A Segmented Attractor Network for Neuromorphic Associative Learning

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ABSTRACT

This work describes a segmented attractor network that records memories across different sets of information. Unlike typical attractor networks that can associate any given inputs with one another, the attractor network presented here tracks information across multiple sets where each set possesses one or more features. Hyperparameter analysis is performed on the network's recall capability by altering factors such as the network's size, the number of memories within the network, the amount of information required to perform recall, and the amount of feedback a synapse can provide. The analysis done shows an improved hit rate when increasing the features per set and allows the network to obtain a memory capacity higher than other standard attractor networks.

CCS CONCEPTS

• **Computer systems organization** → Architectures → Other architectures → Neural networks

KEYWORDS

Attractor network, associative learning, neuromorphic, simulation

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1 INTRODUCTION

The concept of associative learning has been studied for decades as a method of forming memories by creating correlations between information [1]. One of the most popular methods of implementing associative memory is the attractor network [2], which is a neural network that often stores memories in the form of connections between neurons [2-4]. Attractor networks are able to store information in the form of patterns of neuron outputs within the network either firing or not firing, or firing at different magnitudes [2-4]. Different sets of neurons firing simultaneously correspond to different memories within the network, and can be encoded to represent different forms of external stimulus. Different types of attractor networks including the Hopfield network [3], Storkey network [5], and others [4,6-8] have been previously studied to understand these neuronal dynamics.

Many facets of attractor networks have been studied over the years, including the original implementation of the Hopfield net [3], the maximum capacity of networks [8], and other more application-specific studies [9-12]. One facet of attractor networks that must be studied in greater detail is their capability to recall memories across multiple sets of information where each set possesses one of many features it can exhibit at any given moment in time. This capability can be useful to perform tasks such as state estimation for systems possessing large arrays of sensor/analysis input to perform navigation or track a system's overall health. This paper proposes a novel architecture that standardizes sparsity for associative learning across multiple sets of information. The proposed design is suitable for both software and custom neuromorphic implementations.

2 ARCHITECTURE

A basic form of the segmented attractor network architecture is shown in Figure 1. The objective of this architecture is to be able to associate various features belonging to different sets to one another. The network consists of the standard core components of a neural network and is similar in design to Hopfield networks [3]. The key difference in the design proposed here is how portions of the network are segmented.

Figure 1: Basic example architecture of the segmented attractor network used in this work.

Within the architecture, the neurons are segmented into an array of sets defined as s where the scalar magnitude of s is defined as $y(t)$

$$s = \begin{bmatrix} s_1 \\ s_2 \\ \vdots \\ s_n \end{bmatrix}_{n \times 1}, \text{ where } |s| = y(t) = \begin{bmatrix} y_1(t) \\ y_2(t) \\ \vdots \\ y_n(t) \end{bmatrix}_{n \times 1} \quad (1),$$

where n represents the total number of sets within the network. In an application space, these sets can be used to define different types of incoming inputs such as sensor input for temperature, altitude, light intensity, air pressure, etc. Each sensing modality would belong to its own set. Each of these sets (s_i) then possess an array of neurons that describe all features of the set. For example, the features of a set describing temperature could be each neuron within the set describing a certain temperature value (10°C , 15°C , etc.). It is not necessary for each set to have the same number of features, nor is it necessary for the feature count of the sets to be the same as n . Any set within s is defined as

$$s_i = \begin{bmatrix} f_{i1} \\ f_{i2} \\ \vdots \\ f_{im} \end{bmatrix}_{m \times 1}, \text{ where } |s_i| = y_i(t) = \begin{bmatrix} y_{i1}(t) \\ y_{i2}(t) \\ \vdots \\ y_{im}(t) \end{bmatrix}_{m \times 1} \quad (2),$$

where f_{ij} denotes an individual feature within the set. The indexing term, i , denotes the set to which f_{ij} belongs, and therefore does not change from feature to feature within a set. The indexing term, j , differentiates each feature from one another within s_i . The term m represents the total number of features within s_i . As mentioned before, m does not have to be the same for each s_i . If m is different

for separate s_i , each m can be indexed as m_i . For simplicity in equation definition in this work, it is assumed that all m are equal. The architecture example shown in Figure 1 possesses two sets of neurons ($n = 2$) where each set has a feature count of three ($m = 3$). External sensory input is also provided to each feature within each set via an input line. The entire collection of external input provided to the network is defined by E

$$E = \begin{bmatrix} E_1 \\ E_2 \\ \vdots \\ E_n \end{bmatrix}_{n \times 1} \quad (3),$$

where the external inputs are segmented into subsets (E_i) that correspond to the same pattern of segmenting within s . Each individual set within E can be defined as

$$E_i = \begin{bmatrix} E_{i1} \\ E_{i2} \\ \vdots \\ E_{im} \end{bmatrix}_{m \times 1} \quad (4),$$

where the indexing term i once again indicates which set within E each E_{ij} belongs. For the architecture analyzed within this work, a one-hot encoding scheme was used for all E_{ij} 's within a given E_i . This environmental rule was defined as it was assumed only one value could be provided to the network at any given time per s_i since each s_i could correspond to basic input values such as temperature, altitude, air pressure, etc. if the network was being used in an application space such as state estimation based upon a set of given input sensor data. This rule does not have to hold true for all use-cases of the architecture, however. Within the artificial network analyzed here, E can be provided to the system as a simple numerical input. If the system was to be implemented in a hardware-based neuromorphic architecture, E could be provided to the system via a network of demultiplexers that receive their input from an array of sensors or other circuits analyzing the system's state.

Using these defined sets of features across all s_i , recurrent connections can be made between features of each s_i to form paths where associations between features can be made during use. These paths are defined as synapses ($w_{ik,jl}$). Each synapse within the network belongs to one of the synapse blocks (W_{ij}) within the block matrix of synapses W

$$W = \begin{bmatrix} W_{1,1} & \cdots & W_{1,n} \\ \vdots & \ddots & \vdots \\ W_{n,1} & \cdots & W_{n,n} \end{bmatrix}_{n \times n}, \text{ where } W_{i,j} = 0 \text{ if } i = j \quad (5),$$

where each W_{ij} within W corresponds to a set of synapses that connects set i with set j . A critical element of W_{ij} is the defined conditional where if $i = j$, all synapses within the W_{ij} matrix are forever fixed at 0. This rule defines this network's "empty" diagonal across s , which is similar to how a traditional Hopfield network possesses an empty diagonal [3]. However, this network's empty diagonal exists across all locations where each f_{ij} within a given s_i could associate with one another. This architecture design choice allows any feature within a given s_i to associate itself with any feature from another set, but not associate itself with any feature within s_i . This expanded empty diagonal helps define the

segments within the network and distinguish each set from one another during memory formation and recall. Within any specific $W_{i,j}$, the collection of synapses can be further broken down into individual synapses

$$W_{i,j} = \begin{bmatrix} w_{i1,j1} & \cdots & w_{i1,jm} \\ \vdots & \ddots & \vdots \\ w_{im,j1} & \cdots & w_{im,jm} \end{bmatrix}_{m \times m} \quad (6),$$

where ik refers to the post synaptic feature connected to the synapse and jl refers to the pre-synaptic feature connected to the synapse. With each $W_{i,j}$ being an m by m matrix of synapses, the total number of synapses within the network is defined by the equation

$$N = n^2 m^2 \quad (7),$$

where N represents the total number of synapses within the network.

3 MEMORY FORMATION AND RECALL

3.1 Memory Formation

The first step in utilizing the segmented attractor network is populating its memory space with memories that correspond to relations between different values of E_{ij} within each E_i firing simultaneously. To form a basic memory within the architecture, a sensory input value is provided to two or more E_{ij} 's where at least two E_{ij} 's actively being provided input do not belong to the same s_i . In terms of the generic network example being defined in this work, this input value is simply expressed as a value of one. This input of one along each E_{ij} causes its corresponding feature, f_{ij} , to fire with a given output magnitude, $y_{ij}(t)$. In this defined network, $y_{ij}(t)$ simply shares a 1:1 relationship to the input being provided to f_{ij} . This relationship could be made nonlinear depending on the use-case defined for the network. Once f_{ij} fires, any synaptic connections between features receiving both pre-synaptic and post synaptic stimulation simultaneously increases the weight of the synapse ($\uparrow w_{ik,jl}$) from its initial value of zero to a non-zero value (w_{ON}). This change in the synapse's weight is defined as $\Delta w_{ik,jl}$ and follows the given rule

$$\Delta w_{ik,jl} = \begin{cases} w_{ON}, & y_{jl}(t) * E_{ik} \geq 1 \\ 0, & y_{jl}(t) * E_{ik} < 1 \text{ or } w_{ik,jl} = w_{ON} \end{cases} \quad (8),$$

where $y_{jl}(t)$ is the recurrent input to synapse $w_{ik,jl}$ shown in Figure 1. The rule dictates that if $y_{jl}(t)$ and E_{ik} are both one simultaneously, $w_{ik,jl}$ is set to the value of w_{ON} . If $y_{jl}(t)$ or E_{ik} are less than one or $w_{ik,jl}$ is already set to w_{ON} , $w_{ik,jl}$ will not change. This rule defines each $w_{ik,jl}$ as a one of two binary values: zero or w_{ON} . This ruleset could be modified depending on the network's application to instead use a non-binary synaptic weight value if needed where $w_{ik,jl}$ can assume multiple discrete values or a continuous analog range of values.

A visual example of a memory being formed within the defined example network is found in Figure 2. In Figure 2, the inputs E_{12} and E_{21} are both simultaneously being provided with input, which therefore increase the weights of synapses $w_{21,12}$ and $w_{12,21}$.

Figure 2: A demonstration of how a memory is formed within the example architecture to associate the two features E_{12} and E_{21} .

A final note on memory formation within the network is that over time not reversing any weights from a value of w_{ON} to zero could lead to an over saturation of the network. Inhibition can be used to counteract this over saturation, however forgetting memories was not a primary focus of this work. Therefore, no inhibition was used at any point during network evaluation.

3.2 Memory Recall

Once the segmented attractor network is populated with memories, the architecture should be able to perform recall. This network's definition of recall refers to giving input to one or more E_{ij} 's that all belong to at least one memory already existing within the network. If all E_{ij} values for a given memory are provided to the network, the network should in theory always provide a correct state estimation (unless the network has become over saturated). If not all E_{ij} 's for a given memory are provided, the network must perform analysis to recall missing info.

The network follows an iterative process to determine its estimate for the correctly recalled state. To understand this process, $y(t)$ for the whole system must be defined and calculated. For all sets in the network, the output of all features is equal to

$$y(t) = P + E \quad (9),$$

where P represents the post synaptic feedback provided to all features and E still represents all external input provided to features. The term P can be defined as

$$P = Wy(t-1) \quad (10),$$

where $y(t-1)$ represents the output of all features at the previous iteration of the network $t-1$. W still represents all synaptic weights

within the network. When Equation 9 is fully expanded (including expanding W into its full nm by nm matrix form), one obtains

$$\begin{bmatrix} y_{11}(t) \\ y_{12}(t) \\ \vdots \\ y_{nm}(t) \end{bmatrix} = \begin{bmatrix} w_{11,11} & \cdots & w_{11,nm} \\ \vdots & \ddots & \vdots \\ w_{nm,11} & \cdots & w_{nm,nm} \end{bmatrix} \begin{bmatrix} y_{11}(t-1) \\ y_{12}(t-1) \\ \vdots \\ y_{nm}(t-1) \end{bmatrix} + \begin{bmatrix} E_{11} \\ E_{12} \\ \vdots \\ E_{nm} \end{bmatrix} \quad (11).$$

Once the magnitude for all $y_{ij}(t)$ has been found for given E_{ij} 's to the network, the network attempts to look for a maximum $y_{ij}(t)$ within each s_i . If the network finds a $y_{ij}(t)$ within each s_i that is larger than its counterparts, the network interprets that given set of maximum magnitude features as its state estimation. If any s_i is determined to have no single maximum $y_{ij}(t)$, all values of $y(t)$ are applied to the presynaptic connections of W and Equation 11 is calculated once again. This process continues until a maximum value for each s_i is found or until it is clear no maximum value can be found for a given s_i . A diagram of a simple example of this process can be seen in Figure 3.

Figure 3: A demonstration on how memories are recalled within the example architecture by recalling the association between E_{12} and E_{21} by only giving input to E_{21} .

When multiple memories exist within a network, more complex situations can arise when attempting to perform recall. For example, if two memories exist within the example architecture where one associates E_{11} and E_{21} with one another and the other associates E_{11} and E_{22} , if only E_{21} receives input to the network (Figure 4), which of the two features should be chosen for the proper recall state? If there is a true state to be recalled in this case (i.e. there is a correct answer to the hidden feature set), there is a 50% chance the correct one is guessed in this case if it's guessed randomly. The network doesn't simply outright guess to determine the recalled state, however. It will indeed randomly pick one of the two tied maximum $y_{ij}(t)$ values from the given s_i , but it will perform one intermediary step prior to determining the recalled state. The randomly chosen f_{ij} will have external input applied to it via its E_{ij} , and then the network will be reevaluated via Equation 11 to observe the maximum $y_{ij}(t)$ values once again. In the case of the example in Figure 4, this step really is not needed, but it is needed when performing state estimation on networks possessing higher amounts of sets where multiple memories exist. If the network could randomly guess between every set of tied $y_{ij}(t)$ values independently, it might accidentally guess a memory that was never

formed in the network if reevaluation is not performed. This process of randomly selecting one of multiple tied $y_{ij}(t)$ is needed in software implementations when all initial weights are zero but might not be needed in non-zero weight initialization implementations where all initial weights are random. It might not be needed in hardware implementation as well due to device/circuit variability from synapse to synapse giving the synapses naturally-occurring initial random weights.

Figure 4: Example showing how the architecture needs to reevaluate at times when recalling memories and multiple features of the same set have similar magnitudes.

When performing analysis on larger networks with more memories within them, the architecture must be able to successfully recall despite having a higher complexity. Often, this requires the network to perform the iterative summation process of solving for the recalled state described previously. This process is akin to how other attractor networks converge to a normal answer by navigating through a set of network states [2,4]. In this architecture's case, the states are instead the value of each feature's output magnitude at the end of each iteration. A basic example of a network having to iterate multiple times can be seen in Figure 5.

In the example shown in Figure 5, the network has a total of three sets ($n=3$) with two features per set. The value of w_{ON} for this network is set as 0.1. In this network, three memories have already been created. The memories within the network are as follows:

Memory #1: E_{11}, E_{21}, E_{32} are associated

Memory #2: E_{11}, E_{21}, E_{31} are associated

Memory #3: E_{11}, E_{22}, E_{31} are associated

In Figure 5(a), the network is provided with input at E_{11} . The network's job is to recall the memory it is most likely currently observing based upon its input provided. Since E_{11} is associated with all three memories within the network (along with other features such as E_{21} and E_{31} each belonging to multiple memories), it must perform multiple feedback iterations in the network to solve for the recalled state. Initially, the external input at E_{11} causes the

feature f_{11} to have an output magnitude of one. This output is the only output to have a non-zero magnitude at this point in time. Others will receive input from feedback on the following iterations, however.

Once f_{11} receives input and emits its output, its output then travels through all activated synapses with which it's connected (black synapses) and provides a feedback signal to any associated features (Figure 5(b)). In this case, that is f_{21} , f_{22} , f_{31} , and f_{32} . The magnitude of each synapse's feedback to its post synaptic neuron will be a product of w_{ON} and the magnitude of its pre-synaptic value, as dictated by Equation 11. For this iteration, the feedback provided by each activated synapse will be 0.1, since f_{11} has a magnitude of one and $w_{ON} = 0.1$. Each of the features receiving feedback now have an output magnitude of 0.1. A final answer still hasn't been found yet, however. The network must iterate once again to find its solution.

On this iteration for the network, feedback will be calculated in accordance to the outputs that received a feedback signal on the previous iteration. f_{21} , f_{22} , f_{31} , and f_{32} will provide feedback to neurons within the network via any connected, activated synapses (Figure 5(c)). Each of these features has a magnitude of 0.1 and will pass feedback to the network's features once again as a product of their magnitude and w_{ON} . On this iteration, each synapse will provide feedback of 0.01 since $y_{ji}(t)w_{ik,jl} = 0.01$. Due to the configuration of activated synapses within the network, f_{11} , f_{21} , f_{22} , f_{31} , and f_{32} will all receive feedback on this iteration of the network. f_{11} will receive feedback of 0.04, since it's receiving input from four activated synapses. f_{21} and f_{31} will both receive feedback of 0.02 since they both are connected to two activated synapses previously receiving feedback. f_{22} and f_{32} will only receive feedback of 0.01 since they are each only connected to a single activated neuron that is previously receiving feedback.

At this point in the analysis, three clear winning features (one per set) have emerged from the network. f_{11} has a clear winning output amongst the first set since it was receiving external input. f_{21} and f_{31} have been evaluated as higher values from the other two sets within the network with respect to their counterparts f_{22} and f_{32} . With this answer, the network has converged to the estimate of 11, 21, and 31 being the three features that it's observing given only external input for 11. This estimate corresponds to the second memory given to the network, which associated E_{11} , E_{21} , and E_{31} . This estimation is the most probabilistic solution from the network's perspective, as it has a 2/3 chance of being correct if it guesses feature 21 from the features available within s_2 and a 2/3 chance of being correct if it guesses feature 31 from the features in the s_3 . Since both of these most probable estimates also happened to share a common memory, the system naturally estimated this memory to be the most likely answer. If those two features didn't share a common memory, the system would most likely have to perform an educated random guess as was shown in Figure 4 to find its final answer.

Figure 5: (a) A network with three pre-populated memories is provided stimulus along E_{11} . This stimulus causes f_{11} to fire with a magnitude of 1. (b) Feedback from f_{11} propagates through any activated (black) synapses to other outputs within the network. This output is a product of the pre-synaptic output's magnitude and w_{ON} . (c) A final iterative pass is performed on the network by using the feedback from the outputs that just received feedback previously and passing it once again through any activated pre-synaptic connections. With this iteration of the network, one output from each set of neurons has emerged as a maximum value. These maximum values act as the network's most probable state estimation in accordance to the input it initially received and the memories that already existed within the architecture's synaptic connections.

4 SIMULATION RESULTS AND DISCUSSION

To simulate the attractor network, the simulation model was split into two parts: populating the network with memories and testing the network's recall capability once populated. Figure 6 portrays the full MATLAB simulation of the network from initialization of the network to analyzing the output during recall. To determine the hit rate during the second half of simulation, a random memory from the uniform distribution of memories across the network was chosen to be recalled. Before showing this memory to the network, a certain amount of non-zero E_{ij} 's given to the network would be hidden from the network (defined as I_{hid}). For example, if $n=10$ (therefore each memory associated ten E_{ij} 's) and $I_{hid}=1$, one of the ten E_{ij} values from each memory would not be shown to the network. This is analogous to how networks such as the Hopfield network define the Hamming distance between network states [3]. The network must then properly recall what these hidden values are in the form of $max(y(t))$. During simulation, many different hyperparameters were analyzed to observe the network's behavior including n and m , memories within the network, w_{on} , and I_{hid} .

Figure 6: Algorithm used for analyzing the network. It should be noted that all work shown here uses a test point amount of 200 and convergence was detected to a change of 0.0001.

The first analysis of the network looks at the network's hit rate as the network expands in size and the number of memories within the network increases. The results from this analysis can be seen in Figure 7. It should be noted that I_{hid} and w_{on} were fixed at 1 and 0.1, respectively, for all simulations in Figure 7. In addition, for simplicity in displaying results, n and m were set to equal values. Naturally, as the network's size increases, its capacity for maintaining high hit rates on larger amounts of memories increases. This increase in capacity is primarily due to two factors. The first is due to memories having more space to stay separate from one another in larger networks. This extra space decreases the possibility that memories share features. A similar type of behavior has been seen in other studies [13]. The second is that I_{hid} remains at a fixed value of one even for larger networks, which decreases the Hamming distance between the input and the original memory along the y-axis.

One feature of note within Figure 7 is the lighter blue tail that extends to higher memory counts from $n=m=6$ to $n=m=10$. The hit rate begins to increase for higher memory counts at this point, but then suddenly dies past 10 and 11 in the y-direction. The cause of this tail formation can be explained once one considers the value of w_{on} for all simulations shown in Figure 7. Once n increases to ~ 10 -11, synapses can override external input to a neuron. If a f_{ij} has a non-zero E_{ij} (meaning that the system is certain that feature is the correct one), synapses to another f_{ij} within that f_{ij} 's s_i can potentially override E_{ij} to the correct feature within the set. This problem has immediate negative impact on the network's hit rate at higher memory counts. This characteristic means that the product of the network's set count (minus one) and w_{on} is not allowed to exceed the magnitude of the external input provided to an individual feature. In other words,

$$E > w_{on}(n - 1) \quad (12).$$

Figure 7: Colormap showing the average hit rate of the segmented attractor network as the number of sets/features and memories within the network are increased when $I_{hid} = 1$ and $w_{on} = 0.1$. $n=m$ for all networks.

If the value of w_{on} is decreased to a value to ensure that Equation 12 holds true, the performance of the network for larger set/feature counts increases. This increase in performance can be seen in Figure 8. For the results in Figure 8, w_{on} was set to a value of 0.04. If the set and feature count was further increased past 20, a similar dip in performance would be seen in the network's performance as is seen in Figure 7. If this network was ever implemented in hardware, this limitation shows the network's size is limited by the magnitude of feedback the synaptic circuit/device provides.

Figure 8: Colormap showing the average hit rate of the segmented attractor network as the number of sets/features and memories within the network are increased when $I_{hid} = 1$ and $w_{on} = 0.04$. $n=m$ for all networks.

To expand on the study of w_{ON} , the size of the network can be fixed with w_{ON} increased to observe how it alters the network's hit rate. For this setup, a network of eight sets where each set contains eight features was chosen with $I_{hid}=1$. The value of w_{ON} was then increased from a value of 0.025 to 0.25 to observe how it affected hit rate. The results of this analysis are shown in Figure 9. A couple points can be drawn from what the figure shows. Firstly, at higher memory count values, a drop in the hit rate does occur past the point where $w_{ON} = 0.125$, which is where Equation 12 predicts a drop in the hit rate would occur. Secondly, the overall hit rate appears to decrease even at lower values of w_{ON} . This observed behavior of the network requires a slight adjustment of Equation 12 if the desire is to maximize hit rate.

$$E \gg w_{ON}(n-1) \quad (13).$$

According to Figure 9, the difference between a single synapse's feedback input to the neuron versus the input applied by the external stimulus should be much larger than previously shown. Equation 13 and Figure 9 show that w_{ON} should be made as small as feasibly possible when implementing this type of network.

Figure 9: Colormap showing the average hit rate of an $n=8$, $m=8$ network where $I_{hid}=1$ as the memories within the network and w_{ON} are increased.

Feedback provided from synapses is not the only parameter that alters the network's hit rate. One of the primary parameters that affects hit rate is the amount of information that is hidden from the system (I_{hid}). The effect of increasing I_{hid} was tested on a network with 20 sets of features where each set possessed 20 features. The value of w_{ON} was set to 0.04 to prevent feedback overriding external inputs. The results of I_{hid} being varied from zero to 19 can be seen in Figure 10. As can be naturally expected, the network's performance decreases as I_{hid} increases. High performance can still be attained at high values of I_{hid} as long as the memory count within the network remains low. This performance disappears quickly when I_{hid} approaches its highest possible values, however.

Figure 10: Colormap showing the average hit rate of a $n=20$, $m=20$ network as I_{hid} and the number of memories within the network are increased. The value of w_{ON} was set to 0.04.

The final hyperparameters to analyze are n and m . The values of n and m and their effect on the network were studied in two separate simulations. For the first simulation (Figure 11), n was fixed to ten with $I_{hid} = 5$. Values of $I_{hid} = 5$ and $n = 10$ sets the Hamming distance between memories and test points to the network to the value that other previously defined attractor networks use to evaluate memory capacity [3,5,8]. The value of m was then increased from two to 40. As m increases, the number of memories able to be perfectly recalled from within the network increases at a quicker pace than observed in Figure 7. According to Figure 11, a ten-set network with 40 features per set can store ~140-150 simultaneous memories near-perfectly. This increase in memory capacity is primarily due to the increased space within the feature space within each set. The increase in m increases the likelihood that memories have separate features from one another if each memory within the network has equal probability of occurring (assuming uniform memory distribution). This result shows that given a high enough m , the network's memory capacity can surpass previously defined networks such as the Hopfield network ($0.12n-0.15n$) [3,8,14] or Storkey's network ($0.3n$) [5,8], where n is the number of neurons within the network [15]. For this particular set count (10), the network surpasses the capacity of the Hopfield and Storkey networks when m reaches ~15 and ~33, respectively. At the highest feature per set count in Figure 11, the memory capacity reaches $\sim 0.375n$.

Figure 11: Colormap showing the average hit rate of the network as the features per set and memories within the network are increased. The values of w_{ON} , I_{hid} , and n were set to 0.01, 5, and 10, respectively.

If the set count within a network is changed in size while the feature count of each set remains fixed, different patterns in performance emerge from the network. In Figure 12, the number of features per set was fixed to a value of ten while the number of sets within the network was increased from two to 20. The value of I_{hid} remains at one. As the set count increases, the memory capacity of the network increases to an extent. Once the network reaches a set count of approximately seven, the capability of the network to performance perfect recall effectively stalls. Recall at lower probabilities does continue to increase until the set count reaches ten. After this point, the capability of recalling higher memory counts at lower probabilities begins to diminish. This performance decrease is likely due to the number of features within each set not increasing, which causes the amount of space the increasing amount of sets are having to share to become increasingly cramped. This smaller space increases the likelihood that memories share overlapping features, which can lead to incorrect recall and oversaturation of the network. The stagnation of perfect recall at higher set counts also likely contributes to the reason why the memory capacity of the largest network in Figure 8 (20 sets, 20 features per set) is less than the largest network in Figure 12. If this hypothesis is correct, it would also likely mean that the relationship between increasing the number of features per set and increasing the number of sets to maximize memory capacity is nonlinear.

Figure 12: Colormap showing the average hit rate of the network as n and memories within the network are increased. The values of w_{ON} , I_{hid} , and m are 0.04, 1, and 10, respectively.

5 CONCLUSION

This work analyzed a segmented attractor network that tracked associations between multiple sets of input where each set possessed its own set of features. A set of hyperparameters was altered to observe the architecture's hit rate and how it evolved with network size, number of memories within the system, the weight of synapses, and the amount of input hidden from the network during recall. The hyperparameter analysis revealed a list of behaviors that the network exhibits under different conditions. These behaviors are summarized in Table 1. These results show this type of attractor network is useful for any system that is using multiple types of input information to form associations between a large number of features. These associations can then be used to perform tasks such as state estimation when given partial input. If given enough features per set within the network, the network surpasses the memory capacity of other previously defined attractor networks.

Table 1: Summary of behaviors exhibited by the network studied.

Behavior	Source
The feedback provided from an individual synapse (w_{ON}) caps memory capacity.	Figures 7-8
Hit rate decreases if feedback from synapses can exceed the input provided to neurons that external input provides.	Figures 7-9, Equations 12-13
Increasing the amount of information hidden from the network (I_{hid}) decreases hit rate.	Figure 10
Increasing m increases hit rate.	Figure 11
Given high enough m , the memory capacity of the network surpasses both the Hopfield [3] and Storkey [5] networks.	Figure 11
Hit rate saturates and diminishes at higher memory counts if m is not increased while n increases.	Figure 12
Hit rate saturates for near-perfect memory recall if m is not increased while the set count increases.	Figure 12

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