WiP Abstract: On the Need for Sensor and Actuator Placement Algorithms in Nonlinear Systems

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ABSTRACT

In this work, we consider the problem of selecting—or placing—sensors and control nodes in nonlinear dynamic networks (NDN) which model the evolution of many cyber-physical systems. Specifically, we illustrate here the need for computational methods that obtain the optimal combinations of sensors and actuators, while considering the nonlinear behavior of NDN and and systems-theoretic performance metrics. This work-in-progress abstract briefly shows cases our recent work in this problem, which is divided into two steps: (1) Characterizing the classes of nonlinearities encompassed in NDN; (2) Developing tractable computational algorithms to solve the joint problem of selecting control nodes/sensors and designing feedback control/state estimators.

CCS CONCEPTS

• **Mathematics of computing** → **Mixed discrete-continuous optimization.**

KEYWORDS

Control nodes selection, sensors selection, nonlinear dynamic networks, cyber-physical systems control and estimation.

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1 INTRODUCTION AND MOTIVATION

Intelligent transportation systems, water networks, power grids, and many other cyber-physical systems (CPS) are naturally assembled into nonlinear dynamic networks (NDN)—intertwined networks that evolve *nonlinearly* in time and space. A defining feature of these networks is the prevalence and ubiquity of sensors and control nodes/actuators: sensors collecting data while ensuring network observability, and controllers (or *actuators*) driving networks to desired states. The selection/placement of sensors and control

Table 1: SCSP in various CPS.

Application	Actuator Selection	Sensor Selection
<i>Transportation Networks</i> [1, 3]	Closure/opening of roads, traffic lights	Placement of sensors monitoring traffic flow
<i>Power Grids</i> [7]	Selection of generators, batteries	Placement of phasor measurement units

nodes (SC) is one of the main steps in the network design and control. From this perspective, the SC selection problems (SCSP) can be split into: (i) finding SC selection/placement satisfying system constraints and performance metrics, while (ii) designing localized control/estimation routines. Tab. 1 describes two examples of SCSP in CPS.

To illustrate the importance of studying SCSP for say, any nonlinear dynamical systems, suppose that we want to perform state estimation while obtaining the minimal number of sensor on a permanent magnet stepper motor [8] which dynamics are expressed as $\dot{\mathbf{x}}(t) = \phi(\mathbf{x}) + \mathbf{B}\mathbf{u}(t)$, $\mathbf{y}(t) = \mathbf{C}\mathbf{x}(t)$, where $\phi : \mathbb{R}^4 \rightarrow \mathbb{R}^4$ represents the nonlinear dynamics specified as

$$\phi(\mathbf{x}) = \begin{bmatrix} x_2(t) \\ -K_1 x_2(t) - K_2 x_3 \sin(K_3 x_1(t)) + K_2 x_4 \cos(K_3 x_1(t)) \\ K_4 x_2(t) \sin(K_3 x_1(t)) - K_5 x_3(t) \\ -K_4 x_2(t) \cos(K_3 x_1(t)) - K_5 x_4(t) \end{bmatrix}.$$

The simplest approach to solve this problem is to linearize the above NDN around known equilibrium points, in which methods to solve SCSP in linear systems can be applied. Let $\mathbf{x}_{e,1} = [\pi/200 \ 0 \ 1 \ 1]^T$, and $\mathbf{x}_{e,2} = [-\pi \ 0 \ 0 \ 0]^T$ be two equilibria of the desired operating regions. After linearizing the system around these points, we obtain linear dynamics $\dot{\mathbf{x}}(t) = \mathbf{A}_{e,i}\mathbf{x}(t) + \mathbf{B}\mathbf{u}(t)$, for $i = 1, 2$. The observer design along with sensors selection can be lumped into the following nonconvex feasibility problem

$$\mathbf{A}_{e,i}^T \mathbf{P} + \mathbf{P} \mathbf{A}_{e,i} - \mathbf{C}^T \mathbf{\Gamma} \mathbf{Y}^T - \mathbf{Y} \mathbf{C} < 0, \quad \mathbf{P} > 0, \quad (1)$$

where $\mathbf{P}$, $\mathbf{Y}$, and $\mathbf{\Gamma}$ are matrix variables to be solved. Specifically, $\mathbf{\Gamma} = \text{diag}([\gamma_1, \dots, \gamma_4])$ is a diagonal matrix representing the binary sensors selection, with $\gamma_i = 1$ if sensor i is selected and zero otherwise. After solving the above mixed-integer semi-definite program (MI-SDP) using branch-and-bound (BnB) algorithm, we obtain two distinct locations of sensor for the two linear models: $\gamma_{(1)}^* = \{0, 1, 0, 0\}$, $\gamma_{(2)}^* = \{1, 0, 0, 0\}$. It is revealed that the second sensor selection, $\gamma_{(2)}^*$, can be used to design observers for linear systems linearized around both equilibria, but not otherwise. Interestingly, the sensor selection for the original nonlinear system using Lipschitz-based observer—see Section 2.2—is identical to $\gamma_{(2)}^*$. This simple example empirically illustrates that the solution to a SCSP depends on the *local* behavior of the network. In contrast, the

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biggest advantage of solving SCSP for NDN is the independence of the sensor/actuator selection from the NDN's operating point, which consequently offers practicality in situations where operating points of NDN are frequently changing. It is also important to mention that for NDN comprising of hundreds/thousands of subsystems, SCSP is highly **non** trivial, particularly when a specific metric such as robustness is considered.

2 RESEARCH METHODOLOGY

The optimal selection/placement of SC in **linear** networks has been thoroughly studied in the literature through various approaches [6, 7]. Despite these advances, the SCSP for NDN remains to a large extent an open research problem. To the best of our knowledge, there are only two main approaches in the literature that are devoted to solve SCSP for NDN. The first approach is based on *empirical* Gramians in low-dimensional systems [5], while the other presents a method for reconstructing the initial states $\mathbf{x}(t_0)$ of NDN, given a NDN model, and optimally selecting sensors given an observation window of fixed length [2]. Still, these approaches are limited in their applicability to large-scale, unstable networks, and difficulty to incorporate SC selection or robustness metrics. To that end, the objective of this work is to develop control-theoretic based methods for some classes of SCSP in various NDN while addressing the limitations of the aforementioned approaches. The following sections give a more detailed description of our approaches.

2.1 Classification of The Nonlinear Dynamics

We start this section by exploiting the following *attributes* of NDN: (i) physical states are almost always bounded, and (ii) the nonlinearities present in the NDN can be shown to belong to various classes of nonlinear functions. For instance, NDN with continuous partial derivatives on compact domains are indeed locally Lipschitz continuous. Knowing that a NDN satisfies the Lipschitz property is crucial as it allow us to design state estimator/controller for Lipschitz nonlinear systems, for which strategies for solving SCSP can be applied—see Section 2.2. However, this method often relies on the knowledge of the corresponding Lipschitz constant. Although computing Lipschitz constant analytically for NDN of medium size and complexity can still be manageable (see our recent work on analytical computation of Lipschitz constants [3, 4]), for high-dimensional NDN with complex nonlinearities (e.g. multi-machine power networks), computing Lipschitz constants can be daunting. This observation motivates us to develop a new strategy to determine Lipschitz constants for high-dimensional NDN. To begin with, let $f : \mathbb{R}^n \rightarrow \mathbb{R}$ be a differentiable function with domain of interest $\mathcal{X} \subset \mathbb{R}^n$. Since f is locally Lipschitz in $\mathcal{X}$, then we have

$$\|f(\mathbf{x}) - f(\hat{\mathbf{x}})\| \leq \left(\max_{\mathbf{z} \in \mathcal{X}} \|\nabla f(\mathbf{z})\| \right) \|\mathbf{x} - \hat{\mathbf{x}}\|, \quad \forall \mathbf{x}, \hat{\mathbf{x}} \in \mathcal{X}.$$

To solve this problem, we consider both deterministic and stochastic approaches. The deterministic approach utilize interval arithmetic for global optimization to find the best interval for $g(\mathbf{z}) := \max_{\mathbf{z} \in \mathcal{X}} \|\nabla f(\mathbf{z})\|$. The stochastic approach uses quasi random sampling with low discrepancy sequences [4] to approximate the global solution of $g(\mathbf{z})$. Other than Lipschitz nonlinearity, we plan to propose a unified framework to find such constants for

other classes of nonlinear functions, including one-sided Lipschitz, quadratically inner-bounded, and bounded Jacobian.

2.2 SCSP Strategies for NDN

After the NDN's nonlinearity is identified, Lyapunov theory can then be exploited to construct SCSP formulations, given that most NDN can be expressed in the following form

$$\dot{\mathbf{x}}(t) = \mathbf{A}\mathbf{x}(t) + \mathbf{G}\mathbf{f}(\mathbf{x}) + \mathbf{B}\mathbf{U}(t), \quad \mathbf{y}(t) = \mathbf{C}\mathbf{x}(t), \quad (2)$$

where the network is comprised of N subsystems, each having its own states, control inputs, and outputs. The nonlinearity is represented by $\mathbf{f}(\mathbf{x})$ while matrices $\mathbf{A}, \mathbf{B}, \mathbf{C}, \mathbf{G}$ define the topology and parameters of the NDN. Matrices $\mathbf{\Pi}$ and $\mathbf{\Gamma}$ are diagonal with binary variables defining the selection of SC. As an illustration, let us revisit the previous example with β as the Lipschitz constant of $\mathbf{f}$ where $\phi(\mathbf{x}) = \mathbf{A}\mathbf{x}(t) + \mathbf{G}\mathbf{f}(\mathbf{x})$. The observer design for this system with sensor selection problem can be posed as

$$\underset{\mathbf{\Gamma}, \mathbf{P}, \mathbf{Y}, \kappa}{\text{minimize}} \quad \mathbf{1}^\top \boldsymbol{\gamma} \quad (3a)$$

$$\text{subject to} \quad \begin{bmatrix} \mathbf{A}^\top \mathbf{P} + \mathbf{P}\mathbf{A} - \mathbf{C}^\top \mathbf{\Gamma} \mathbf{Y}^\top - \mathbf{Y}\mathbf{C} + \kappa \beta^2 \mathbf{I} & \mathbf{P}\mathbf{G} \\ \mathbf{G}^\top \mathbf{P} & -\kappa \mathbf{I} \end{bmatrix} < 0 \quad (3b)$$

$$\mathbf{P} > 0, \quad \kappa \geq 0, \quad \boldsymbol{\gamma} \in \{0, 1\}^N, \quad (3c)$$

where $\boldsymbol{\gamma} = \text{vec}(\mathbf{\Gamma})$ and in the case of our example, $N = 4$. This problem can be cast as a MI-SDP by invoking the disjunctive programming principles and the Big-M method [6, 7], in which the resulting problem is similar to (3) by replacing $\mathbf{Y}\mathbf{C}$ with a new variable $\mathbf{Q}$ and adding the constraints $|\mathbf{Y}_{ij} - \mathbf{Q}_{ij}| \leq M(1 - \gamma_j)$ and $|\mathbf{Q}_{ij}| \leq M\gamma_j$ for all i, j with M as a large positive constant. This problem can be solved with various methods, with BnB algorithm being the popular one. Although BnB algorithm theoretically is able to give optimal solutions, unfortunately, its computational time does not scale well with the number of variables, which in turn expose the limitation of this approach. With that in mind, we plan to explore different methods as well as develop faster computational algorithms which is tractable for large-scale CPS.

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