from 0.7V to 1.3V, the resistance first rises and then converges to a certain value around $1.5k\Omega$, while the capacitance keeps incrementing. For the high performance model, as the supply voltage increases from 0.5V to 0.95V, the resistance first decreases and then converges to a certain value around $1.2k\Omega$, while the capacitance keeps incrementing. For both models, the nonlinear fitting curves, the resistance and capacitance in the noise model, are polynomials of second order fitting piece-wisely and the tolerances are empirically determined.

4 PERFORMANCE EVALUATION

The inverter chain with fluctuating supply voltage simulation results by HSPICE are the metrics to compare with. Our model generates the results within 3% error in both high performance and low power models as shown in Tab.1, Tab. 2 and Fig. 6a, 6b. The calculation of the error follows Equation 10. In both tables, the numbers represent the relative error rate between the inverter chain current load value and the current load value of our simulation. The equation applied in this research is Eq. 10. In Tab. 1 and Tab. 2, each row refers to the supply voltage, and each column defines the averaged transient cycle number from the activity current load. Besides the less than 3% relative error rates, the trends of the current load of our simulation results are consistent with HSPICE in Fig. 6a, 6b. For both high performance and low power models, the current load nonlinearly increases with the increment of the supply voltage.

$$\text{Error Rate} = \frac{I_{\text{MODEL}} - I_{\text{HSPICE}}}{I_{\text{HSPICE}}} \times 100\% \quad (10)$$

Low Power	1 cycle	2 cycles	3 cycles	4 cycles	5 cycles
0.75V	-1.56	-1.75	-1.81	-1.85	-1.87
0.80V	-1.69	-1.96	-2.05	-2.09	-2.12
0.85V	1.42	1.07	0.95	0.90	0.86
0.90V	-1.77	-2.14	-2.27	-2.33	-2.37
0.95V	0.52	0.13	-0.01	-0.07	-0.11
1.00V	-1.54	-1.94	-2.07	-2.14	-2.65
1.05V	-2.18	-2.59	-2.73	-2.80	-2.84
1.10V	-0.96	-1.40	-1.54	-1.61	-1.66

Table 1: Low Power Model: Error in Percentage (%)

High Performance	1 cycle	2 cycles	3 cycles	4 cycles	5 cycles
0.60V	0.17	-0.10	-0.19	-0.24	-0.26
0.65V	1.49	1.27	1.19	1.16	1.13
0.70V	-2.03	-2.20	-2.25	-2.28	-2.29
0.75V	-2.42	-2.54	-2.58	-2.60	-2.61
0.80V	-2.38	-2.46	-2.49	-2.50	-2.51
0.85V	-1.63	-1.69	-1.71	-1.72	-1.72
0.90V	-1.74	-1.78	-1.79	-1.80	-1.80

Table 2: High Performance Model: Error in Percentage (%)

Referring to sensitivity analysis, we use low power model as demonstration. The sensitivity analysis ($\frac{di}{dv_{dd}}$) for the total current is exhibited in Fig. 7a and 7b. In Fig. 7a, the X axis is the supply voltage, and Y axis is the derivative of each parameter of interest. From top to bottom of each subplot, we have $\frac{dI}{dv_{dd}} = \frac{dI_o}{dv_{dd}} + \frac{d(Gv_{dd})}{dv_{dd}}$, $\frac{dR}{dv_{dd}}$, and $\frac{dC}{dv_{dd}}$. For the activity current load, the sensitivity enlarges with the increase of v_{dd} . For the resistance in noise model, the sensitivity converges to zero with the increase of v_{dd} ; regarding the capacitance, the sensitivity does not show the trend of convergence. These results are consistent with the results in Fig. 4b. As for Fig. 7b, the X axis is time, and the Y axis is $\frac{dI}{dv_{dd}}$. Since the noise fluctuates periodically, and given the fact that R, C are determined at initial stage, it is reasonable that the sensitivity of the current load changes periodically.

Figure 6: Results of the proposed power load model

Figure 4: Results of Low Power model simulation

Figure 5: Results of High Performance model simulation

Figure 7: Results of sensitivity analysis

5 CONCLUSIONS

A nonlinear power load modeling method as a function of the supply voltage is proposed in this paper. To speed up the calculation,

the algorithms to adaptively slice piecewise linear regions are also provided. Moreover, sensitivity analysis methods are given and relying to the sensitivity equations, any parameter of interest can be investigated. In the inverter chain test cases, we reach the performance within 3% of error in the operating regions of both low power and high performance models. The future work suggested are a more generalized noise model instead of sinusoidal waves, thermal analysis of the current load, and the application of the proposed model to the investigation of the subthreshold regions.

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