A Stochastic Geometry Approach to Modeling Time Hopping Based TDOA in 3D Indoor localization

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ABSTRACT

Indoor localization has greatly leveraged applications regarding to location based service (LBS), which witnesses ever-increasing impact on human life. Time difference of arrival (TDOA) is one of the most widely used technique for indoor localization because of its low complexity and high positioning accuracy. However, the strong multiple access interference (MAI) is a crucial issue that affects the accuracy of TDOA in indoor positioning, in order to mitigate interference, we incorporate time-hopping (TH) in TDOA technology, which can effectively reduce the interference at the transmitting side. Particularly, we propose an analytical model to evaluate the performance of the TH based TDOA in 3D indoor scenario by using the tool of stochastic geometry. Through the proposed model, we obtain some easy-to-use expressions for key performance metrics. The analytical results include the average interference, the success probability (i.e. the probability of a base station (BS) that can successfully participate in the localization procedure), and the number of effective serving BSs that has successfully participated in. In addition, some special cases are analyzed in detail, which reveals interesting insights. All the analytical results in this paper are validated through Monte Carlo simulations.

CCS Concepts

•Networks → Network performance modeling; Network performance analysis; Network simulations; Physical topologies;

Keywords

indoor localization; time hopping; time difference of arrival; 3D stochastic geometry

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1. INTRODUCTION

Indoor localization is an important component of many applications in smart city like navigation in hospital, shopping mall navigation, fire emergency rescue, etc. It locates objects or people inside a building using radio waves, magnetic fields, acoustic signals, or other sensory information collected by mobile devices [1]. Due to building occlusion and other reasons, it is difficult for global positioning system (GPS) to achieve high-precision indoor positioning. Therefore, the solution with the characteristics of high reliability, high precision, easy installation and cheap integrated for indoor localization is imperatively needed.

A lot of techniques for indoor localization are proposed, such as methods based on received signal strength (RSS), time-of-arrival (TOA), time-difference-of-arrival (TDOA), or angle-of-arrival (AOA). Among all these techniques, TDOA has been widely used since it is easy to install and cheap to integrate. However, TDOA based indoor location is seriously disturbed by multiple access interference (MAI) [2]. To overcome the MAI problem, lots of researches have been done. In [3], the authors analyze the performance of general self correlation peak detection, and introduce a new method of anti-MAI self correlation peak detection, which can improve 15dB of the receiver's capability of anti MAI. Authors in [4] introduce a new initial acquisition and cell search algorithm with implicit MAI cancelation. However, all these researches focus on anti-MAI at the receiver (the object to be positioned), while it is also necessary to make improvements from the transmitter to reduce the effects of MAI. Therefore, time-hopping (TH) is introduced as an effective and easy implemented way to reduce MAI at the transmitter in this paper.

Stochastic geometry has been widely used as an analytical approach to model and quantify the key metrics in cellular networks for its mathematical tractability and modeling accuracy. In stochastic geometry analysis, positions of base stations (BSs) are modeled from certain stochastic point process [5]. Authors of [6] have proved the accuracy of stochastic geometry model, and they point out that the stochastic geometry model may better capture the increasingly opportunistic and dense placement of base stations in future networks. With the rapid development of economy and society, the evolution trend of future network is extending to indoor three dimensional (3D) space. In [7], a new 3D model for cellular network which considers the attenuation effect of floors and walls is proposed. The stochastic geometry is introduced to analyze the localization fundamentals in [8] and it

makes evident that localization performance is limited by the interference from the BSs participating in the localization procedure. However, the research in [8] is in 2D space. In our work, we give a specific analysis of TDOA technology for indoor localization in 3D space.

In this paper, we conduct a stochastic geometry analysis to model and evaluate the performance of indoor localization with TH based TDOA technology in 3D space. Firstly, we derive the expression of average interference to prove the proposed TH based TDOA is effective for reducing the interference. Then we define two performance metrics: the success probability and the average number of effective serving BSs, and the closed-form expressions of the two metrics are derived. Particularly, some special cases are researched which illustrate interesting insights. This paper is organized as follows. Section II describes the system model. The localization performances which include the average interference, success probability and the average effective serving BS number are analyzed in Section III. Simulation results are presented in Section IV. Finally, Section V concludes this paper.

2. SYSTEM MODEL

2.1 Network Model

The cellular network model consists of BSs are arranged according to a homogeneous Poisson point process (PPP) Φ of intensity λ in 3D Euclidean space $\mathbb{R}^3$. Fig. 1 has shown a sketch of the distribution of BSs.

Figure 1. A topology of 3D Poisson distributed base stations.

The standard power loss propagation model is used with path loss exponent α and $\alpha > 3$ [9]. Considering an indoor environment in 3D, we use the path loss model refined in [10], which takes into account the penetration loss of floors and walls. The signal frequency is assumed to be 2.4 GHz. In this case, the path loss is expressed in dB as follows

$$PL(r) = PL(r_0) + 10\alpha \log_{10}\left(\frac{r}{r_0}\right) + AF(r), \quad (1)$$

where r is the distance between the tagged receiver and tagged BS. $PL(r_0)$ is the path loss of the first meter ($r_0 = 1$ m), $PL(r_0) = 40$ dB at 2.4 GHz. $AF(r)$ is the average walls and floors attenuation factor [11], which is given in dB as

$$AF(r) = 10\log_{10}(ur), \quad (2)$$

where u is a constant attenuation per unit of path length. Based on equation (1-2), the value of the signal attenuation due to the path

loss can be presented as $Kr^{-(\alpha+1)}$, where K is a coefficient given by $K = r_0^\alpha / (u10^{PL(r_0)/10})$.

As far as random channel effects such as fading and shadowing, we assume that the tagged BS and tagged receiver experience only Rayleigh fading h and $h \sim \exp(1)$. In this case, the received power at the typical receiver at distance r from the tagged BS is

$$P_r = P_t \cdot h \cdot K \cdot r^{-(\alpha+1)}. \quad (3)$$

2.2 Time Hopping based TDOA

In the TDOA system, the transmitted pulses from spatially separated BSs arrive at the object to be positioned o_p at different times, and the absolute transmission time of the signal from each BS to o_p is measured to calculate the distance difference between each pair of BSs. The distance difference further convert into a hyperboloid. In 3D positioning, at least three pairs of BSs (four different BSs) are required. The position of o_p is the intersection of the corresponding hyperboloids.

By using time hopping, the channel interval is divided into n_{sf} subframes $\{SF_1, SF_2, \dots, SF_{n_{sf}}\}$, and the BSs are also divided into n_{sf} collections $\{C_1, C_2, \dots, C_{n_{sf}}\}$. All the BSs in collection C_i transmit in the corresponding subframe SF_i . The mechanism of the time hopping based TDOA in this paper can be depicted as follows

- A random mark, which is uniformly distributed between 1 and n_{sf} , is given to each BS at the beginning of every TH period. The BSs with mark i belong to C_i and will transmit on subframe SF_i . C_i follows PPP distribution of intensity $\lambda_i = \frac{\lambda}{n_{sf}}$ [5]. As shown in Fig. 2 and Fig. 1, the different colors represent different subframes and the BSs in the corresponding collections. There is no interference between BSs in different collections since their corresponding subframes are orthogonal.
- According to equation (3), the closer the BS to the receiver, the stronger the signal is, which means the location information will be more accuracy. In this case, we choose the BSs in each collection which are closest to the positioned object as the serving BSs. Then, in the 3D scenario, n_{sf} can not be less than 4.

Figure 2. Sketch of subframe allocation, $n_{sf} = 4$.

3. LOCALIZATION PERFORMANCE

Localization performance fundamentally depends on the number of serving BSs and their accuracies. These two factors are highly interdependent. Both the number of effective serving BSs whose signals arrive with sufficiently - high SINRs to participate in the localization procedure and the quality of their positioning observations are strongly correlated to the interference. Therefore, in this section, firstly, we will analyze the average interference, then the quality of the serving BSs' positioning observations is

evaluated by defining a metric, success probability. The success probability is defined as the probability that the signal from a serving BS is successfully detected. It can also be considered as the probability of a serving BS can successfully participate in the localization procedure. Finally, we calculate the number of effective serving BSs whose signals arrive with sufficiently - high SINRs. Since the positions of BSs in different collections are independent identically distributed, therefore, a typical collection denoted as C_0 is analyzed for representation. Assume there is a object o_p to be positioned, without loss of generation, we assume o_p is the origin.

3.1 Interference Analysis

According to the system model, the interference received at o_p from the BSs in collection C_0 can be expressed as

$$I_r = \sum_{BS_i \in C_0 \setminus BS_1} P_t h_i K r_i^{-(\alpha+1)}, \quad (4)$$

where BS_1 is the closest BS to the object, which also means BS_1 is the serving BS of collection C_0 , and the distance between BS_1 and o_p is denoted as r_1 . h_i and r_i are respectively the fading and distance between other BSs in C_0 (interference BSs) and o_p .

The average interference received at o_p can be calculated as [12]

$$E[I_r] = E_r \left[- \left| \frac{d}{ds} \mathcal{L}_{I_r}(s) \right|_{s=0} \right], \quad (5)$$

where $\mathcal{L}_{I_r}(s)$ is the Laplace transform of interference, and it can be calculated as follows

$$\begin{aligned} \mathcal{L}_{I_r}(s) &= E[e^{-sI_r}] \\ &\stackrel{(a)}{=} E_{C_0, h_1} \left[e^{-s \sum_{BS_i \in C_0 / BS_1} P_t K h_i r_i^{-(\alpha+1)}} \right] \\ &= E_{C_0, h_1} \left[\prod_{BS_i \in C_0 / BS_1} e^{-s P_t K h_i r_i^{-(\alpha+1)}} \right] \\ &= E_{C_0} \left[\prod_{BS_i \in C_0 / BS_1} E_{h_i} \left[e^{-s P_t K h_i r_i^{-(\alpha+1)}} \right] \right] \\ &\stackrel{(b)}{=} E_{C_0} \left[\prod_{BS_i \in C_0 / BS_1} \frac{1}{s P_t K r_i^{-(\alpha+1)} + 1} \right] \\ &= e^{-4\pi\lambda_0 \int_{r_1}^{\infty} \frac{s P_t K}{s P_t K + t_i^{(\alpha+1)}} t_i^2 dr_1}, \end{aligned} \quad (6)$$

where step (a) comes after plugging in the expression of I_r in equation (4). Step (b) follows the i.i.d. distribution of h_i . The last step follows from the probability generating functional (PGFL) [5] of the PPP, which states for some function $f(x)$ that $E[\prod_{x \in \Phi} f(x)] = \exp(-\lambda \int_{\mathbb{R}} (1 - f(x)) dx)$. The integration limits are from r_1 to ∞ since the closest interferer is at least at a distance r_1 from o_p . λ_0 in the last step represents the density of C_0 , and according to section I, $\lambda_0 = \frac{\lambda}{n_{sf}}$.

Therefore, the average interference can be derived as

$$\begin{aligned} E[I_r] &= E_{r_1} \left[- \left| \frac{d}{ds} \mathcal{L}_{I_r}(s) \right|_{s=0} \right] \\ &= E_{r_1} \left[\frac{4\pi\lambda_0 P_t K r_1^{-(\alpha-2)}}{\alpha-2} \right] \\ &= \int_0^{\infty} \frac{4\pi\lambda_0 P_t K r_1^{-(\alpha-2)}}{\alpha-2} f(r_1) dr_1 \end{aligned}$$

$$\begin{aligned} &\stackrel{(a)}{=} \int_0^{\infty} 4\pi r_1^2 \lambda_0 \frac{4\pi\lambda_0 P_t K r_1^{-(\alpha-2)}}{\alpha-2} e^{-\frac{4}{3}\pi r_1^3 \lambda_0} dr_1 \\ &\stackrel{(b)}{=} \int_0^{\infty} \frac{16\pi^2 \lambda_0^2 P_t K r_1^{-(\alpha-4)}}{\alpha-2} \cdot e^{-\frac{4}{3}\pi r_1^3 \lambda_0} dr_1, \end{aligned} \quad (7)$$

where step (a) is after expressing the probability density function (PDF) of r_1 , and step (b) comes from $\lambda_0 = \frac{\lambda}{n_{sf}}$. According to [5], the PDF of r_1 is expressed as

$$f_r(r_1) = 4\pi r_1^2 \lambda_0 \cdot e^{-\frac{4}{3}\pi r_1^3 \lambda_0}. \quad (8)$$

It can be proved that the average interference is a decreasing function with respect to the subframe number n_{sf} by calculating the partial derivative of the average interference with respect to n_{sf} , and $\frac{\partial E[I_r]}{\partial n_{sf}}$ is derived as follows

$$\begin{aligned} \frac{\partial E[I_r]}{\partial n_{sf}} &= \int_0^{\infty} \frac{16\pi^2 \lambda_0^2 P_t K r_1^{-(\alpha-4)}}{\alpha-2} \cdot e^{-\frac{4}{3}\pi r_1^3 \lambda_0} \\ &\quad \cdot \left(\frac{4}{3} \frac{\pi r_1^3 \lambda_0}{n_{sf}} - 2 \right) dr_1. \end{aligned} \quad (9)$$

Since $\frac{4}{3}\pi r_1^3 \lambda_0$ is the average number of BSs in volume of $\frac{4}{3}\pi r_1^3$ in 3D space [2], and r_1 is the closest BS, therefore $\frac{4}{3}\pi r_1^3 \lambda_0 = 1$, then, $\frac{\partial E[I_r]}{\partial n_{sf}} < 0$, which means the average interference is a decreasing function with respect to the subframe number n_{sf} .

3.2 Success Probability and Effective Serving BS Number Evaluation

Based on the definition of success probability, it is modeled as the probability of SINR higher than a pre-defined threshold γ , which is expressed as follows

$$p_s = P[\text{SINR} > \gamma]. \quad (10)$$

The SINR received at o_p is expressed as

$$\text{SINR} = \frac{P_t h_1 K r_1^{-(\alpha+1)}}{I_r + \sigma^2}, \quad (11)$$

where h_1 represents the channel fading between the serving BS BS_1 and the object o_p , and r_1 is the distance between them. I_r is the interference in equation (4), and σ^2 is the noise power.

By plugging equation (11) to equation (10), the success probability is calculated as

$$\begin{aligned} p_s &= P[\text{SINR} > \gamma | r_1] \\ &= E_{h_1, I_r} \left[\frac{P_t K h_1 r_1^{-(\alpha+1)}}{I_r + \sigma^2} > \gamma \middle| r_1 \right] \\ &= E_{I_r} \left[E_{h_1} \left[h_1 > \frac{\gamma(I_r + \sigma^2) r_1^{\alpha+1}}{P_t K} \right] \middle| r_1 \right] \\ &\stackrel{(a)}{=} E_{I_r} \left[e^{-\frac{\gamma(I_r + \sigma^2) r_1^{\alpha+1}}{P_t K}} \middle| r_1 \right] \\ &= \int_0^{\infty} e^{-\frac{\gamma \sigma^2 r_1^{\alpha+1}}{P_t K} \mathcal{L}_{I_r}} \left(\frac{\gamma r_1^{\alpha+1}}{P_t K} \right) f(r_1) dr_1 \\ &\stackrel{(b)}{=} \int_0^{\infty} 4\pi r_1^2 \lambda_0 \cdot e^{-\frac{\gamma \sigma^2 r_1^{\alpha+1}}{P_t K} \mathcal{L}_{I_r}} \left(\frac{\gamma r_1^{\alpha+1}}{P_t K} \right) \cdot e^{-\frac{4}{3}\pi r_1^3 \lambda_0} dr_1, \end{aligned} \quad (12)$$

where step (a) follows from $h_1 \sim \exp(1)$, and step (b) comes after expressing the PDF of r_1 which has been given in equation (8).

$\mathcal{L}_{J_r} \left(\frac{\gamma r_1^{\alpha+1}}{P_t K} \right)$ is the Laplace transform of the interference, and it can be obtained by plugging $s = \frac{\gamma r_1^{\alpha+1}}{P_t K}$ to equation (6).

The expression of success probability is obtained as

$$p_s = \int_0^\infty 4\pi r_1^2 \lambda_0 \cdot e^{-\frac{4}{3}\pi r_1^3 \lambda_0} \cdot e^{-\frac{\gamma \sigma^2 r_1^{\alpha+1}}{P_t K}} \cdot e^{-4\pi \lambda_0 \int_{r_1}^\infty \frac{\gamma r_1^{\alpha+1}}{\gamma r_1^{\alpha+1} + r_i^{\alpha+1}} r_i^2 dr_i} dr_1. \quad (13)$$

When $\alpha = 5$, the success probability can be simplified as

$$p_s = \int_0^\infty 4\pi r^2 \lambda_0 \cdot e^{-\frac{4}{3}\pi r^3 \lambda_0} \cdot e^{-\frac{\gamma \sigma^2 r^6}{P_t K}} \cdot e^{-\frac{4}{3}\pi \lambda_0 \sqrt{r} \left(\frac{\pi}{2} - \arctan \frac{1}{\sqrt{r}} \right)} dr_1. \quad (14)$$

Proof: When $\alpha = 5$, p_s can be expressed as

$$\begin{aligned} p_s &= \int_0^\infty 4\pi r^2 \lambda_0 \cdot e^{-\frac{4}{3}\pi r^3 \lambda_0} \cdot e^{-\frac{\gamma \sigma^2 r^6}{P_t K}} \\ &\quad \cdot e^{-4\pi \lambda_0 \int_{r_1}^\infty \frac{\gamma r_1^6}{\gamma r_1^6 + r_i^6} r_i^2 dr_i} dr_1 \\ &\stackrel{(a)}{=} \int_0^\infty 4\pi r^2 \lambda_0 \cdot e^{-\frac{4}{3}\pi r^3 \lambda_0} \cdot e^{-\frac{\gamma \sigma^2 r^6}{P_t K}} \\ &\quad \cdot e^{-\frac{4}{3}\pi \lambda_0 \int_{r_1}^\infty \frac{\gamma r_1^6}{\gamma r_1^6 + x^2} dx} dr_1 \\ &= \int_0^\infty 4\pi r^2 \lambda_0 \cdot e^{-\frac{4}{3}\pi r^3 \lambda_0} \cdot e^{-\frac{\gamma \sigma^2 r^6}{P_t K}} \\ &\quad \cdot e^{-\frac{4}{3}\pi \lambda_0 \sqrt{r} \left(\frac{\pi}{2} - \arctan \frac{1}{\sqrt{r}} \right)} dr_1, \end{aligned} \quad (15)$$

where step (a) follows by using the change of variables $r_i^3 \rightarrow x$. When $\sigma^2 = 0$, the success probability is only related to the SINR threshold γ , and it is expressed as

$$p_s = \int_0^\infty e^{-\int_y^\infty \frac{\frac{\gamma y^{\frac{\alpha+1}{3}}}{\gamma y^{\frac{\alpha+1}{3}} + x^{\frac{\alpha+1}{3}}}} \cdot e^{-y} dy. \quad (16)$$

Proof: When $\sigma^2 = 0$, p_s can be expressed as

$$\begin{aligned} p_s &= \int_0^\infty 4\pi r_1^2 \lambda_0 \cdot e^{-4\pi \lambda_0 \int_{r_1}^\infty \frac{\gamma r_1^{\alpha+1}}{\gamma r_1^{\alpha+1} + r_i^{\alpha+1}} r_i^2 dr_i} \\ &\quad \cdot e^{-\frac{4}{3}\pi r_1^3 \lambda_0} dr_1 \\ &= \int_0^\infty e^{-\int_{r_1}^\infty \frac{\gamma r_1^{\alpha+1}}{\gamma r_1^{\alpha+1} + r_i^{\alpha+1}} d\left(\frac{4}{3}\pi r_i^3 \lambda_0\right)} \\ &\quad \cdot e^{-\frac{4}{3}\pi r_1^3 \lambda_0} d\left(\frac{4}{3}\pi r_1^3 \lambda_0\right) \\ &\stackrel{(a)}{=} \int_0^\infty e^{-\int_y^\infty \frac{\frac{\gamma y^{\frac{\alpha+1}{3}}}{\gamma y^{\frac{\alpha+1}{3}} + x^{\frac{\alpha+1}{3}}}} \cdot e^{-y} dy, \end{aligned} \quad (17)$$

step (a) is obtained by using the change of variables $\frac{4\pi r_1^3 \lambda_0}{3} \rightarrow x$ and $\frac{4}{3}\pi r_i^3 \lambda_0 \rightarrow y$.

The *no noise* ($\sigma^2 = 0$) result is the upper bound of success probability since the noise-related part in success probability is $e^{-\frac{\gamma \sigma^2 r_1^{\alpha+1}}{P_t K}}$, and $e^{-\frac{\gamma \sigma^2 r_1^{\alpha+1}}{P_t K}} \leq e^{-\frac{\gamma \sigma^2 r_1^{\alpha+1}}{P_t K}}|_{\sigma^2=0} = 1$.

As discussed at the beginning of this section, the number of effective serving BSs whose signals arrive with sufficiently - high SINRs also strongly affects the accuracy of the localization. The expression of success probability, i.e. the probability for a typical serving BS to successfully participate in the localization procedure is derived, and based on the system model, there is n_{sf} serving BSs, therefore, the average number of effective serving BSs which can successfully participate in the localization procedure can be calculated as follows

$$E[N_{bs}] = p_s \cdot n_{sf}. \quad (18)$$

4. SIMULATION RESULTS

Here, both numerical and Monte Carlo simulations are carried out to validate conclusions, and we use the default values in Table I obtained from [3] unless otherwise stated.

Table 1. Parameters of simulation

Parameter	Value
Path loss exponent	$\alpha = 5$
Transmit power	$P_t = 10 \text{ W}$
Constant attenuation per unit of path length	$\mu = 0.5$
Noise power	$\sigma^2 = -70 \text{ dBm}$

Fig. 3 compares the performance of average interference with different subframe numbers n_{sf} . It is shown that the average interference is an increasing function with BS density, which is because as the BS density increases, the number of BSs that may cause interference increases. In addition, when the number of subframe n_{sf} increases, the average interference decreases, the

Figure 3. Average interference of different number of subframes vs. BS density.

Figure 4. Success probability of different number of subframes vs. BS density. $\gamma = -10 \text{ dB}$

reason is that when the BS density is the same, the increase in n_{sf} can reduce the number of BS transmitting at the same time. Particularly, $n_{sf} = 1$ can be regarded as the traditional no-time hopping condition. Therefore, it proves the conclusion that time hopping can effectively reduce the interference in TDOA. The analytical results match well with the corresponding simulation results.

Fig. 4 and Fig. 5 show the success probability performance with different value of n_{sf} . All the analytical results are validated with the simulation results.

Fig. 4 shows the success probability performances versus BS densities. It is shown that the success probability is an increasing function with BS density, which is because the useful signal will be stronger as the BS density increases. As the BS density keeps increasing, there will be a region, where a great increment in the BS density λ leads to a negligible growth of the success probability, which is because when the BS density is large enough, not only the useful signal becomes stronger, the interference will also getting significantly stronger. It's illustrates in this figure that the success probability converges to the fixed value of p_s without

Figure 5. Success probability of different number of subframes vs. SINR threshold. $\lambda = 0.0004 \text{ m}^{-3}$

Figure 6. Average number of effective serving BSs of different number of subframes vs. BS density. $\gamma = -10 \text{ dB}$ Figure 1. A topology of 3D Poisson distributed base

any noise as we have proved. The reason is as BS density continued to increase, the interference will become the main influence to the success probability and the noise can be neglected. In addition, the success probability is not improving with the increase of n_{sf} . The reason is when n_{sf} increases the BSs density in a collection will decrease, in this case, although the interference is reduced, the useful signal is also reduced.

Fig. 5 shows the relationship between success probability and SINR threshold. As shown in this figure, the success probability decreases monotonically with respect to SINR threshold γ . The reason is that when γ increases, it is more difficult to meet the SINR restriction for successfully participating in the localization process. In addition, the no noise case is the upper bound of the success probability as we have proved.

In Fig. 6 and Fig. 7, the average number of effective serving BS with sufficiently-high SINR is shown, and all the analytical results matches well with the Monte Carlo simulation results.

Fig. 6 shows the average number of effective serving BS versus BS density. The average number of effective serving BS is an increasing function with respect to BS density, which is because when the BS density increases, the success probability increases as shown in Fig. 4, and when the number of standby serving BS (the closest BSs in each collection, i.e. the BSs are selected as the serving BSs) is the same, the higher the success probability is, the greater the number of effective serving BS will be. It illustrates in this figure that the increase of n_{sf} can improve the average number of effective serving BS. The reason is as n_{sf} increases, the number of standby serving BS will increase.

In Fig. 7, it is shown the average number of effective serving BS is a decreasing function with respect to SINR threshold, which is because when the SINR threshold increases, less standby serving BS can meet the SINR restriction.

Figure 7. Average number of serving BSs of different number of subframes vs. SINR threshold. $\lambda = 0.0004 \text{ m}^{-3}$

5. CONCLUSION

In this paper, an analytical model of time hopping based TDOA technology is proposed, and the performance of the proposed technology in 3D wireless networks is mathematically formulated. Firstly, the expression of average interference is derived. Based on that, the success probability and the number of effective

serving BS are modeled and analyzed. In addition, some special cases are researched, which reveals interesting insights. Numerical results demonstrate that the time hopping can effectively reduce the interference, and a greater n_{sf} can bring more benefits. All the results in this paper are validated with the Monte Carlo simulation results.

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