Demo Abstract: SReachTools: A MATLAB Stochastic Reachability Toolbox*

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CCS CONCEPTS

• **General and reference** → **Verification**; • **Mathematics of computing** → **Stochastic control and optimization**; **Solvers**;

KEYWORDS

Reachability, Stochastic optimal control, Linear systems

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1 INTRODUCTION

In this demo, we present SReachTools, an open-source MATLAB toolbox for performing stochastic reachability of linear, potentially time-varying, discrete-time systems that are perturbed by a stochastic disturbance [8]. The toolbox addresses the problem of stochastic reachability of a target tube, which also encompasses the terminal-time hitting reach-avoid [7] and viability problems [1]. As illustrated in Figure 1, the stochastic reachability of a target tube problem maximizes the likelihood that the state of a stochastic system will remain within a collection of time-varying target sets for a give time horizon, while respecting the system dynamics and bounded control authority [9]. We are interested in the computation of the stochastic reach set, denoted by $\mathcal{L}_{SR}(\alpha)$, which is the set of initial states that satisfy the reach/safety specification with a likelihood above α , and the associated optimal, admissible controller.

SReachTools implements several new algorithms based on convex optimization, computational geometry, set-based methods, and Fourier transforms, to efficiently compute over- and under-approximations of the stochastic reach set. These algorithms do not grid

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the state space, which partially mitigates the curse of dimensionality. SReachTools can be used to perform probabilistic verification of closed-loop systems, and it can synthesize open-loop, affine, and state-feedback controllers.

SReachTools is released under GNU General Public License v3. It is well-documented, user-friendly, and extensible. The toolbox is available at <https://github.com/unm-hscl/SReachTools>. Figure 2 shows the documentation webpage for SReachTools.

2 OVERVIEW OF SREACHTOOLS

SReachTools provides four MATLAB functions for the user to analyze and verify stochastic linear systems and perform controller synthesis.

- (1) SReachPoint: Given an initial state, this function computes the maximum likelihood of reachability of the target tube and an associated open-loop or affine disturbance feedback controller. For Gaussian perturbed systems, it provides approaches based on chance constraints, Fourier transforms, and sampling (particle filters). For non-Gaussian perturbed systems, it relies on sampling-based approaches.
- (2) SReachSet: Given a probability threshold, this function computes over- and under-approximations of the stochastic reach set. For Gaussian perturbed systems, it provides approaches based on chance constraints, Fourier transforms, and Lagrangian (set-theoretic) techniques. For non-Gaussian perturbed systems, it relies on Lagrangian techniques. Lagrangian analysis also synthesizes a set-based state feedback controller.
- (3) SReachDyn: This function provides a naive implementation of the grid-based dynamic programming for benchmarking results from SReachPoint and SReachSet
- (4) SReachFwd: Given an initial state, this function characterizes the stochasticity of the state or the trajectory of an uncontrolled system at or up to a future time of interest, respectively.

SReachTools also provides auxiliary classes—LtvSystem, LtiSystem, Tube, SReachEllipsoid, and RandomVector—to set up and solve the problem of stochastic reachability of a target tube. It uses MPT3 [5] for polytopic computational geometry, LRS [2, 6] or CDD [3] for vertex-facet enumeration, and CVX [4] for formulation of convex problems.

3 OBJECTIVES OF THE DEMO

In this demo, we will demonstrate various examples analyzed using SReachTools. Specifically, we demonstrate the scalability of the MATLAB functions provided by SReachTools, and use it to verify

Figure 1: The problem of stochastic reachability of the target tube. We wish to identify the stochastic reach set $\mathcal{L}_{\text{SR}}(\alpha)$, the set of initial states from which the state can be driven to stay within the target tube $\mathcal{T} = \{\mathcal{T}_k\}_{k=0}^N$ with a probability above α under bounded control authority. We also wish to synthesize admissible controllers that achieve this specification. This image is licensed by the authors under the Creative Commons Attribution-ShareAlike 4.0 International License.

Figure 2: Documentation webpage of SReachTools, <https://unm-hscl.github.io/SReachTools/>.

the satellite rendezvous and docking problem as well as controlling a Dubin's vehicle in a target tube [8]. Figure 3 shows approximative stochastic reach sets of a Dubin's vehicle, computed using various methods of SReachSet. The sets obtained from chance-open and lag-under underapproximate the set of initial states that can be driven to achieve a reach probability ≥ 0.8 .

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Figure 3: Verification of Dubin's vehicle