Incomplete Tests for Undetectable Faults to Improve Test Set Quality

IRITH POMERANZ, Purdue University

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The presence of undetectable faults in a set of target faults implies that tests, which may be important for detecting defects, are missing from the test set. This article suggests an approach for addressing missing tests that fits with the rationale for computing an n -detection test set. The article defines the concept of an incomplete test that is relevant when a target fault is undetectable. An incomplete test activates the fault but fails to detect it because of one or more assignments that are missing from the test. The procedure described in this article improves the quality of a test set by attempting to ensure that every undetectable fault has n incomplete tests with the smallest possible numbers of missing assignments, for a constant $n \geq 1$. The incomplete tests are expected to contribute to the detection of detectable defects around the site of the undetectable fault. The computation of missing assignments for a test is performed in linear time by avoiding fault simulation and considering all the undetectable faults simultaneously. Experimental results demonstrate the extent to which a given test set can be improved without increasing the number of tests.

CCS Concepts: • **Hardware** → **Test-pattern generation and fault simulation**;

Additional Key Words and Phrases: n -detection test set, test generation, test set quality, undetectable faults

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1 INTRODUCTION

A high-quality test set can be generated by targeting faults from more than one fault model. For example, a test set can target the detection of bridging [1–7] and open [8–16] faults. The fault models can be matched to the expected behaviors of defects in a specific technology [17–20]. An understanding of the causes for defects is also important for the definition of cell-aware faults [21–24].

Gate-exhaustive [25–29] and n -detection [30–35] test generation procedures attempt to address the fact that not all the possible defects are known in advance and understood well enough to select fault models for them. To ensure that the test set covers unexpected defects, a gate-exhaustive

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Author's address: I. Pomeranz, School of Electrical and Computer Engineering, Purdue University, West Lafayette, IN 47907, USA; email: pomeranz@ecn.purdue.edu.

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Fig. 1. Example of an incomplete test.

approach tests gates exhaustively, and an n -detection test set includes n different tests for every target fault. The tests create different conditions around a gate or fault site, thus contributing to the detection of defects in these areas. This is expected to compensate for the possibility that defect behavior is not captured accurately by a fault model.

Considering specific fault models, the procedures described in References [36, 37] address the presence of undetectable faults based on the following observation. Let f be a target fault. A test for f is expected to detect the fault f if it occurs in the circuit, as well as defects around the site of f . If f is undetectable, then a test that would have detected it is missing from the test set. As a result, the test set may fail to detect defects around the site of f , even if these defects are detectable. The issue of a missing test for an undetectable fault f is addressed in References [36, 37] by adding to the test set tests for different faults around the site of f . In Reference [36], a bridging fault model is defined to address the sites of undetectable transition faults. In Reference [37], what are called unspecified transition faults are used for addressing undetectable transition faults under multicycle tests.

To remove the reliance that exists in References [36, 37] on specific fault models, it is possible to use gate-exhaustive faults to cover the sites of undetectable target faults. This article takes a more general approach that fits with the rationale for computing an n -detection test set. The article defines the concept of an incomplete test that is relevant when a target fault is undetectable. An incomplete test activates the fault but fails to detect it because of one or more assignments that are missing from the test.

An example is shown in Figure 1, where the fault g_2 stuck-at 0 is undetectable. The test shown in Figure 1 activates the fault by assigning the value 1 to line g_2 in the fault-free circuit. The test also propagates the fault effect 1/0 to line g_4 . However, the value one on g_3 prevents the fault effect from propagating further. The value $g_5 = 0$ would have allowed the fault effect to propagate, but $g_7 = 0$ would have stopped its propagation. The test is referred to as an incomplete test with two missing assignments, where the missing assignments are caused by $g_3 = 1$ and $g_7 = 0$.

The procedure described in this article improves the quality of a test set by attempting to ensure that every undetectable fault has n incomplete tests with the smallest possible numbers of missing assignments, for a constant $n \geq 1$. The incomplete tests are expected to contribute to the detection of detectable defects around the site of the undetectable fault. Such defects are expected to require propagation paths that are similar to the ones required for the undetectable fault. Therefore, even missing assignments that are far from the fault site are important. The importance of a small number of missing assignments is that it makes it more likely that detectable defects will be detected by an incomplete test. Specifically, with a smaller number of missing assignments, it is more likely that a propagation path will exist for a detectable defect. The main challenge in the computation of missing assignments for a test is to avoid repeated fault simulation to determine which assignments are missing. The procedure described in this article addresses this

challenge by counting the number of missing assignments based only on fault-free values and traversing the circuit three times for this purpose. This results in a computational complexity that is linear in the number of lines in the circuit for computing the missing assignments of a test.

The procedure is designed such that it does not increase the number of tests in the test set given to it. This approach is used by the n -detection test generation procedure from Reference [34]. It is suitable here, because incomplete tests do not detect known faults. In contrast, in References [36, 37], as well as when gate-exhaustive faults are considered, tests are added to the test set to detect known faults.

Only single stuck-at faults are considered in this article. The concepts of missing assignments and incomplete tests can be applied to other fault models.

The article is organized as follows. The procedure for counting the number of missing assignments for a test is described in Section 2. Section 3 describes the procedure for improving a given test set by increasing the numbers of incomplete tests for undetectable faults and reducing the numbers of missing assignments for these tests. Section 4 presents experimental results.

2 COUNTING MISSING ASSIGNMENTS

Let F be the set of target faults. Let the faults in $U \subseteq F$ be undetectable. This section considers a test t as an incomplete test for the faults in U . For every fault $f \in U$, the procedure described in this section computes the number of missing assignments under t . This number is denoted by $\mu(f, t)$. This section also defines parameters related to the numbers of missing assignments under a test set T .

The procedure requires three traversals of the circuit. Each traversal has a computational complexity that is linear in the number of lines in the circuit. In the first traversal, the procedure performs logic simulation of t to find a value $v(g)$ for every line g . In the second traversal, the procedure performs a local analysis of missing assignments. In the third traversal, the procedure performs a global analysis of missing assignments that considers paths for propagating fault effects.

The second and third traversals are described next.

2.1 Local Missing Assignments

In the second traversal, the procedure computes a number of missing assignments $\mu_{lcl}(g)$ locally for every line g . Initially, $\mu_{lcl}(g) = 0$ for every line g . If g is not a gate input, then $\mu_{lcl}(g) = 0$ remains. Suppose that g is the input of a gate with a controlling value c , and inputs $g, g_0, g_1, \dots, g_{k-2}$. For $0 \leq i < k - 1$, the procedure adds one to $\mu_{lcl}(g)$ for every gate input g_i such that $g_i = c$. In this way, the procedure counts in $\mu_{lcl}(g)$ the number of assignments to other inputs of the same gate that prevent the propagation of fault effects from g .

For illustration, Figure 2 shows the values of $\mu_{lcl}(g)$ for the circuit and test from Figure 1. For every line g , the values are shown in the format $v(g), \mu_{lcl}(g)$. For example, $\mu_{lcl}(g_2) = 0$ because of the non-controlling value on line g_0 ; $\mu_{lcl}(g_4) = 1$ because of the controlling value on line g_3 , and the non-controlling value on line g_5 ; and $\mu_{lcl}(g_5) = 2$ because of the controlling values on lines g_3 and g_4 .

The computation of local missing assignments can be performed in linear time as follows. Every gate output is considered separately. For every gate output, the number of controlling input values is counted, and assigned to a variable denoted by n_c . For a gate input g with a non-controlling value, $\mu_{lcl}(g) = n_c$. For a gate input g with a controlling value, $\mu_{lcl}(g) = n_c - 1$. Since every line is considered at most twice, once for counting n_c and once for assigning $\mu_{lcl}(g)$, the computational complexity is linear in the number of lines.

Fig. 2. Example of local missing assignments.

Fig. 3. Example of global missing assignments.

2.2 Global Missing Assignments

The third traversal of the circuit computes a number of missing assignments $\mu_{glb}(g)$ for every line g taking into consideration that fault effects need to propagate through a path from g to an output. The computation proceeds from the outputs to the inputs. It results in the minimum number of missing assignments on a path from g to an output under the given test. The computation uses the numbers of missing assignments $\mu_{lcl}(g)$ that were computed locally. The computation is not accurate, because it does not take into consideration interactions between values, and the possibility of propagating fault effects through multiple paths. However, accuracy is not necessary, since the goal is not to generate a test that detects detectable target faults, but to obtain a metric for how far a test is from detecting an undetectable target fault. A test with fewer missing assignments is expected to be better, because it is more likely to detect detectable defects. The advantage of the metric as it is defined here is that it can be computed in linear time, and thus can be used as part of a test generation procedure that computes incomplete tests.

The third traversal initially assigns $\mu_{glb}(g) = 0$ for every output g of the circuit, and $\mu_{glb}(g) = \infty$ for every other line. When a line g is considered in the order from outputs to inputs, the procedure makes the following updates. If g is a gate output, then the procedure considers the gate inputs $g_0, g_1, \dots, g_{k-1}$. If g is a fanout branch, then the stem g_0 is considered its only input, and $k = 1$. For $0 \leq i < k$, the procedure compares the current value of $\mu_{glb}(g_i)$ with the value that would be assigned to it by considering a propagation path that goes through g . The number of missing assignments for a propagation path through g is equal to $\mu_{glb}(g) + \mu_{lcl}(g_i)$, where $\mu_{glb}(g)$ accounts for the path from g to an output, and $\mu_{lcl}(g_i)$ accounts for the local missing assignment of g_i . If $\mu_{glb}(g) + \mu_{lcl}(g_i) < \mu_{glb}(g_i)$, then the path through g is preferred, and the procedure assigns $\mu_{glb}(g_i) = \mu_{glb}(g) + \mu_{lcl}(g_i)$.

For illustration, Figure 3 shows the values of $\mu_{glb}(g)$ for the circuit and test from Figure 2. For every line g , the values are shown in the format $v(g), \mu_{lcl}(g), \mu_{glb}(g)$. Considering the fault g_2 stuck-at 0 from Figure 1, we have that $\mu_{glb}(g_2) = 2$. This matches the fact that the values of g_3 and

Fig. 4. Example without missing assignments.

g_7 stop the propagation of the fault effect. The controlling value of g_7 is reflected in $\mu_{icl}(g_6)$, and results in $\mu_{glb}(g_6) = 1$. The controlling value of g_3 is reflected in $\mu_{icl}(g_4)$. With $\mu_{glb}(g_6) = 1$ and $\mu_{icl}(g_4) = 1$ the result is $\mu_{glb}(g_4) = 2$. With $\mu_{glb}(g_4) = 2$ and $\mu_{icl}(g_2) = 0$ the result is $\mu_{glb}(g_2) = 2$.

Since every line is considered twice in this traversal, once as a gate output and once as a gate input, the computational complexity is linear in the number of lines.

2.3 Missing Assignments for Undetectable Faults

After the three traversals, for every fault $f \in U$, $\mu(f, t)$ is computed as follows. Let the fault f be the fault g stuck-at a . If t assigns $g = a$, then it does not activate the fault. In this case, $\mu(f, t) = \infty$. If t assigns $g = a'$, then $\mu(f, t) = \mu_{glb}(g)$.

Given a test set T and a set of undetectable faults U , the subset of undetectable faults that are activated by T is denoted by A . For a fault $f \in A$, every test $t \in T$ that activates f yields a number $\mu(f, t)$ of missing assignments. Let $M(f) = \{\mu(f, t) < \infty : t \in T\}$. For a constant $n \geq 1$, if the number of tests that activate f is larger than n , then the procedure keeps in $M(f)$ only the n smallest values.

For example, suppose that a test set contains seven tests that activate a fault f , and $M(f) = \{1, 1, 1, 2, 3, 3, 5\}$. With $n = 4$, the procedure uses $M(f) = \{1, 1, 1, 2\}$. We have that $|M(f)| = 4$ and the average value of an entry of $M(f)$ is $\bar{M}(f) = 1.25$.

Finally, it is interesting to note that the inaccuracy in the computation of the metric $\mu(f, t)$ can lead to a situation where $\mu(f, t) = 0$ for an undetectable fault. An example to illustrate this possibility is shown in Figure 4. The fault under consideration is g_1 stuck-at 1. The test in Figure 4 activates the fault, and creates a propagation path g_1 - g_5 - g_7 - g_8 that results in $\mu(f, t) = 0$. However, fault effects propagate to g_6 and g_7 , and cancel each other. As a result, the fault is not detected. The fault is undetectable, since $g_0 = 1$ and $g_1 = 0$ are both necessary for activating and propagating the fault.

3 PROCEDURE FOR TEST SET IMPROVEMENT

This section describes a procedure that accepts a test set $T = \{t_0, t_1, \dots, t_{K-1}\}$, which may be generated by any test generation procedure and target any number of detections. The procedure improves the quality of the test set as an n -detection test set, where $n \geq 1$, by: (1) preserving the numbers of detections for detectable faults, and (2) for undetectable faults, increasing the numbers of incomplete tests, and reducing their numbers of missing assignments.

The procedure does not increase the number of tests in the test set T given to it. Instead, it modifies the tests that are already included in T to achieve its objectives. In addition, the procedure does not attempt to increase the numbers of detections of detectable faults.

The procedure considers the tests in T for modification one by one in the order $t_{K-1}, t_{K-2}, \dots, t_0$. The premise is that the tests at the end of T are easier to modify, since they were generated after more of the detectable faults have already been detected the required number of times.

Table 1. Example of Test Modification

j	$\mu(f, t_i)$ for $f =$													
	f_{180}	f_{477}	f_{575}	f_{638}	f_{717}	f_{908}	f_{983}	f_{1008}	f_{1023}	f_{1028}	f_{1217}	f_{1336}	f_{1435}	f_{1437}
—	—	3	2	2	2	—	—	2	—	2	—	—	—	—
0	—	3	2	2	2	6	—	2	—	2	—	—	—	—
1	—	3	2	2	2	5	—	2	—	2	—	—	—	—
2	—	3	2	2	2	3	—	2	—	2	—	—	—	—
37	3	3	2	2	2	3	—	2	—	2	—	—	—	—
81	3	3	2	1	2	3	—	2	—	2	—	—	—	—

To modify a test $t_i \in T$, the procedure moves t_i to the end of the test set. For the reordered test set, it performs n -detection fault simulation of T . For the undetectable faults in U , it computes the sets $M(f)$. The procedure considers the following two subsets of target faults for the modification. With t_i at the end of T , fewer target faults are obtained, and it is easier to modify t_i .

A fault $f \in F - U$ that is detected by t_i is included in a set of target faults D_i .

A fault $f \in U$ is included in a set of target faults U_i if $|M(f)| < n$, or $\overline{M}(f) > 1$, or t_i contributes an entry to $M(f)$. In these cases, it is important for t_i to be an incomplete test for f with the smallest possible number of missing assignments. For the faults in U_i , the procedure records the numbers of missing assignments $\mu(f, t_i)$.

The procedure modifies t_i by complementing its bits one at a time. After complementing bit j , let the test t_i^j be obtained. The procedure decides whether or not to accept the complementation as follows.

It simulates D_i under t_i^j . If any of the faults in D_i is not detected, then the complementation is not accepted.

It computes $\mu(f, t_i^j)$ for every fault $f \in U_i$. If $\mu(f, t_i^j) > \mu(f, t_i)$ for any fault $f \in U_i$, then the complementation is not accepted.

If all the faults in D_i are detected, and $\mu(f, t_i^j) \leq \mu(f, t_i)$ is obtained for every fault $f \in U_i$, then the complementation is accepted. In this case, the procedure assigns $t_i = t_i^j$.

An example is given in Table 1, where a test t_i of benchmark circuit s1423 is modified based on 14 undetectable stuck-at faults. The faults are shown as column headers. Column j shows the bits whose complementation is accepted. The dash corresponds to the initial test, before any bits are complemented.

Table 1 shows the values of $\mu(f, t_i)$ for the initial test, and after the complementation of bit j is accepted, for $j = 0, 1, 2, 37$ and 81 . A dash for $\mu(f, t_i)$ indicates that the fault is not activated. For example, fault f_{908} is not activated by the initial test. It is activated by complementing bit $j = 0$, with six missing assignments. The number of missing assignments is reduced to five after complementing bit $j = 1$, and then to three after complementing bit $j = 2$.

An improvement in the test set occurs after t_i is modified if new faults are activated (the size of the set A increases), or the number of activating tests for a fault in U is increased ($|M(f)|$ increases for at least one fault $f \in A$), or the average number of missing assignments for an activating test is decreased ($\overline{M}(f)$ is decreased for at least one fault $f \in A$).

The procedure is summarized next.

Procedure 1: Test set improvement

- (1) For $i = K - 1, K - 2, \dots, 0$:
 - (a) Move t_i to the end of the test set.
 - (b) Perform n -detection fault simulation.

- (c) Compute the sets $M(f)$ for $f \in U$.
- (d) Assign $D_i = \{f \in F - U : f \text{ is detected by } t_i\}$.
- (e) Assign $U_i = \{f \in U : |M(f)| < n \text{ or } \overline{M(f)} > 1 \text{ or } t_i \text{ contributes an entry to } M(f)\}$.
- (f) For every bit j of t_i :
 - (i) Complement bit j to obtain the test t_i^j .
 - (ii) Simulate D_i under t_i^j .
 - (iii) Compute $\mu(f, t_i^j)$ for every fault $f \in U_i$.
 - (iv) If all the faults in D_i are detected, and $\mu(f, t_i^j) \leq \mu(f, t_i)$ for every fault $f \in U_i$, then assign $t_i = t_i^j$.

The worst-case computational complexity of Procedure 1 is determined by the modification of the tests from T . Let the number of bits in a test be B . The procedure simulates B modified tests for every test in T . The union of all the subsets of faults D_i that the procedure simulates is at most F . This implies that the procedure performs fault simulation for $O(B|F|)$ tests and faults.

4 EXPERIMENTAL RESULTS

The results of the test set improvement procedure for single stuck-at faults in benchmark circuits are presented in this section.

4.1 Setup

The procedure is applied to compact one-detection and 10-detection test sets for single stuck-at faults. For further illustration, for several circuits, the procedure is also applied to a one-detection test set that is extended with tests for gate-exhaustive faults to improve the coverage of sites of undetectable faults.

Most of the test sets detect all the detectable faults. If not, then the undetected faults are included in the set U together with the undetectable faults. While the procedure attempts to improve the test set, it may detect such a fault accidentally.

The target n for the number of incomplete tests for an undetectable fault $f \in U$ is $n = 16$. In addition, n -detection fault simulation for detectable faults is carried out using $n = 16$. This allows the numbers of detections of target faults to be used as an independent quality metric. With $n = 16$, the modification of the tests is expected to increase the numbers of detections of detectable faults even under the ten-detection test set.

4.2 Results

Tables 2 and 3 show the results of the test set improvement procedure. There are two rows for every circuit and test set. The first row describes the initial test set. The second row describes the final test set obtained by the test set improvement procedure. This test set is obtained after modifying the last test that results in an improvement of the test set.

For every test set, column *pi* shows the number of primary inputs. Column *ts* shows whether the one-detection or 10-detection test set is considered. A one-detection test set that is extended with tests for gate-exhaustive faults has 1G under column *ts*. Column *tests* shows the number of tests in the test set. The number of tests does not change by the improvement procedure. Column *mod* shows the index of the modified test.

Column *f.c.* shows the fault coverage. Column *16det* shows the average number of tests for a detectable fault when 16-detection fault simulation is carried out.

Column *undet* shows the number of undetectable faults. This is the size of the set U . Column *act* shows the number of undetectable faults that are activated by the test set. This is the size of

the set A . Column *targ* shows the number of undetectable faults that are targeted by the test set improvement procedure. Column *16act* shows the average number of incomplete tests for a fault in A . Column *miss* shows the average number of missing assignments, or the average value of $\overline{M(f)}$, considering all the faults in A .

Column *ntime* shows the normalized run time as follows. Let the run time for n -detection fault simulation of the initial test set be ρ_{base} . For the initial test set, the procedure also computes the sets $M(f)$ for the faults in U . Let the total run time for the initial test set be ρ_{init} . Let the total cumulative run time of the improvement procedure be ρ_{improv} . The normalized run time is ρ_{init}/ρ_{base} in the first row, and $\rho_{improv}/\rho_{base}$ in the second row.

4.3 Discussion

The following points can be seen from Tables 2 and 3. In several cases, the test set improvement procedure increases the average number of incomplete tests for a fault in A . In most of the cases, it reduces the average number of missing assignments.

The improvement in the quality of the test set can be seen from these parameters, as well as the average number of detections of detectable faults. This number increases even though the procedure does not target it directly. The importance of increasing the numbers of detections of target faults to improve the ability of a test set to detect defects is discussed in References [30–35].

As may be expected, when the 1-detection test set is extended with tests for gate-exhaustive faults, or when the 10-detection test set is used, the test set has larger numbers of incomplete tests for undetectable faults, and fewer missing assignments. Nevertheless, the improvement procedure is able to improve these test sets as well.

The normalized run time for the initial test set shows that the computation of the numbers of missing assignments for the undetectable faults has a negligible contribution to the run time. This is important for making it possible to use this metric during test generation. Overall, the normalized runtime does not increase with the size of the circuit. This implies that the procedure scales similar to a fault simulation procedure. To further demonstrate this point, Figure 5 shows the normalized run time as a function of the number of primary inputs for the one-detection test set.

The normalized run time is higher for the 10-detection test set because of the larger number of tests that the procedure considers for modification. The normalized runtime is also higher when the last modified test is closer to the beginning of the test set, implying that more tests are considered for modification. This occurs, for example, for *b14* and *tv80*.

4.4 Bridging Faults

As an additional metric for the quality of the test set, a bridging fault coverage is computed in this section. Bridging faults are not targeted by the test sets or procedure in this article. Therefore, bridging faults can be used for representing unmodeled faults, and defects.

The four-way bridging fault model is used in this section. A four-way bridging fault is associated with two lines, g and h , and a value $a \in \{0, 1\}$. In the presence of the fault, the value a on line h dominates the value of line g . For every line g in the circuit, and every value $a \in \{0, 1\}$, eight lines are selected randomly as the line h that dominates g , and eight bridging faults are defined. Feedback bridging faults are avoided.

The set of bridging faults defined in this way is simulated under the one-detection test set, the improved one-detection test set, the ten-detection test set, and the improved ten-detection test set. The numbers of tests and bridging fault coverages are shown in Table 4 for circuits where all four test sets are available.

Table 2. Experimental Results (ISCAS-89)

circuit	pi	ts	tests	mod	f.c.	16det	undet	act	targ	16act	miss	ntime
s1423	91	1	38	—	99.076	8.422	14	14	—	12.000	1.601	1.05
s1423	91	1	38	11	99.076	8.550	14	14	13	12.000	1.548	65.05
s1423	91	1G	41	—	99.076	8.718	14	14	—	12.214	1.588	1.00
s1423	91	1G	41	14	99.076	8.865	14	14	13	12.214	1.539	61.58
s1423	91	10	269	—	99.076	15.376	14	14	—	16.000	1.143	1.05
s1423	91	10	269	3	99.076	15.538	14	14	2	16.000	1.089	586.77
s5378	214	1	111	—	99.131	11.442	40	30	—	13.300	1.252	1.01
s5378	214	1	111	5	99.131	11.691	40	30	25	13.433	1.188	227.93
s5378	214	1G	117	—	99.131	11.656	40	30	—	13.467	1.234	1.01
s5378	214	1G	117	5	99.131	11.922	40	30	24	13.600	1.174	242.12
s5378	214	10	992	—	99.131	15.512	40	30	—	15.967	1.043	1.03
s5378	214	10	992	883	99.131	15.525	40	30	11	16.000	1.000	227.88
s9234	247	1	143	—	93.475	9.280	452	446	—	15.265	1.890	1.00
s9234	247	1	143	3	93.475	9.554	452	446	359	15.271	1.845	294.72
s9234	247	1G	214	—	93.475	10.379	452	446	—	15.343	1.614	1.01
s9234	247	1G	214	0	93.475	10.678	452	446	319	15.350	1.570	444.94
s9234	247	10	1,132	—	93.475	15.145	452	447	—	15.767	1.107	1.01
s9234	247	10	1,132	42	93.475	15.298	452	447	150	15.781	1.032	2,225.25
s13207	700	1	238	—	98.462	11.409	151	145	—	14.469	1.152	1.00
s13207	700	1	238	2	98.462	11.830	151	145	53	15.690	1.082	507.82
s13207	700	1G	305	—	98.462	12.079	151	145	—	15.945	1.159	1.01
s13207	700	1G	305	11	98.462	12.435	151	145	43	16.000	1.034	592.99
s13207	700	10	2,341	—	98.462	15.134	151	145	—	15.834	0.943	1.01
s13207	700	10	2,341	7	98.462	15.165	151	145	15	15.855	0.911	4,774.37
s15850	611	1	118	—	96.682	11.183	389	365	—	14.227	1.560	1.00
s15850	611	1	118	0	96.682	11.597	389	365	196	14.293	1.359	271.19
s15850	611	1G	302	—	96.682	13.284	389	365	—	15.918	1.344	1.01
s15850	611	1G	302	1	96.682	13.623	389	365	106	15.918	1.104	624.61
s15850	611	10	983	—	96.682	15.441	389	365	—	15.449	1.203	1.01
s15850	611	10	983	4	96.682	15.504	389	365	84	15.449	1.098	2,094.26
s35932	1,763	1	20	—	89.809	4.893	3,984	3,983	—	7.475	1.998	1.00
s35932	1,763	10	129	—	89.809	15.188	3,984	3,984	—	16.000	1.362	1.00
s35932	1,763	10	129	0	89.809	15.296	3,984	3,984	2,309	16.000	1.328	365.29
s38417	1,664	1	103	—	99.471	11.462	165	152	—	14.099	1.582	1.00
s38417	1,664	1	103	0	99.471	12.022	165	152	125	14.099	1.516	330.20
s38417	1,664	10	784	—	99.471	15.586	165	152	—	16.000	1.147	1.01
s38417	1,664	10	784	4	99.471	15.678	165	152	27	16.000	1.061	1,725.92
s38584	1,464	1	142	—	95.852	11.038	1,506	1,363	—	11.014	1.526	1.00
s38584	1,464	1	142	1	95.852	11.518	1,506	1,363	1,029	11.066	1.501	335.06
s38584	1,464	10	1,191	—	95.852	15.676	1,506	1,363	—	15.996	1.444	1.00
s38584	1,464	10	1,191	181	95.852	15.770	1,506	1,363	796	15.996	1.442	2,175.88

Table 3. Experimental Results (ITC-99 and IWLS-05)

circuit	pi	ts	tests	mod	f.c.	16det	undet	act	targ	16act	miss	ntime
b04	78	1	44	—	99.851	9.029	2	2	—	16.000	3.031	1.00
b04	78	1	44	3	99.851	9.421	2	2	2	16.000	2.562	95.75
b04	78	1G	47	—	99.851	9.260	2	2	—	16.000	2.594	1.06
b04	78	1G	47	10	99.851	9.622	2	2	2	16.000	2.219	85.83
b04	78	10	348	—	99.851	15.518	2	2	—	16.000	1.000	1.07
b05	36	1	61	—	95.209	8.671	87	74	—	13.743	2.226	1.02
b05	36	1	61	1	95.209	8.728	87	74	84	13.770	2.128	125.97
b05	36	1G	83	—	95.209	9.961	87	74	—	13.946	1.948	1.02
b05	36	1G	83	6	95.209	10.049	87	74	80	13.959	1.846	158.64
b05	36	10	514	—	95.209	15.548	87	74	—	16.000	1.329	1.07
b05	36	10	514	41	95.209	15.696	87	74	32	16.000	1.232	1,062.10
b11	38	1	59	—	99.174	7.878	9	7	—	10.571	2.738	1.00
b11	38	1	59	22	99.174	8.032	9	7	9	10.571	2.729	78.54
b11	38	1G	68	—	99.174	8.312	9	7	—	11.714	2.411	1.00
b11	38	1G	68	12	99.174	8.563	9	7	9	11.714	2.384	115.19
b11	38	10	572	—	99.174	14.912	9	7	—	16.000	1.589	1.07
b11	38	10	572	88	99.174	15.056	9	7	7	16.000	1.527	1,063.40
b14	280	1	332	—	94.870	10.607	512	472	—	15.015	2.241	1.01
b14	280	1	332	2	94.880	10.746	511	471	373	15.025	2.068	704.99
b14	280	1G	656	—	95.241	11.449	475	435	—	14.963	1.590	1.01
b14	280	1G	656	6	95.291	11.659	470	430	248	14.965	1.440	1,360.53
b14	280	10	3,058	—	94.870	15.490	512	463	—	15.577	1.397	1.02
b14	280	10	3,058	0	95.131	15.519	486	437	159	15.586	1.208	5,379.47
b15	483	1	393	—	98.445	11.014	312	295	—	14.264	1.445	1.00
b15	483	1	393	27	98.445	11.546	312	296	187	14.412	1.405	723.99
b15	483	1G	418	—	98.445	11.135	312	295	—	14.342	1.435	1.00
b15	483	1G	418	7	98.445	11.673	312	296	177	14.601	1.390	837.41
b20	527	1	302	—	94.056	11.115	1,342	1266	—	15.115	2.559	1.00
b20	527	1	302	0	94.056	11.264	1,342	1266	1,037	15.122	2.487	660.99
tv80	372	1	489	—	99.329	10.787	105	95	—	15.895	2.236	1.00
tv80	372	1	489	15	99.329	11.148	105	95	88	15.895	2.154	974.35
tv80	372	10	4,095	—	99.348	15.487	102	92	—	16.000	1.424	1.00
tv80	372	10	4,095	428	99.361	15.588	100	90	50	16.000	1.344	7,580.50

In most of the cases, the improved test set achieves a higher bridging fault coverage than the initial test set to which the improvement procedure is applied. This is consistent with the increase in the average number of detections reported in Tables 2 and 3. The improvement is not large, because the number of tests is not increased, and the initial bridging fault coverage is already high. Nevertheless, it provides further evidence to the improvement in the quality of the test set and its ability to detect defects.

5 CONCLUDING REMARKS

This article defined the concept of an incomplete test that is relevant when a target fault is undetectable. An incomplete test activates the fault, but fails to detect it because of one or more

Fig. 5. Normalized runtime.

Table 4. Bridging Fault Coverage

circuit	1-det			10-det		
	tests	init	improv	tests	init	improv
s1423	38	85.594	86.028	269	94.805	95.235
s5378	111	90.099	90.449	992	97.560	97.575
s9234	143	85.957	86.350	1,132	95.677	95.696
s13207	238	88.667	89.227	2,341	97.375	97.318
s15850	118	90.756	91.179	983	97.728	97.650
s38417	103	93.698	94.260	784	99.449	99.486
s38584	142	85.048	85.691	1,191	92.805	92.822
b05	61	83.733	83.876	514	93.702	93.980
b11	59	76.108	76.293	572	82.375	82.925
b14	332	82.923	83.102	3,058	87.916	88.057
tv80	489	83.734	84.092	4,095	91.026	91.269

assignments that are missing from the test. A smaller number of missing assignments makes it more likely that detectable defects, which should have been detected by a test for the undetectable fault, will be detected. Based on this concept, the article described a procedure that improves the quality of a test set in a way that fits with the rationale for computing an n -detection test set. For a constant $n \geq 1$, the procedure attempts to ensure that every undetectable fault has n incomplete tests with the smallest possible numbers of missing assignments. The computation of missing assignments was performed in linear time by avoiding fault simulation and considering all the undetectable faults simultaneously. Experimental results were presented to demonstrate the extent to which a given test set can be improved without increasing the number of tests. Both one-detection and ten-detection test sets were considered, as well as one-detection test sets that are extended with tests for gate-exhaustive faults.

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