A Simple Yet Effective Graph Partition Model for GPU Computing

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ABSTRACT

Graph **edge** partition models have recently become an appealing alternative to graph **vertex** partition models for parallel and distributed computing due to their flexibility in balancing loads and their performance in reducing communication cost [1, 3]. In this paper, we introduce a simple yet effective graph edge partitioning model for GPU computing. In practice, our model yields high partition quality (better than or the same as the state-of-the-art edge partition approaches, at least for power-law graphs) with low partition overhead. In theory, previous work [1] showed that an approximation factor of $O(d_{max} \sqrt{\log n \log k})$ apply to the graphs with $m = \Omega(k^2)$ edges (k is the number of partitions). Our model extends this result to all graphs.

We demonstrate how graph edge partition model can be applied to GPU computing. We draw our examples from GPU program for locality enhancement both over time and (processor) space. For the first time, we demonstrate the effectiveness of edge partition for modeling data reuse in a many-core processors, both in theory and in practice.

CCS CONCEPTS

• **Mathematics of computing** → **Graph algorithms**; • **Theory of computation** → **Parallel computing models**; • **Computing methodologies** → *Modeling and simulation*;

KEYWORDS

Graph partition model; GPU; data reuse; locality

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1 MODEL OVERVIEW

Graph **edge** partition models have recently become an appealing alternative to graph **vertex** partition models for parallel and distributed computing due to their flexibility in balancing workloads and their performance in reducing communication cost [1, 3]. In the edge partition model, a graph is partitioned based on the edges

as opposed to vertices in the vertex partition problem. The edge partition model optimizes the number of vertex copies, while the vertex partition model optimizes the number of edge cuts.

We use the computation fluid dynamics (CFD) program to demonstrate use of edge partition model for parallel computing (in Figure 1). In CFD, the main computation is to calculate the interaction between fluid elements bounded by certain spatial distances and to use the interaction information to update the status of every fluid element for the next time step.

A fluid element is modeled as a vertex. An interaction is calculated between a pair of fluid elements, modeled as an edge. Figure 1 shows six interaction edges among six fluid elements.

Figure 1(a) shows one possible way to partition the computation (edges) to two multi-core processors, in which case, edges e_1, e_2, e_3 are mapped to one multi-core processor and e_4, e_5, e_6 are mapped to the other multi-core processor, requiring three fluid elements to be copied into both multi-core processors (marked as solid circles). However, with an optimized partition as illustrated in Figure 1(b), with e_1, e_2, e_4 mapped to one multi-core processor and e_3, e_5, e_6 mapped to the other multi-core processor, only one fluid element needs to be copied into both multi-core processors. In total, Figure 1(b) reduces the number of vertex copies by 66%, corresponding to a significant memory communication cost reduction.

Graph **edge** partition has been less studied in literature than graph **vertex** partition. The Powergraph [3] work is the first work that showed edge partition improves work balancing and reduces communication cost in practice, especially for power-law graph. Bourse, Lelarge, and Vojnovic [1] established the first theoretical approximation guarantee and developed an efficient *streaming edge partition* algorithm. The target application for both studies is data analytics in large-scale distributed computing clusters, in which case single-pass streaming edge partition algorithms are desired.

The *hypergraph* partition approach [4] is an indirect approach to solve edge partition problem. The target application for *hypergraph* partition is parallel program that has potentially good data reuse and has multi-pass computation phases, for instance, sparse linear algebra solvers for optimization problems.

The *streaming edge partition* algorithms [1, 3] are for scenarios when the input graph is given as a stream or only one pass through the input graph is allowed. In these settings their use is much preferred even when they do not yield the same partition quality as the *hypergraph* algorithm, their non-streaming counterpart. The reason is the large overhead of the latter.

We introduce a novel edge partition model that yields similar or improved partition quality than the *hypergraph* model, while in the meantime, keeps partition overhead low. Our approach improves previous work in both theory and practice, in the following ways.

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Figure 1: Mapping of *cfd* interaction computation into two multi-core processors.** Assuming we have two multi-core processors and each multi-core processor is in charge of three edges. In (a) there are three fluid elements that need to be copied into both processors while in (b) there is only one.

First, our approach is practical. It has significantly lower partition overhead than the *hypergraph* partition approach. Partitioning a power-law graph with 2 million vertices and 16 million edges takes about 400 seconds if we apply the best *hypergraph* algorithm we are aware of, while less than 18 seconds if we apply our model.

Second, our method is simple and easy to implement. It is based on a transformation procedure name *split-and-connect* procedure we developed in this work. The *split-and-connect* procedure improves the partition quality (the number of vertex copies in different partitions) over the *streaming* and the *hypergraph* approaches or is at least similar to the better of the two. In particular, when compared with the *streaming* edge partition algorithms, the partition quality improvement is the most pronounced in power-law graphs.

Third, it provides a theoretical guarantee for graphs with all ranges of parameters. Similar to the work by Bourse, Lelarge, and Vojnovic [1], by formulating the edge partition problem into vertex partition problem, we establish an approximation guarantee of $O(d_{\max} \sqrt{\log n \log k})$, where n is the number of vertices, m is the number of edges, k is the number of partitions, and $d_{\max}$ is the maximum degree. The work by Bourse, Lelarge and Vojnovic [1] showed that this approximation factor holds for graphs with the constraint $m = \Omega(k^2)$. We rigorously proved this approximation factor holds for all graphs.

2 USE CASES

We demonstrate how to use edge partition model to improve data reuse in GPU programs under two scenarios: (1) **load-balanced work partition** model, and (2) **data-balanced work partition** model. The former corresponds to the **Edge-balanced Edge-Partition (E-EP) Problem** **Vertex-balanced Edge-Partition (V-EP) Problem** For both models, the input is a list of 2-tuple modeled as a work graph and the output is a set of work (edge) partitions. We formally define these two *work partition* models as follows.

Edge-balanced Edge-Partition (E-EP) Problem Given a graph $G = (V, E)$ with the set of vertices V and the set of edges E , and vertex capacity constraint T . Let $x = \{e_1, e_2, \dots, e_k\}$ denote a partition of the edges of G into k disjoint subsets, let $|e_i|$ denote the number of unique edges in e_i . $\forall n \in V$ and $|E|$ denote the number of edges in E . $P(n)$ is defined as the number of subsets that n 's incident edges fall into. We optimize the total vertex replication cost:

$$\begin{aligned} \underset{x}{\text{minimize}} \quad & R(x) = \sum_{n \in V} (P(n) - 1) \\ \text{subject to} \quad & \forall i \in [1..k], |e_i| \leq |E|/k \end{aligned} \quad (1)$$

The *Edge-balanced Edge-Partition (E-EP)* problem has been used to minimize communication while balancing the load between computation threads or nodes. It has been used in the distributed graph processing setting [1, 3]. We use it to model workload balancing across thread blocks on GPUs [5].

Vertex-balanced Edge-Partition (V-EP) Problem Given a graph $G = (V, E)$ with the set of vertices V and the set of edges E , and vertex capacity constraint T . Let $x = \{e_1, e_2, \dots, e_k\}$ denote a partition of the edges of G into k disjoint subsets, and let $V(e_i)$ denote the set of unique vertices in e_i . $\forall n \in V$, let $P(n)$ denote the number of subsets that n 's incident edges fall into. We optimize the total vertex replication cost:

$$\begin{aligned} \underset{x}{\text{minimize}} \quad & R(x) = \sum_{n \in V} (P(n) - 1) \\ \text{subject to} \quad & \forall i \in [1..k], |V(e_i)| \leq T \end{aligned} \quad (2)$$

The V-EP model also minimizes vertex replication cost. While the E-EP model aims to balance the workload among threads/processors in space, the V-EP model aims to keep data fit into cache (the capacity constraint T) and maximize data reuse over time. We use the E-EP model to perform software throttling on GPUs [2].

3 CONCLUSION

We evaluate the effectiveness of our edge partition model over GPU applications from the domains of linear algebra, graph processing, machine learning and scientific simulation. As far as we know, this is the first systematic study of edge partition model for GPU computing. In both settings: load-balanced work partition and data-balanced partition, our approach significantly improves data reuse in cache and greatly enhanced program performance. Evaluated on over 200 real-world large sparse matrices and graphs that suffer from cache contention, our technique achieves up to 6.45X speedup, and a maximum performance loss $\leq 5\%$.

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