A Case Study of Integer Sum Reduction using Atomics

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ABSTRACT

This paper presents the implementations of integer sum reduction using atomic functions on FPGA, CPU, and GPU platforms. We explain the implementations and optimizations of the kernel using an OpenCL-based high-level synthesis flow for an FPGA. In addition, we describe the optimizations of the reduction using directives for a multi-core CPU and a GPU. The experimental results show that the reduction on an Nvidia K80 GPU is 3.4X and 6.7X faster than an Intel Xeon 16-core CPU and an Arria 10 GX1150 FPGA, respectively. However, the FPGA consumes 4.4X and 2.3X less power than the CPU and GPU, respectively. The performance per watt on the FPGA is 2.2X higher than that on the CPU and 2.9X lower than that on the GPU.

1. INTRODUCTION

Reduction is an important type of parallelism exploited in many applications that include map reduce applications, machine learning, numerical analysis, and scientific simulation applications. Although graphics processing units (GPUs) and/or central processing units (CPUs) are ubiquitously used for achieving high-performance reductions, there are few studies on a reduction kernel using atomic functions (atomics) in a high-level synthesis (HLS) flow for field-programmable gate arrays (FPGAs). Atomics facilitate fine-grained concurrency in which threads can execute without blocking other threads. Atomics have been demonstrated empirically to be a highly efficient synchronization method for conventional multiprocessors [1], but their support of Open Computing Language (OpenCL) standard for FPGAs is limited. Intel's FPGA software development kit (SDK) for OpenCL provides 32-bit atomic operations, but the user guide warns that they could decrease kernel performance when there are different atomic operations in an OpenCL kernel [2]. Atomics are currently not supported by Xilinx's SDAccel SDK.

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In this paper, we explore the implementations of the sum reduction using atomics on FPGA, CPU, and GPU computing platforms. We seek to better understand the characteristics of an atomic operation and its performance impact on these computing platforms. To that end, we describe the implementations and optimizations of the reduction using OpenCL atomics on an FPGA. Then we present the optimized implementations of the reduction using OpenMP and OpenACC directives. Using directives for accelerator programming is a productive and portable way to optimize a kernel in parallel computing. Then we measure the raw performance, power, and performance per watt of the reduction on an Arria10-based FPGA platform, an Intel Xeon 16-core CPU, and an Nvidia Tesla K80 GPU.

The experimental results show that fully utilizing the bandwidth of the data bus on the target FPGA system can improve the kernel performance by a factor of 14.4. Using local memories to store the partial reductions, we can achieve 6.7X performance speedup over the reduction with a global memory. When both optimizations are enabled, they can lead to 112X speedup. We observe that kernel duplication can further improve the performance by a factor of 2.9. Although the raw performance of the reduction on the FPGA is the lowest, the FPGA consumes 4.4X and 2.3X less power than the CPU and GPU, respectively. The performance per watt of the kernel implementation on the FPGA is 2.2X higher than that on the CPU and 2.9X lower than that on the GPU. The FPGA results based on a HLS flow are promising; FPGAs will become a major heterogeneous computing component for energy-efficient high-performance computing.

The rest of the paper is organized as follows. The background, which we will briefly present in Section 2, is mainly about the OpenCL essentials. Section 3 defines the scope of the reduction for the paper. Section 4 describes the implementations and optimizations of the kernel. The experimental results are presented in Section 5. Section 6 summarizes the studies on atomics using a high-level synthesis flow. Section 7 concludes the paper.

2. BACKGROUND

The traditional hardware development flow using hardware description languages (HDLs) bars a wider audience from evaluating applications on FPGA-based heterogeneous computing systems. Emerging HLS tools [3,4] could provide an alternative for the scientific computing community. As most researchers have little experience with FPGA-based hardware development, using high-level

programming language, particularly OpenCL for FPGA, could allow them to take advantage of the potential benefits of FPGAs.

An OpenCL application consists of host and kernel programs. The host program is written in standard C/C++ that runs on most of modern microprocessors. A host program needs to manage a set of OpenCL-related resources that facilitate kernel execution. A kernel is a function declared in a program and executes on an OpenCL-enabled device such as a GPU or an FPGA. Kernels can be executed in multiple instances. Each instance is defined as a work-item. These work-items are organized into multiple work-groups, and can be identified in one, two or three dimensions referred to as the N-Dimension Range (NDRange). A work-item is executed as part of a work-group executing on a compute unit that comprises processing elements and memory interfaces. The

Figure 1. Memory model in OpenCL

execution order of different workgroups within a kernel is unsynchronized. A user can use explicit barriers to ensure the memory consistency for work-items within a work-group. All the work-items belonging to that work-group must reach it before any of them can proceed beyond it. In addition, OpenCL defines a four-level memory hierarchy as shown in Figure 1.

3. SUM REDUCTION

Sum reduction is a common data parallel primitive. Listing 1 shows the sequential integer sum reduction in our study. The integer operations can be done in any order due to the associativity of the integer add. We can take advantage of associativity to divide the reduction into independent partial sums, and then combine results from the partial sums. This can be generalized to reductions on vectors of arbitrary size. This paper is focused on the unsegmented form of the sum reduction. It takes a binary associative operator “+” and an array of M data as inputs, and returns as output one value.

```
int sum = 0;
for ( int i = 0; i < M; i++ )
    sum += input[i];
```

Listing 1. The sequential sum reduction

Sum reduction can be realized sequentially or in parallel. For the parallel reduction, we assume that the array length is large enough that is unfit for a single work-group. To reduce over large vectors, there are three general ways described in [5]. We choose atomic operations for reductions to evaluate the performance of reduction primitive on heterogeneous computing platforms. Atomic operations are useful in safely updating a shared numeric variable by multiple threads, such as on hardware platforms that support atomic operation. They are also useful in implementing barrier synchronization within a kernel [6], constructing complex data structures such as hash tables [7] or building high-level programming frameworks such as MapReduce [8]. The details of the atomic function are described in [9,10]. In this paper, the 64-bit sum reduction is beyond the scope of the paper because the 64-bit atomic functions are not supported in current HLS tools.

4. IMPLEMENTATIONS OF THE REDUCTION IN OPENCL

4.1 Baseline

The baseline reduction kernel B1 is shown in Listing 2. Each work-item reads a unique element from the input vector in the global memory space implied by the `__global` qualifier. The `restrict` keyword is recommended for each global address space to prevent the compiler from creating unnecessary memory dependencies between nonconflicting memory accesses. The OpenCL API function `atomic_add()` performs an atomic addition of input data to the content of memory. The kernel is an NDRange kernel with the global work size equal to the length of the input vector.

```
__kernel
void reduce( __global int* restrict input,
             __global int* restrict output )
{
    int gid = get_global_id(0);
    atomic_add(output, input[gid]);
}
```

Listing 2. The baseline reduction kernel B1

An alternative kernel B2 is shown in Listing 3. Compared to B1, we explicitly use the identifiers (IDs) of a work-group and a work-item in a work-group to index the input array. The identifiers and the size of a work-group (WGS) are queried with the OpenCL API functions.

```
__kernel
void reduce( __global int* restrict input,
             __global int* restrict output )
{
    int gid = get_group_id(0);
    int lid = get_local_id(0);
    int WGS = get_local_size(0);
    atomic_add(output, input[gid*WGS+lid]);
}
```

Listing 3. An alternative OpenCL reduction kernel B2

4.2 Optimizations of Memory Accesses

On our target FPGA platform, the data width between the DDR memory controller and the user FPGA design is 512-

bit. To fully utilize the bandwidth of the data bus, we make use of the OpenCL API function `vload16` to read 16 integers as a vector for each memory transaction. Listing 4 shows the kernel B3. It reads 16 integers as a vector from the global memory for each load, and then atomically adds the sum of the integers to the output.

```
__kernel
void reduce ( __global int* restrict input,
              __global int* restrict output )
{
    int gid = get_global_id(0);
    int16 acc = vload16(gid, input);
    int sum = ((acc.s0+acc.s1)+(acc.s2+acc.s3))+
              ((acc.s4+acc.s5)+(acc.s6+acc.s7))+
              ((acc.s8+acc.s9)+(acc.sa+acc.sb))+
              ((acc.sc+acc.sd)+(acc.se+acc.sf));
    atomic_add(output, sum);
}
```

Listing 4. Kernel B3 loads 16 integers as a vector

Current FPGAs provide a large amount of on-chip memories. For example, Intel’s Arria 10 series have approximately 8 Mbytes of internal memory. To reduce the overhead of memory accesses from the external memory, a local memory can be statically allocated for reusing the data. A local memory corresponds to an on-chip memory that utilizes the Block RAMs (BRAMs) in FPGAs. The BRAM blocks are configurable memory modules, and they are physically located near the kernel computation compared to the global memory. They allow single-cycle random memory accesses, thus significantly improving memory access performance.

Listing 5 shows the kernel B4 that utilizes a local memory for storing partial sum results. The first work-item in each work-group resets the local sum. Then a barrier synchronizes the work-items to use the initialized local sum. After the atomic add is performed by all work-items in a work-group, the last work-item in the group atomically adds the partial sum to the output. As mentioned in [2], the default maximum work-group size is 256 when a kernel contains a barrier.

```
__kernel void
reduce(__global int* restrict input,
       __global int* restrict output)
{
    __local int sum;
    int gid = get_global_id(0);
    int lid = get_local_id(0);
    int WGS = get_local_size(0);

    if (lid == 0) sum = 0;
    barrier(CLK_LOCAL_MEM_FENCE);
    atomic_add(&sum, input[gid]);
    barrier(CLK_LOCAL_MEM_FENCE);
    if (lid == WGS-1)
        atomic_add(output, sum);
}
```

Listing 5. Kernel B4 performs atomics over local memory

4.3 Reduction with Local Memories

So far we have performed atomic operations over a single variable. Could we mitigate the pressure of atomic operations on a single memory by distributing the operations over multiple memory locations?

Listing 6 shows the generalized kernel using multiple local memories. The variable “N” represents the number of local memory locations. The modulo operator intends to distribute the atomic adds evenly for the N memory locations among all the work-items. After all partial sums are computed atomically, they are accumulated to compute atomically the final sum. Furthermore, the kernel fully utilizes the data bandwidth between the memory controller and user logics.

```
__kernel void
reduce(__global int* restrict input,
       __global int* restrict output)
{
    __local int sum[N];
    int gid = get_global_id(0);
    int lid = get_local_id(0);
    int WGS = get_local_size(0);
    int16 acc = vload16(gid, input);
    int r = ((acc.s0 + acc.s1) + (acc.s2 + acc.s3))
            + ... +
            ((acc.sc + acc.sd) + (acc.se + acc.sf));

    if (lid < N) sum[lid] = 0;
    barrier(CLK_LOCAL_MEM_FENCE);
    atomic_add(&sum[gid%N], r);
    barrier(CLK_LOCAL_MEM_FENCE);
    if (lid == WGS-1)
        atomic_add(output, sum[0]+sum[1]+...+sum[N-1]);
}
```

Listing 6. Optimized kernel based on B1, B3, and B4

4.4 Compute Unit Duplication

Compute unit replication (kernel duplication) is an approach to increasing kernel performance by instantiating multiple compute units for each kernel. Each compute unit has its own memory access interface for the kernel function. Each compute unit is capable of executing multiple work-groups simultaneously to increase data throughput. However, multiplying the number of compute units increases global memory bandwidth contention among compute units, which can degrade the performance of memory-bound kernels that have non-streaming memory access patterns.

5. EXPERIMENTS

5.1 Experimental Setup

In this study, we used Nallatech 385A FPGA card that features an Arria 10 GX1150 FPGA chip and two 4GB DDR3L-2133 memory channels. The 385A card is installed on a host machine using a PCIe x8 3.0 interface. The theoretical DDR3 memory bandwidth of the card is approximately 34 GB/s. The kernel execution time is measured with the OpenCL API function `clGetEventProfilingInfo` that returns the time in nano-seconds consumed by the kernel execution on the hardware. In our experiment, the length of the input vector ranges from 2^9 (512) to 2^{27} (128 million) integers. Table 1 lists the characteristics of the OpenCL kernels in our experiment. When `vload16` is used in a kernel, the global work size is divided by 16 as each work-item loads 16 input integers from global memory. The lower bound to the work-

Table 1. Characteristics of OpenCL reduction kernels

Kernel name	Work-group size range	Vector load width	Local memory size
B1	[1, 2^{27}]	1	0
B2	[1, 2^{27}]	1	0
B3	[1, 2^{23}]	16	0
B4	[1, 256]	1	1
B5	[8, 256]	1	8
B6	[16, 256]	1	16
B7	[1, 256]	16	1
B8	[8, 256]	16	8
B9	[16, 256]	16	16

group size of the kernels (B3 to B9) is constrained by the number of local memories. The work-group size, which must be a power of two, is a variable that will be evaluated in our experiment.

5.2 Resource Usage of the Implementations

The resource utilizations for all the kernels except kernel duplication described in the last section are almost the same. The logic utilization is 12%, and the number of RAM blocks increase from 307 to 329 due to the increase of local memories in the kernels. The dedicated arithmetic processing unit (DSP) is not utilized by any kernels. When the kernel is duplicated by a factor of eight, the logic utilization increases from 12% to 17%, and the number of RAM blocks is 547.

5.3 Kernel Performance on the FPGA

Figure 2 shows the execution time of the two baseline kernels with respect to the work-group size using an array size of 2^{27} . When the input array is indexed with the global ID in B1, the kernel time stays at approximately 6229 ms, insensitive to the work-group sizes. However, the work-group size has a large impact upon the performance of B2. The execution time decreases by a factor of 4 when the work-group size increases from 1 to 512. When the size is 512, the execution time of B2 is 4.6% slower than B1. The execution time of B2 levels off at 6519 ms when the work-

Figure 2. Performance of the two baseline kernels

group size is larger than 512. The result suggests that the application does not effectively hide the overhead of the memory address generation using explicit work-group size and local ID until a work-group size of 64 or more. As the performance of B1 is better than B2 across the work-group sizes in our experiment, we will evaluate the performance of other kernels that index the input vector using global IDs.

Figure 3 shows the execution time of the kernels (B4, B5, B6) that contain one, eight, and sixteen local memories, respectively. The bars in the graph shows that the lowest execution time is approximately 939 ms when the work-group size is larger than or equal to 64. On the other hand, the execution time starts to saturate for work-group sizes larger than 64. Compared to B1 that performs atomic operation over global memory, using local memory can reduce the execution time by a factor of 6.7. Figure 3 also reveals that using more local memories slightly increases the execution time from 939 ms (N=1) to 951 ms (N=16) due to the arbitration of local memory accesses.

Figure 3. Performance of the kernel implementations with respect to work-group size and local memories.

Figure 4 shows the execution time of the four kernels (B3, B7, B8, B9) that fully utilize the 512-bit data bus. The kernels (B7, B8, B9) contain one, eight, and sixteen local memories for atomic operations, respectively. Compared to B1, the execution time of B3 stays at around 433 ms for different work-group sizes, achieving 14.4X reduction in execution time. On the other hand, the combination of using local memory and vectorized load can lower the execution time by a factor of 16.2 and 7.4 respectively compared to each individual optimization. The results show the

Figure 4. Performance of the kernel implementations with respect to work-group sizes and local memories. Each kernel reads 16 integers as a vector.

importance of using on-chip memories and fully utilizing the bandwidth of the data bus for significantly improving the kernel performance.

We observe that kernel duplication is effective in further improving the kernel performance by a factor of 2.9, which is achieved with a duplication factor of 8. Due to the conflicting global memory accesses among compute units, kernel performance decreases as more compute units are instantiated in the hardware design.

5.4 Comparison with the CPU and GPU

The parallel reduction using OpenMP atomic operations for a multi-core CPU is shown in Listing 7. Based on our experiment of various OpenMP directives, the directive “#pragma omp simd” in the first version achieves the lowest execution time on the target CPU. In the second version, each OpenMP thread accumulates over the vectorized loop, and then adds the local sum to the final sum atomically without waiting for other threads to finish their local sums.

```
// the first version v1
#pragma omp simd
for (int i = 0; i < M; i++) {
    #pragma omp atomic
    sum += din[i];
}
// the second version v2
#pragma omp parallel
{
    int local_sum = 0;
    #pragma omp for simd nowait
    for (int i = 0; i < M; i++) {
        local_sum += din[i];
    }
    #pragma omp atomic
    sum += local_sum;
}
```

Listing 7. Integer sum reduction using OpenMP atomic operations

We use OpenACC [11] to implement two versions of the reduction for a GPU as shown in Listing 8. In the second version, the number of local sums is L . The local sums are initialized in the beginning. Then the modulo operator distributes the atomic adds over L local sums. The final sum is computed atomically using the local sums.

```
// the first version v1
#pragma acc parallel copy(sum) copyin(din[0:M])
#pragma acc loop independent
for (int i = 0; i < M; i++) {
    #pragma acc atomic update
    sum += din[i];
}
// the second version v2
#pragma acc data copy(sum) copyin(din[0:M])
create(local_sum [0:L])
{
    #pragma acc parallel loop
    for (int i = 0; i < L; i++) {
        local_sum[i] = 0;
    }
    #pragma acc parallel loop
    for (int i = 0; i < M; i++) {
        #pragma acc atomic update
        local_sum[i % L] += din[i];
    }
}
```

```
#pragma acc parallel loop
for (int i = 0; i < L; i++) {
    #pragma acc atomic update
    sum += local_sum[i];
}
}
```

Listing 8. Integer sum reduction using OpenACC atomic operations

In our system evaluation environment, the host has two CPU sockets, each with an Intel Xeon E5-2687W processor (16 total cores, each 2-way hyper-threaded), clocking at 3.4 GHz, and a total of 64 GB of DDR3 RAM. The theoretical memory bandwidth is 119.4 GB/s. The program is compiled with an Intel C compiler, version 2018. The compiler optimization option “-O3 -xAVX -qopenmp” is enabled and the program is executed with 32 threads. We chose an Nvidia Tesla K80 with 2,496 cores as the target GPU. The theoretical memory bandwidth is 240 GB/s. The OpenACC program is compiled with a PGI compiler, version 16.10. The compiler options are “-acc -ta=tesla:cc35 -O3”.

Figure 5 shows the CPU execution time of the two reduction implementations for the array sizes from 2^9 to 2^{27} . The y axis is scaled logarithmically with a base of 2 while the x axis only shows the exponent for brevity. The size of 8192 is a threshold beyond which v2 is faster than v1, which suggests the overhead of using local sums for small input array sizes. When the array size is 2^{27} , the execution time of v2 is 9.99 ms, almost 74X faster than v1 (741 ms).

Figure 5. The execution time of the reduction using OpenMP atomic operations on the CPU. The x axis shows only the exponent.

To evaluate the impact of the number of local sums L on the performance of the reduction, we use an input array of size 2^{27} and observe that the execution time decreases from $L = 4$ to $L = 256$ and then nearly levels off for larger values of L . Based on the findings, Figure 6 shows the elapsed

Figure 6. The execution time of the reduction using OpenACC atomic operations on the GPU. The x axis shows only the exponent.

compute time of the two kernel implementations when the input array sizes range from 2^9 to 2^{27} . The number of local sums is 256. The y axis is scaled logarithmically with a base of 2 while the x axis only shows the exponent. The threshold is 2^{21} , above which v2 is better than v1. When the array size is 2^{27} , the lowest time of v2 is 2.957 ms, approximately 1.87X faster than v1.

To compare the performance, power, and performance per watt on the three platforms, we choose the implementations that achieve the highest performance for the largest array size on the three platforms. Figure 7 shows that the reduction on the GPU takes approximately 2.957 ms, 3.4X and 6.7X faster than the CPU and FPGA, respectively. However, the FPGA consumes approximately 30 W, 4.4X and 2.3X lower than the CPU and GPU, respectively. The performance per watt, defined as the million number of array elements that are reduced per second per watt, is 223.7 on the FPGA, 2.2X higher than that on the CPU and 2.9X lower than that on the GPU. There are a few factors that lead to the low FPGA performance. The kernel is memory-bound and the theoretical peak memory bandwidth on the FPGA is 3.5X and 7.05X lower than that on the CPU and GPU, respectively. The work-items on an FPGA are executed in a pipeline to save the limited memory bandwidth, which is different from the execution of work-items on a GPU where they are processed in a data parallel manner. The upper bound to the kernel clock frequency on the FPGA is around 300 MHz compared to 562 MHz on the GPU and 3.4 GHz on the CPU.

Figure 7. Performance, power, and performance per watt on the CPU, GPU, and FPGA.

6. RELATED WORK

There are extensive studies on the performance characterizations of reductions and atomic functions on GPUs [12,13,14,15,16]. However, understanding the performance of reduction using atomic functions on an OpenCL FPGA platform is important as FPGAs are becoming promising heterogeneous computing components for HPC. Early research was focused on maximizing the floating-point performance under the constraint of memory bandwidth and resource usage using HDL [17]. Recently, the author evaluated an integer reduction kernel using OpenCL on a Xilinx ZYNQ FPGA board [18]. They did not fully utilize the data bus on the target FPGA platform, and implemented the segmented version of reduction. It was reported that atomics are ‘expensive’ when implementing

locks in their study of data partition problem within an FPGA [19]. However, the authors applied atomics to the OpenCL implementation of a K-means clustering algorithm and achieved 1.5X-1.9X speedup [20]. In this paper, we are focused on the performance of the atomics and its performance implications on heterogeneous platforms.

7. CONCLUSION

The experimental results show that current HLS tools do not take advantage of platform-specific optimizations automatically; simple porting would not guarantee performance portability. Although there is performance and power tradeoff when using atomics, they can simplify a kernel design, and allow the creation of more sophisticated algorithms. They also promote productivity and program portability. Hence, it will be worthwhile for HLS tools to adopt atomics and enhance their functionality, performance, and verification.

8. ACKNOWLEDGMENTS

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