Topic Set Size Design for Paired and Unpaired Data

Tetsuya Sakai

Waseda University, Tokyo, Japan

tetsuyasakai@acm.org

ABSTRACT

Topic set size design is an approach to determining the sample sizes of an experiment (e.g., number of topics) based on a statistical requirement, namely a desired statistical power or a cap on the confidence interval (CI) width for the difference in means. Previous work considered paired data cases for a desired power of the t -test and for a cap on CI width, as well as unpaired data cases for a desired power of one-way ANOVA. In the present study, we consider unpaired (i.e., two-sample) cases for the t -test and for the CI width. Since one-way ANOVA with two groups is strictly equivalent to the two-sample t -test, we compare the outcomes of the topic set size design results based on these two approaches, and show that the one-way ANOVA-based approach actually returns tighter sample sizes than the two-sample t -test approach. Moreover, we compare the paired and unpaired cases for both t -test-based and CI-based topic set size design approaches. Because estimating the variance of the score differences for the paired data setting is problematic, we recommend the use of our unpaired-data versions of t -test-based and CI-based topic set size design tools, as they only require a variance estimate for individual scores and the appropriate sample sizes for unpaired data are also large enough for paired data.

KEYWORDS

confidence intervals; effect sizes; evaluation; sample sizes; statistical power; statistical significance; test collections

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1 INTRODUCTION

Sakai's Microsoft Excel tools for *topic set size design* [7] are simple implementations of sample size design techniques described in Nagata [5]. They aim to provide a straight answer to test collection builders' frequently asked question: "How many topics should I create in order to guarantee reliable experiments?" His first tool computes the required topic set size to ensure high *statistical power* for the paired t -test; his second tool does the same for one-way ANOVA with m groups (e.g., systems); his third tool provides the required topic set size to ensure a small confidence interval (CI)

width for the difference between any two systems for paired data. As the t -test and the CI-based tools are for paired data, they require a variance estimate $\hat{\sigma}_d^2$ of the *difference* between two systems in terms of a particular evaluation measure. Whereas, the ANOVA-based tool is for unpaired data, and requires a variance estimate $\hat{\sigma}^2$ of each system under the homoscedasticity assumption: that is, the scores of each system are assumed to obey possibly different normal distributions but with a common variance σ^2 .

In the present study, we introduce two topic set size design tools for *unpaired* (i.e., *two-sample*) data, one based on a desired power of t -test and one based on a cap on the CI width of the differences in means. Sakai [7] considered the paired data case only as IR researchers often conduct paired data experiments using test collections, where different systems are evaluated using a common topic set. However, unpaired settings are also possible, for example, when User Group A rates System 1 and User Group B rates System 2, and we are interested in the appropriate group sizes. We then directly compare topic set size design based on the unpaired t -test with the existing one-way ANOVA-based approach, by leveraging the fact that one-way ANOVA with $m = 2$ groups is strictly equivalent to the unpaired t -test. Based on our analysis, we recommend the use of the ANOVA-based tool with $m = 2$ groups even when the researcher is interested in topic set size design for paired or unpaired t -tests, as it provides the tightest sample size estimates. Moreover, we compare the paired and unpaired cases for both t -test-based and CI-based topic set size design approaches. Because estimating the variance $\hat{\sigma}_d^2$ of the score differences for the paired data setting is problematic, we recommend the use of our unpaired-data versions of t -test-based and CI-based topic set size design tools over the paired-data ones, as they only require a variance estimate σ^2 for individual scores, and the appropriate sample sizes for unpaired data are always large enough for paired data as well.

2 PRIOR ART

Until around early 1990s, IR researchers were on the whole reluctant to apply parametric significance tests such as t -tests and ANOVAs to IR evaluation, and preferred the nonparametric alternatives such as the sign test. For example, Van Rijsbergen remarked in 1979 [10] (p.136): "*Parametric tests are inappropriate because we do not know the form of the underlying distribution.*" In the same year, in their discussion of designing "ideal" IR test collections, Gilbert and Sparck Jones [3] reported on a table that shows the required number of topics as a function of the number of relevant or retrieved documents per topic, also based on the sign test. They remarked: "*Since there is some doubt about the feasibility of getting 1000 requests, or the convenience of such a large set for future experiments, we consider 500 requests.*" In 1993, Hull [4] advocated the use of parametric significance tests; perhaps from around that time, more and more IR researchers started to accept these tests. With the advent to computer-based, distribution-free tests, namely,

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the bootstrap [6] and the randomisation test [9], it is now easy for researchers to demonstrate the robustness of the classical parametric tests to distributional assumption violations. While statistical significance testing is just a tool (See, for example, Carterette [1]), and harm can be done if used inappropriately (See, for example, Ziliak and McCloskey [13]), our view is that it can be a useful tool if researchers use it appropriately by considering and reporting statistical power, effect sizes, and confidence intervals.

With the exception of the TREC Million Query track which once had about 1,800 “sparsely-judged” topics to evaluate a given set of participating runs [2], the *de facto* standard in IR evaluation has been to use test collections that each contain a set of 50 topics, despite the aforementioned considerations for “ideal” test collections in the 1970s. Researchers who worried about this default topic set size have conducted *topic set splitting* experiments to empirically examine the reliability of such topic sets. For example, Voorhees [11] concluded that “Fifty-topic sets are clearly too small to have confidence in a conclusion when using a measure as unstable as $P(10)$.”

Webber, Moffat, and Zobel [12] proposed an incremental method for constructing a topic set for a test collection where a topic (with relevance assessments) is added to the set one by one until the desired statistical power for the paired t -test is achieved. Whereas, Sakai’s topic set size design approach [7] aims to provide simple answers to questions such as: “How many topics should I prepare if I want the paired t -test for any system pair with an effect size of 0.2 to achieve 80% statistical power while ensuring that the Type I error probability is no more than 5%?” The present study builds on this approach.

3 POWER-BASED TOPIC SET SIZE DESIGN

This section first introduces a topic set size design tool based on the *two-sample t -test*. This enables us to directly compare one-way ANOVA-based t -test-based approaches, for one-way ANOVA with $m = 2$ groups is exactly equivalent to the two-sample t -test. More specifically, it is known that if a random variable t obeys a t distribution with ϕ degrees of freedom, t^2 obeys an F distribution with $(1, \phi)$ degrees of freedom. Moreover, when $m = 2$, the F statistic for one-way ANOVA is exactly the square of the t statistic for the two-sided, two-sample t -test. Whereas, the power calculations for t -tests and ANOVA are done differently [5], so we should examine which approach gives us tighter sample size estimates. This is discussed in the latter half of this section. In addition, we also compare the results with the *paired t -test*-based approach.

3.1 Topic Set Size Design based on the Two-Sided, Two-Sample t -test

The model behind the two-sample t -test is as follows. The scores x_{1j} (e.g., nDCGs) of System 1 independently obey $N(\mu_1, \sigma^2)$ ($j = 1, \dots, n_1$); the scores x_{2j} of System 2 independently obey $N(\mu_2, \sigma^2)$ ($j = 1, \dots, n_2$), where n_1, n_2 are the sample sizes (e.g., topic set sizes, user group sizes). The null hypothesis is $H_0 : \mu_1 = \mu_2$, and the alternative hypothesis is $H_1 : \mu_1 \neq \mu_2$ (a two-sided test). Let $\bar{x}_{1\bullet} = \sum_{j=1}^{n_1} x_{1j}/n_1$, $\bar{x}_{2\bullet} = \sum_{j=1}^{n_2} x_{2j}/n_2$ and

$$V_p = \frac{\sum_{j=1}^{n_1} (x_{1j} - \bar{x}_{1\bullet})^2 + \sum_{j=1}^{n_2} (x_{2j} - \bar{x}_{2\bullet})^2}{\phi}, \quad \phi = n_1 + n_2 - 2 \quad (1)$$

Given the above distributional assumptions, the following random variable obeys a *noncentral t distribution* with ϕ degrees of freedom with a *noncentrality parameter* λ_t [5]:

$$t_0 = \frac{\bar{x}_{1\bullet} - \bar{x}_{2\bullet}}{\sqrt{V_p(1/n_1 + 1/n_2)}}, \quad (2)$$

where

$$\lambda_t = \sqrt{\frac{n_1 n_2}{n_1 + n_2}} \Delta_t, \quad \Delta_t = \frac{\mu_1 - \mu_2}{\sigma}. \quad (3)$$

Eq. 2 is the usual t -value we compute when conducting a two-sample t -test. Under H_0 , the noncentrality parameter $\lambda = 0$ and therefore t_0 obeys the *central t -distribution*, a special case of the noncentral t distribution. On the other hand, Under H_1 , $\lambda \neq 0$ and t_0 obeys a (truly) noncentral t distribution. We are interested in the latter situation, for statistical power is the ability to detect a difference *when there in fact is a difference*.

For any random variable t' that obeys the aforementioned non-central t distribution, $Pr(t' \leq w)$ for any w can be approximated as follows [5], using a random variable u that obeys a standard normal distribution:

$$Pr\left(u \leq \frac{wc^* - \lambda_t}{\sqrt{1 + w^2(1 - c^{*2})}}\right) \approx Pr\left(u \leq \frac{w(1 - 1/4\phi) - \lambda_t}{\sqrt{1 + w^2/2\phi}}\right), \quad (4)$$

where

$$c^* = \frac{\sqrt{2}\Gamma((\phi + 1)/2)}{\sqrt{\phi}\Gamma(\phi/2)}, \quad (5)$$

and $\Gamma(\bullet)$ denotes the gamma function [5]. Hence, the statistical power of a two-sided, two-sample t -test can be approximated as:

$$Pr\left(u \leq \frac{-w(1 - 1/4\phi) - \lambda_t}{\sqrt{1 + w^2/2\phi}}\right) + 1 - Pr\left(u \leq \frac{w(1 - 1/4\phi) - \lambda_t}{\sqrt{1 + w^2/2\phi}}\right), \quad (6)$$

where $w = t_{inv}(\phi; \alpha)$, the two-sided 100 $\alpha\%$ t -value for ϕ degrees of freedom, since we make decisions based on a *central t distribution* in significance testing, regardless of what t_0 *actually* obeys.

To discuss appropriate topic set sizes for the two-sample t -test, we consider a balanced design, $n = n_1 = n_2$, so that:

$$\phi = 2n - 2, \quad \lambda = \sqrt{\frac{n}{2}} \Delta_t \quad (7)$$

from Eqs. 1 and 3. Given α (Type I error probability), β (Type II error probability, i.e., one minus statistical power), and the *minimum detectable difference* $minD_t$ between two systems, a good initial sample size estimate can be obtained as [5]:

$$n \approx 2 \left(\frac{z_{inv}(\alpha/2) - z_{inv}(1 - \beta)}{minD_t / \sqrt{\hat{\sigma}^2}} \right)^2 + \frac{z_{inv}(\alpha/2)^2}{4}, \quad (8)$$

where $z_{inv}(P)$ denotes the upper 100 $P\%$ z -value and $\hat{\sigma}^2$ is an estimate of the aforementioned common variance for a particular evaluation measure. That is, we try to ensure 100 $(1 - \beta)\%$ power whenever the true difference $(\mu_1 - \mu_2)$ between any two systems is $minD_t$ or larger. Given $(\alpha, \beta, minD_t, \hat{\sigma}^2)$, we first compute Eq. 8 as an initial estimate, and check with Eq. 6 to see whether the desired power is actually achieved. If not, we increment the estimate by one, and repeat the process. Our Excel tool automates this process.

3.2 One-way ANOVA vs. t -tests

One-way ANOVA-based topic set size design requires the following input: α and β as before, the number of systems to be compared (m), and the *minimum detectable range* ($\min D$), and the variance estimate ($\hat{\sigma}^2$): we try to guarantee $100(1 - \beta)\%$ power whenever the population means of the best and the worst among the m systems differ by $\min D$. Clearly, when $m = 2$, $\min D$ is equivalent to the aforementioned minimum detectable *difference*, $\min D_t$. Moreover, as was mentioned earlier, one-way ANOVA with $m = 2$ groups (e.g., systems) is equivalent to the two-sample t -test: the only difference is that the F statistic used in one-way ANOVA is a *squared t statistic*. Hence, *theoretically*, the above two-sample t -test-based tool and Sakai's one-way ANOVA-based one (when m is set to 2) should return an identical topic set size for the same setting of $(\alpha, \beta, \min D_t, \hat{\sigma}^2)$. In practice, however, the results slightly differ, as we shall demonstrate below, for the power calculations are done differently: while the t -test power calculation involves approximating a noncentral t distribution as discussed above, ANOVA-based power calculation involves approximating a *noncentral F distribution* [7].

We also compare topic set size results based on the paired and two-sample t -tests. Given accurate variance estimates, the paired t -test should require a small sample size than the two-sample t -test under the same statistical requirement. However, while we can easily obtain an unbiased estimate of the variance $\hat{\sigma}^2$ for both two-sample and one-way ANOVA tests by computing the *residual variance* given a topic-by-run score matrix [7], the paired t -test-based topic set size design requires a variance estimate $\hat{\sigma}_d^2$ for the score *differences*. The paired t -test assumes $N(\mu_1, \sigma_1^2)$ and $N(\mu_2, \sigma_2^2)$ for the scores of the two systems and therefore $N(\mu_1 - \mu_2, \sigma_1^2 + \sigma_2^2)$ for the score differences; hence Sakai [7] recommends letting $\hat{\sigma}_d^2 = 2\hat{\sigma}^2$ without heeding the covariance, to consciously overestimate σ_d^2 and err on the side of oversampling. Under this setting, how will the results based on paired and two-sample t -tests differ?

Following Sakai [7], we address the above questions using residual variances from past TREC data for four different evaluation measures: AP (Average Precision), Q (Q-measure), nDCG (normalised Discounted Cumulative Gain), and nERR (normalised Expected Reciprocal Rank): from the topic-by-run score matrices of the TREC 2003 and 2004 robust tracks (with document cutoff 1000), we obtained the following pooled variances $\hat{\sigma}^2$ for the four measures: 0.047, 0.047, 0.046, 0.115; similarly, from the TREC 2011 and 2012 web adhoc tasks (with document cutoff 10), we obtained 0.082, 0.037, 0.044, 0.086. We then let $\hat{\sigma}_d^2 = \hat{\sigma}^2$ for the paired t -test-based topic set size design.

Table 1 summarises our power-based topic set size design results. The paired t -test-based and ANOVA-based results were obtained using Sakai's Excel tools [7]; the two-sample t -test-based results were obtained using our tool. It can be observed that the ANOVA-based tool returns the tightest estimates despite the fact that the two-sample t -test and one-way ANOVA with $m = 2$ groups are strictly equivalent. The differences arise from the respective power calculation methods. Moreover, the paired and two-sample t -test results are almost identical: the former does not return smaller topic set sizes, due to our overestimation of σ_d^2 . These results suggest that we can use the ANOVA-based tool with $m = 2$ even if we are interested in the power of paired or two-sample t -tests.

Table 1: Power-based topic set size design results: paired t -test/two-sample t -test/one-way ANOVA ($m = 2$).

$\min D_t$	AP	Q	nDCG	nERR
TREC 2003+2004 robust (cutoff=1000)				
0.05	298/297/289	298/297/289	291/290/283	725/724/705
0.10	76/75/73	76/75/73	75/74/71	183/182/177
0.15	35/34/33	35/34/33	35/34/32	83/82/79
0.20	21/20/19	21/20/19	21/20/19	48/47/45
0.25	14/13/12	14/13/12	14/13/12	31/30/29
TREC 2011+2012 web adhoc (cutoff=10)				
0.05	517/516/503	235/234/227	279/278/270	542/541/527
0.10	131/130/126	61/60/58	72/71/68	137/136/133
0.15	60/59/57	28/27/26	33/32/31	62/61/59
0.20	35/34/32	17/16/15	20/19/18	36/35/34
0.25	23/22/21	12/11/10	14/13/12	24/23/22

4 CI-BASED TOPIC SET SIZE DESIGN

This section first introduces a topic set size design method based on a cap on the CI width for the difference between two systems when the data are *unpaired*, under the homoscedasticity assumption. Then a simple approximation is introduced to both paired and unpaired CI-based topic set size design tools, in order to make the tools capable of handling large topic set sizes. Finally, the CI-based topic set size results based on paired and unpaired settings are compared.

4.1 Topic Set Size Design based on the CI Width for Unpaired Data

In CI-based topic set size design, we try to ensure that the expected CI width for the difference between any system pair is no larger than δ . As before, we try to find the right sample size n under a balanced condition: $n = n_1 = n_2$, $\phi = 2n - 2$. In this case, the above condition can be expressed as:

$$2t_{inv}(\phi; \alpha)E(\sqrt{V_p})\sqrt{\frac{1}{n_1} + \frac{1}{n_2}} = 2t_{inv}(\phi; \alpha)c^*\sigma\sqrt{\frac{2}{n}} \leq \delta, \quad (9)$$

where c^* is given by Eq. 5 and σ is the common standard deviation¹.

Although Sakai [7] used Eqs. 5 and 9 to consider the appropriate CI-based topic set size for *paired* data, Eqs. 5 relies on the Gamma function, which is computationally highly expensive². Hence we propose to utilise the following approximation:

$$c^* \approx 1 - 1/4\phi. \quad (10)$$

In fact, this approximation has already been used for t -test-based topic set size design in Eq. 4, when we discussed replacing a random variable that obeys a noncentral t distribution with one that obeys $N(0, 1^2)$. Figure 1 compares c^* as defined in Eq. 5 and its approximation as defined in Eq. 10. It can be observed that the approximation is highly accurate except when ϕ is extremely small. Hence, we use the above approximation for both paired-data and unpaired data cases in CI-based topic set size design.

Given the above approximation, Eq. 9 can be rewritten as:

$$\frac{t_{inv}(\phi; \alpha)(1 - 1/4\phi)}{\sqrt{n}} \leq \frac{\delta}{2\sqrt{2}\sigma} \approx \frac{\delta}{2\sqrt{2}\hat{\sigma}^2}, \quad \phi = 2n - 2. \quad (11)$$

¹ Note that $E(\sqrt{V_p}) = c^*\sigma$ while $E(V_p) = \sigma^2$. That is, V_p is an unbiased estimator of σ^2 , but $\sqrt{V_p}$ is not an unbiased estimator of σ .

² GAMMA(172) is greater than 10^{307} and cannot be computed by Excel.

Figure 1: Accuracy of $c^* \approx 1 - 1/4\phi$.

Table 2: CI-based topic set size design results: paired/unpaired data.

δ	AP	Q	nDCG	nERR
TREC 2003+2004 robust (cutoff=1000)				
0.10	147/146	147/146	144/143	356/355
0.20	39/38	39/38	38/37	91/90
0.30	19/18	19/18	18/17	42/41
0.40	12/11	12/11	11/10	25/24
0.50	8/7	8/7	8/7	17/16
TREC 2011+2012 web adhoc (cutoff=10)				
0.10	254/253	116/115	138/137	267/266
0.20	65/64	31/30	36/35	69/68
0.30	30/29	15/14	18/17	32/31
0.40	18/17	10/9	11/10	19/18
0.50	13/12	7/6	8/7	13/12

The initial topic set size estimate, which is an optimistic estimate that uses a standard normal distribution (σ^2 known) instead of a t -distribution (σ^2 unknown), is given by [5]:

$$n \approx \frac{8z_{inv}(\alpha/2)^2 \hat{\sigma}_d^2}{\delta^2}. \quad (12)$$

Then we check if this n actually satisfies Eq. 11; if not, we increment n and repeat the above. Our Excel tool automates this process.

For the paired data case, the following corresponds to Eq. 11:

$$\frac{t_{inv}(\phi; \alpha)(1 - 1/4\phi)}{\sqrt{n}} \leq \frac{\delta}{2\sigma_d}, \quad \phi = n - 1. \quad (13)$$

Note that, if we let $\sigma_d^2 = 2\sigma^2$ as before, Eq. 13 becomes identical to Eq. 11 except for the degrees of freedom. As for the initial topic set size estimate, it is given by [7]:

$$n \approx \frac{4z_{inv}(\alpha/2)^2 \hat{\sigma}_d^2}{\delta^2}. \quad (14)$$

which again is identical to Eq. 12 if we let $\hat{\sigma}_d^2 = 2\hat{\sigma}^2$. For the paired data case, we have verified that our new CI-based tool returns the same result as that of Sakai [7], and that it can return large topic set sizes where Sakai's cannot.

4.2 Paired vs. Unpaired Data in CI-based Topic Set Size Design

Table 2 compares the CI-based topic set size design results for paired and unpaired data settings, using the aforementioned variance estimates from TREC data. It can be observed that our unpaired data tool returns topic set sizes that are exactly one topic smaller than the corresponding paired data cases, even though theoretically

paired data require smaller sample sizes than unpaired data. Again, this is due to our overestimation of σ_d^2 .

5 CONCLUSIONS AND LIMITATIONS

We discussed the *unpaired data* cases for the t -test-based and CI-based topic set size design approaches. We directly compared the two-sample t -test approach and the one-way ANOVA-based approach (with $m = 2$), and showed that the latter returns tighter sample sizes despite the fact that the two tests are strictly equivalent for $m = 2$. Moreover, we showed that, if the variance σ_d^2 of the score difference between two systems is estimated as $\hat{\sigma}_d^2 = 2\hat{\sigma}^2$, the paired and unpaired results are basically the same for both t -test-based and CI-based topic set size design. This strongly suggests that we are indeed overestimating σ_d^2 's. We recommend the use of our unpaired versions of the t -test-based and CI-based tools over the paired versions because: (a) The unpaired versions give slightly tighter estimates while avoiding the use of $\hat{\sigma}_d^2$; and (b) Topic set sizes that are large enough for unpaired experiments always suffice for paired experiments as well. Finally, our new CI-based tools avoids computing the gamma function and thus can handle large sample sizes, unlike Sakai's earlier tool [7]. Our topic set design tools are available from: <http://www.f.waseda.jp/tetsuya/tools.html>; an extended discussion of this work will be given in a forthcoming book [8]. Limitations of the present study are as follows. Firstly, to estimate the variance of the score differences, we did not go beyond $\hat{\sigma}_d^2 = 2\hat{\sigma}^2$. This should not stop researchers from trying to obtain more accurate estimates of σ_d^2 somehow and thereby obtain tighter sample size estimates for paired data settings. Secondly, topic set size design does not consider *multiple comparison procedures* [1], as statistical power is difficult to define in these settings where there are *multiple* null hypotheses.

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