Poster Abstract: Safety Control with Preview Automaton

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ABSTRACT

In this work we consider control problems for discrete-time dynamical systems with safety constraints. As opposed to the existing work on similar problems, we assume that the system at run-time can *preview* some of the future uncontrolled inputs (e.g., disturbances). In order to develop a control synthesis technique that can leverage such preview information, we introduce a mathematical construct called *Preview Automaton* that incorporates system dynamics with the prior knowledge on the structure of the preview. We propose an algorithm to find the maximum controlled invariant set for the Preview Automaton. Although Preview Automaton can be equivalently represented by a Finite Transition System, for which existing control synthesis methods can be applied, we show that exploiting the structure of the Preview Automaton results in more efficient solution algorithms. These ideas are demonstrated with a lane-keeping problem, where we show that existence of the preview information allows the vehicle to safely navigate roads with larger curvature, whereas the problem becomes infeasible when no preview information is available.

CCS CONCEPTS

• **Theory of computation** → **Timed and hybrid models**; • **Applied computing** → **Engineering**.

KEYWORDS

Safety Control, Preview Automaton, Controlled Invariant Sets

ACM Reference Format:

Zexiang Liu and Necmiye Ozay. 2019. Poster Abstract: Safety Control with Preview Automaton. In *22nd ACM International Conference on Hybrid Systems: Computation and Control (HSCC '19)*, April 16–18, 2019, Montreal, QC, Canada. ACM, New York, NY, USA, 2 pages. <https://doi.org/10.1145/3302504.3313360>

1 INTRODUCTION

Most modern autonomous systems, like self-driving cars, unmanned aerial vehicles, or robots, are equipped with sensors like cameras and LIDAR and corresponding perception systems that provide a window of projecting into what lies ahead. This ability to have a (possibly uncertain) preview of the future is quite different than the traditional sensors in classical control loops that only provide

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HSCC '19, April 16–18, 2019, Montreal, QC, Canada

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ACM ISBN 978-1-4503-6282-5/19/04.

<https://doi.org/10.1145/3302504.3313360>

Figure 1: Example of systems with preview: Once the vehicle enters the orange region, the sensor on the vehicle will alert that in T steps the road grade θ will change to a different range.

information about the current state. The goal of our work is to understand how this preview information can be used for safety control synthesis.

The goal for safety control is to design control strategies such that under some assumptions about the system model, the closed-loop system trajectory is guaranteed not to violate any safety constraints at any time. In this work, the safety constraints we consider are represented by subsets of the state space, which we denote as safety sets. One common way to handle the safety problem is to find a controlled invariant set embedded in the safety set, namely a subset of the safety set that once the system state is in there, there exists control inputs such that the state stays there indefinitely. For some special classes of systems, such as Finite Transition Systems (FTSs) and Piece-Wise Affine Systems (PWAs) with polytopic constraints, the computation of the controlled invariant sets has been well studied in the literature [1, 2].

Generally speaking, the dynamical systems being controlled usually have disturbance terms to capture the uncertainty from the environment or the model mismatch. Any unexpected disturbance can cause the violation of safety constraints. The existing methods deal with this issue by assuming sufficiently large bounds on the potential disturbances and designing invariant sets robust to such disturbances. However, a larger bound on the disturbance usually leads to a more conservative result. On the other hand, in practice, though some disturbances cannot be controlled, it is possible to preview the changes of the disturbance in the future by the sensors the control system is equipped with. For instance, for the vehicle cruise control problem shown in Figure 1, the road grade in the near future, as one disturbance that influences the forward acceleration, can be previewed by a vision system. But the existing methods just assume that the vehicle is “blind” and set a worst-case bound

Figure 2: The Preview Automaton that models the system with preview shown in Figure 1, where T_{ij} is the least preview time for transition $i \rightarrow j$ and H_i is the minimum holding time for discrete state i .

on the disturbance term. If we can take the preview ability of the system into account when computing the controlled invariant sets, the result can be less conservative, which is the motivation of this work.

Based on the observations in the previous paragraph, we develop the Preview Automaton (PA) to model switched systems that have access to some preview information. Roughly speaking, a Preview Automaton is a hybrid automaton with two specific time structures, the holding clock and the preview clock. We assume disturbances come from a finite (or finitely discretized) set, and each mode of the switched system captures the dynamics for one of the disturbance values. For a PA associated with such a switched system, each state in the PA corresponds to a mode and each transition corresponds to switching between two modes. There is a minimum holding time $H_{min}(s)$ associated to each state s and a least preview time $T_{min}(\tau)$ associated with each transition τ . Consider again the example in Figure 1: Instead of modeling the vehicle longitudinal dynamics[3] with the disturbance θ within $[-1, 1]$ to cover all the potential values of road grade, Preview Automaton allows us to model the switched system with three modes corresponding to the cases $\theta \in [-1, -0.5]$, $\theta \in [-0.5, 0.5]$ and $\theta \in [0.5, 1]$, as shown in Figure 2.

The invariant set for the Preview Automaton is dependent on the initial discrete state. It means that for each state in the Preview Automaton, there is a corresponding invariant set assuming that the system initially starts from this state. In the worst-case, Preview Automaton leads to an invariant set that is the same as not using the preview information, but in general preview greatly improves the invariant set size as shown in our case study. From a computational perspective, it can be shown that for a PA corresponding to a switched FTS, a FTS equivalent to the PA exists and can be used to compute the invariant set of the PA using existing methods. However, since the size of the state space of the equivalent FTS is much larger than that of the original system, this can be computationally expensive. Therefore, we develop a specialized algorithm to compute the invariant set for the Preview Automaton in an efficient way. Furthermore, our method can be thought of as a meta-algorithm that works for any switched systems where controlled predecessors for a given set can be computed, such as piecewise affine systems or finite transition systems.

2 CASE STUDY

In this section, the proposed method is used to compute a controlled invariant set for a vehicle lane-keeping problem.

Suppose that the vehicle is running on a highway with a constant speed. The purpose of lane-keeping is to guarantee the lateral displacement of the vehicle is smaller than a safety threshold. We use

Figure 3: Projections of the invariant set of the Preview Automaton into 3D space

the linearized bicycle model for lateral dynamics with parameters taken from [3]. The states $y, v, \Delta\Phi, r$ are the lateral displacement from the center of the lane, lateral velocity, yaw angle and yaw rate respectively. The control input δ_f is the steering angle of the vehicle with constraint $|\delta_f| \leq \pi/2$. The disturbance r_d is a quantity related to the curvature of the road.

The safety set in this study is $\{(y, v, \Delta\Phi, r) : |y| \leq 0.9, |v| \leq 1.2, |\Delta\Phi| \leq 0.05, |r| \leq 0.3\}$. When there is no preview, we assume $|r_d| \leq 0.05$, the maximum recommended road curvature at Michigan[3]. Under this setting, the controlled invariant set is empty. To apply the Preview Automaton to the same problem, we first divide $[-0.5, 0.5]$ into 5 segments evenly (that is $[-0.5, -0.3]$, $[-0.3, -0.1]$, ..., $[0.3, 0.5]$) and create a switched system with 5 modes, where each mode corresponds to the same dynamics with r_d in one of the 5 segments. In the Preview Automaton we set the transitions to only exist between modes with adjacent ranges of r_d (such as $[-0.5, -0.3]$ and $[-0.3, -0.1]$), based on the observation that the curvature of the road changes continuously. Also, we set all the least preview time and minimum holding time to be 1, which implies that the road curvature can change from one range to the other adjacent ranges within 1 time step. For the Preview Automaton described above, the invariant set for the initial $r_d \in [-0.5, -0.3]$ is shown in Figure 3. The invariant sets for the other initial ranges of r_d are similar to Figure 3.

3 CONCLUSIONS AND FUTURE WORK

We believe Preview Automaton provides a very interesting modeling formalism relevant for many safety-critical autonomous systems. We are currently working on understanding theoretical properties of the Preview Automaton as well as its extensions to cases where preview horizon can be state-dependent.

ACKNOWLEDGMENTS

This work was supported in part by the Office of Naval Research under grant N00014-18-1-2501 and the National Science Foundation under grant ECCS-1553873.

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