Optimal Auctions vs. Anonymous Pricing: Beyond Linear Utility *

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In Bayesian mechanism design, a central question “*simple versus optimal*” focuses on how well simple mechanisms can approximate the revenue of the optimal mechanism in complex environments. Optimal mechanisms for asymmetric agents are usually both difficult to derive and implement. For general models (e.g., budgeted agents, risk-averse agents), either no closed form characterization is known in the literature, or the optimal mechanisms take the model too literally and are both fragile and impractical. On the other hand, some simple mechanisms (e.g., posting an anonymous price, second-price auction with anonymous reserve) are prevalent broadly.

Alaei et al. [2018] study the anonymous pricing mechanism where an anonymous price is posted for selling a single good, and the first agent who values the good above the price will buy the good. They upper bound the optimal revenue by considering the ex ante relaxation and derive a tight ϵ -approximation bound for independent but non-identical agents with linear utility and regular valuation distributions. A natural question motivating our work is: *Does the approximate optimality of anonymous pricing generalize to agents with non-linear utility under a suitable generalization of the regularity assumption?*

Regularity is a common assumption in mechanism design that simplifies the derivation of the optimal mechanism and enables approximation mechanisms for agents with linear utility. Fixing any class of mechanisms and a single agent, the *revenue curve* is a mapping from a constraint q on the ex ante probability of sale, over randomness in the agent’s type and the mechanism, to the revenue of the optimal mechanism with the ex ante constraint. Specifically, the *price-posting revenue curve* P is generated by fixing mechanism class to all price-posting mechanisms; and the *ex ante revenue curve* R is by fixing mechanism class to all possible mechanisms. Regularity for agents with linear utility is defined as the equivalence of the price-posting revenue curve and the ex ante revenue curve. These two revenue curves are sufficient to pin down the revenue from anonymous pricing, and ex ante relaxation, respectively.

Following the perspective of Alaei et al. [2013], the methods of our paper can be viewed as reductions in the following two senses. First, we approximately reduce the analysis of revenue

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Fig. 1. The left hand side is (α, β) -close for price posting: P is at least R/α on $[0, 1/\beta]$, i.e., the black thin line is above the black dotted line. The right hand side is ζ -close for ex ante optimization: P is concave, $P' \geq R/\zeta$ with $P'(q) = \max_{q' \leq q} P(q)$, i.e., the gray dashed line is above the black dotted line.

public budget		independent private budget		capacitated utilities
regular, decreasing density	regular	value regular, budget MHR	value regular, budget exceeds expectation w.p. at least $1/\kappa$	regular, support $[0, \bar{v}]$, capacity at least $\bar{v}/\eta$
e	$2e$	$3e$	$\sqrt{2(2 + \kappa)(1 + \kappa)} e$	$(2 + \ln \eta) e$

Table 1. Approximation bounds for anonymous pricing with asymmetric agents. Capacitated utilities are an extreme model of risk aversion considered by Fu et al. [2013].

of anonymous pricings for non-linear-utility agents to the analysis of revenue of anonymous pricings for linear-utility agents. Thus, relative to anonymous pricings, non-linear agent models can be considered well approximated by linear agent models. Second, our analysis reduces the multi-agent question of approximation by an anonymous price to a collection of single-agent approximation questions. These single-agent approximation questions ask whether or not the price-posting revenue curve is a good approximation to the ex ante revenue curve, i.e., whether an *approximate regularity* holds. Relative to Alaei et al. [2013], the single-agent problem to which we reduce gives simpler mechanisms.

Our approximation ratios for anonymous pricing under various models are summarized in Table 1. These results follow from analyses of the approximate regularity of each model (Figure 1). In the full paper, we also provide examples showing that without our assumptions, neither are the single-agent problems approximately regular nor is anonymous pricing a constant approximation for the multi-agent problem.

As an example, our reduction framework applied to agents with private budgets, regular valuation distributions, and MHR budget distributions is as follows. We show that under these assumption, the price posting revenue curve is concave and the agents are 3-close for ex ante optimization (Figure 1, right). It follows that the ex ante revenue on $\{P_i\}_{i=1}^n$ is a 3-approximation to the ex ante revenue on $\{R_i\}_{i=1}^n$. From Alaei et al. [2018] with concavity of $\{P_i\}_{i=1}^n$, anonymous pricing on $\{P_i\}_{i=1}^n$ is e -approximation to the ex ante revenue on $\{P_i\}_{i=1}^n$. Combining, anonymous pricing is a $3e$ -approximation to the optimal ex ante revenue.

REFERENCES

- Saeed Alaei, Hu Fu, Nima Haghpahan, and Jason Hartline. 2013. The Simple Economics of Approximately Optimal Auctions. In *Proc. 54th IEEE Symp. on Foundations of Computer Science*. 628–637.
- Saeed Alaei, Jason Hartline, Rad Niazadeh, Emmanouil Pountourakis, and Yang Yuan. 2018. Optimal auctions vs. anonymous pricing. *Games and Economic Behavior* (2018).
- Hu Fu, Jason Hartline, and Darrell Hoy. 2013. Prior-independent auctions for risk-averse agents. In *Proceedings of the fourteenth ACM conference on Electronic commerce*. 471–488.