Limits on All Known (and Some Unknown) Approaches to Matrix Multiplication

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ABSTRACT

In this talk we will consider the known techniques for designing Matrix Multiplication algorithms. The two main approaches are the Laser method of Strassen, and the Group theoretic approach of Cohn and Umans. We define generalizations that subsume these two approaches: the Galactic and the Universal method; the latter is the most general method there is. We then design a suite of techniques for proving lower bounds on the value of ω , the exponent of matrix multiplication, which can be achieved by algorithms using many tensors T and the Galactic method. Some of our techniques exploit ‘local’ properties of T , like finding a sub-tensor of T which is so ‘weak’ that T itself couldn’t be used to achieve a good bound on ω , while others exploit ‘global’ properties, like T being a monomial degeneration of the structural tensor of a group algebra.

The main result is that there is a universal constant $\ell > 2$ such that a large class of tensors generalizing the Coppersmith-Winograd tensor CW_q cannot be used within the Galactic method to show a bound on ω better than ℓ , for any q . We give evidence that previous lower-bounding techniques were not strong enough to show this.

The talk is based on joint work with Josh Alman, which appeared in FOCS 2018. More recently, Alman showed how to extend our techniques, showing that with some work they apply to the Universal method as well. In particular, Alman shows that the Coppersmith-Winograd tensor cannot yield a better bound on ω than 2.16805 even using the Universal method.

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