Random Walk Based Sampling for Load Balancing in Multi-Server Systems

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ABSTRACT

In multi-server systems, a classical job assignment algorithm works as follows: at the arrival of each job, pick d servers independently and uniformly at random and send the job to the least loaded server among the d servers. This model is known as the *power-of- d choices* algorithm. In this paper, we analyze a variant of this algorithm, where d servers are sampled through d independent non-backtracking random walks on a k -regular graph. The random walkers are periodically reset to independent uniform random positions. Under some assumptions on the underlying graph, we show that the system dynamics under this new algorithm converges to the solution of a deterministic ordinary differential equation (ODE), which is the same ODE as the classical *power-of- d choices*. We also show that the new algorithm stabilizes the system, and the stationary distribution of the system converges to the stationary solution of the ODE. The new scheme can be considered as a derandomized version of *power-of- d choices* as it reduces the use of randomness while maintaining the performance of *power-of- d choices*.

CCS CONCEPTS

• **Mathematics of computing** → **Probability and statistics**; *Discrete mathematics*; • **Theory of computation** → *Randomness, geometry and discrete structures*;

KEYWORDS

Queueing Theory; Random Walks; Load Balancing; Martingale Concentration Inequality; Fluid Limit Approximation

ACM Reference Format:

Dengwang Tang and Vijay G. Subramanian. 2019. Random Walk Based Sampling for Load Balancing in Multi-Server Systems. In *ACM SIGMETRICS / International Conference on Measurement and Modeling of Computer Systems (SIGMETRICS '19 Abstracts)*, June 24–28, 2019, Phoenix, AZ, USA. ACM, New York, NY, USA, 2 pages. <https://doi.org/10.1145/3309697.3331506>

1 INTRODUCTION

In the era of Big Data, the computing systems handling the computations typically consist of multiple servers. These computing systems are able to handle multiple jobs at the same time. As the

scale of such systems grows larger and larger, load balancing algorithms, which determine how to route jobs to servers, require careful design. It is well known that the Join-the-Shortest-Queue (JSQ) Algorithm is optimal in several settings [4]. However, JSQ is not practical since it has a huge messaging cost. Another classical load balancing algorithm, called the *power-of- d choices* algorithm [9, 10, 14], is as follows: At each job arrival, pick d servers independently and uniformly at random, and dispatch the job to the server with the shortest queue. There has been significant work on this algorithm that we briefly outline below. It was shown that, when $d \geq 2$, the queuing delay can be reduced significantly compared to the case where $d = 1$ [7–10, 14]. Subsequent works proposed and analyzed several variants of *power-of- d choices* model. Mitzenmacher et al. proposed a scheme where the dispatcher keeps m servers in memory at the same time of probing d random servers [11]. The fluid limit approximation result for this scheme is established in [6]. Yang, Srikant, and Kang [15] analyzed a variant of *power-of- d choices* for batch arrivals, where slightly more than 1 servers is sampled for each task in a job. Mukherjee, Borst and Leeuwarden [12] and Budhiraja, Mukherjee, and Wu [3] analyzed a scheme where d random servers are probed through a random vertex and its neighbors. Many variants of *power-of- d choices* algorithms are also analyzed in the simplistic and static ball-in-bins model, where the maximum load, or the number of balls in the fullest bin, is analyzed. See [13] for a list of references.

In many of the schemes described above, the sampling is via independent uniformly random variables, which uses $\Theta(\log(n))$ randomness, and hence would be considered inefficient in the computer science literature [5]. In the same literature, random walks on graphs have been suggested as an efficient way of sampling [5]. Alon et al. analyzed the number of visits a non-backtracking random walk on an high-girth expander graph [1] [2]. The result shows that a sample path of a non-backtracking random walk on certain graphs has very similar behaviour to independent random choices in terms of statistics in a time window of certain length, while simple random walks do not share this property. With this as inspiration, in this paper we will analyze a variant of *power-of- d choices* algorithm based on random walks on graphs. More specifically, based on our recent work in [13], in which three variants of *power-of- d choices* are analyzed in balls-in-bins model, we seek to investigate the performance of these schemes in queueing system models. With the dynamic nature of the queueing model owing to service completions, in contrast to the static setting of the balls-in-bins model, it is not clear whether the dependent sampling of random walks will still yield good performance. One of the schemes proposed in [13] is as follows: assuming that n servers are connected through a high-girth k -regular graph G , d bins are

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SIGMETRICS '19 Abstracts, June 24–28, 2019, Phoenix, AZ, USA

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ACM ISBN 978-1-4503-6678-6/19/06.

<https://doi.org/10.1145/3309697.3331506>

sampled using d independent non-backtracking random walks on G , where the random walkers are reset to independent uniform random positions periodically. We will call this scheme the *Non-Backtracking Random Walk with Restart based Power-of- d -choices* (NBRWR-Pod) algorithm. In this paper, we analyze the NBRWR-Pod algorithm in the standard low traffic queuing system setting of load $\rho < 1$. The NBRWR-Pod algorithm can be considered as a derandomization of *power-of- d* algorithm, since it achieves the same performance of *power-of- d* choices while using less randomness.

The performance characterization of the NBRWR-Pod algorithm is presented via the following results:

- (1) We analyze the NBRWR-Pod algorithm with the method of *fluid-limit approximation*. We show that the system dynamics for a large-scale multi-server system up to any finite time $T > 0$ can be approximated by the solution to a deterministic ordinary differential equation (ODE), which is the same ODE for the regular *power-of- d* choices.
- (2) We show that for any finite n , the NBRWR-Pod algorithm stabilizes the system.
- (3) We show that the stationary distribution of the queue lengths under NBRWR-Pod algorithm converges to the stationary solution of the limiting differential equation in L_1 .

2 MODEL

We consider following standard queuing system model: jobs arrives according to a Poisson process with rate λn where $\lambda < 1$ is a constant. There are n servers/queues in the system. At the time of an arrival, the dispatcher samples the queues of a subset of servers and dispatches the arrival to one of the sampled queues to wait for service. The service time for each job at each server is *i.i.d.* exponentially distributed with mean 1. The reset period for the NBRWR-Pod scheme is set to be $\lfloor c \log n \rfloor$, where c is a positive constant.

In order for the NBRWR-Pod dispatching scheme to perform well, we need the following assumption on the graph G :

Assumption 1 (G is of high girth). The girth of the graph G is greater than or equal to $2\lceil \alpha \log_{k-1} n \rceil + 1$ for some constant $\alpha > 0$.

3 MAIN RESULTS

The following results are established under the above model and NBRWR-Pod scheme.

THEOREM 1. Consider the dynamic system $\mathbf{x}(t) \in [0, 1]^{\mathbb{Z}_+}$ described by the following differential equations:

$$\begin{aligned} \frac{dx_i}{dt} &= \lambda(x_{i-1}^d - x_i^d) - (x_i - x_{i+1}) \quad i \geq 1 \\ x_0(t) &\equiv 1 \end{aligned} \quad (1)$$

Let $\mathbf{X}^{(n)}(t) = (X_i^{(n)}(t))_{i \in \mathbb{Z}_+}$ be an infinite dimensional vector, where $X_i^{(n)}(t)$ is the proportion of queues with length exceeding (or equal to) i at time t . Suppose that (a) $\mathbf{X}^{(n)}(0)$ is deterministic; (b) $\lim_{n \rightarrow \infty} \|\mathbf{X}^{(n)}(0) - \mathbf{x}(0)\|_1 = 0$; (c) $\|\mathbf{x}(0)\|_1 < +\infty$, then for every finite $T > 0$

$$\lim_{n \rightarrow \infty} \sup_{0 \leq t \leq T} \|\mathbf{X}^{(n)}(t) - \mathbf{x}(t)\|_1 = 0 \quad \text{a.s.}$$

THEOREM 2. Let $Q_j^{(n)}(t)$ be the length of queue j at time t . For any initial queue length $Q^{(n)}(0)$, $Q^{(n)}(t) \xrightarrow{t \rightarrow \infty} \hat{Q}^{(n)}$ for some random vector $\hat{Q}^{(n)}$.

THEOREM 3. $\lim_{n \rightarrow \infty} E[\|\hat{\mathbf{X}}^{(n)} - \hat{\mathbf{x}}\|_1] = 0$, where $\hat{\mathbf{x}}$ is the unique fixed point of (1).

Given the theorem, we can characterize the expected queuing delay under NBRWR-Pod.

COROLLARY 1. The average waiting time for a job in a stationary system of n servers converges to $\sum_{i=1}^{\infty} \lambda^{\frac{i-d}{d-1}}$ as n goes to infinity.

4 CONCLUSIONS

In this paper we analyzed the performance of a new load balancing algorithm for the balls-in-bins problem proposed in [13], in the context of multi-server queuing systems. The new algorithm is a variant of the classical *power-of- d* choices algorithm, which replaces uniform independent sampling with random walks on k -regular graph based sampling. We show that this new algorithm yields a similar behavior as *power-of- d* choices for large scale systems. Hence, the new algorithm can be considered as a derandomization of *power-of- d* choices.

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